A hybrid elastic one-way propagator for strong-contrast media and its application to subsalt migration

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SUMMARY
To overcome the weak scattering limitation of traditional one-way elastic propagator, we propose a hybrid elastic propagator for strong-contrast media, which combines the thin-slab propagator with RT (reflection/transmission) operator at the sharp boundary. In the framework of the hybrid one-way propagator, the elastic model is separated into two or more domains along the strong contrast boundary and the wave propagation in each domain is realized in a depth-marching fashion. Within the thin-slab of each marching step, the perturbations are relatively weak so are treated as volume scatterings and are solved by local Born approximation. The communications between different domains occurs at the boundary elements within the thin slab and are formulated as boundary scatterings, which are calculated by applying the local reflection/transmission operator to the incident wave with the tangent plane approximation. The wavefields at the exit of the thin slab will be updated by the scattered waves from the volume heterogeneities plus the reflected/transmitted waves from the sharp boundary, and then serve as the incident waves at the entrance of the next thin slab. The hybrid propagator shuttles between wavenumber and space domain: wave propagation in wavenumber domain and heterogeneities interaction in space domain, promising high efficiency and accuracy of the propagator. Numerical tests using two simple salt models demonstrate the validity of the hybrid propagator. Finally the new propagator is applied to subsalt migration using synthetic data generated based on the 2D Subsalt model.

INTRODUCTION
Subsalt exploration can be very challenging due to the issues posed by the often complex geometry of the salt bodies and the large impedance contrasts between salt and surrounding sediment deposits. The large velocity contrast across the sedimentary/salt interfaces together with the frequently rugose character of these interfaces prevent P-waves from penetrating the salt body with sufficient strength to image subsalt structures. However, the mode conversion is quite a significant factor in seismic propagation through the salt body so that converted wave may carry considerable energy to illuminate the subsalt region. In other words, the elastic nature of seismic wave is amplified at the surface of the salt body. So in such kind of strong-contrast media, it is better to utilize elastic waves in order to improve the migration image provide by P-wave survey alone.

Previous attempt of elastic wave imaging adopted scalar wave propagators for both P- and S-waves (e.g., Zhe and Greenhalgh, 1997; Sun and McMechan, 2001; Hou and Marfurt, 2002). However, the use of scalar wave propagator for elastic wave extrapolation is dynamically incorrect. Elastic RTM (e.g., Sun and McMechan, 1986; Chang and McMechan, 1987, 1994) extrapolates the wavefields based on the elastic wave equation, but P- and S- modes are mixed together and hard to separate. Elastic thin-slab and elastic complex screen method (Wu and Xie, 1994; Xie and Wu, 1995, 2001, 2005; Wu and Wu, 2005; for a review, see Wu et al, 2007) are developed for one-way migration. It has its special features and advantages in applying to seismic imaging. First of all, similar to elastic RTM, those one-way migrations use vector wavefield extrapolator so P-S and S-P conversions are well handled. Secondly, mode type is kept during migration, which is especially useful for elastic wave imaging in terms of controlling migration cross-talks and parameter inversion. Thirdly, one-way wave method is much more efficient, often is orders of magnitude faster than the full wave method.
The elastic thin-slab or elastic complex screen methods are based on perturbation theory. They can handle elastic perturbations only up to 30% (Wu and Wu, 2005). The algorithms may become unstable beyond this limit. Although these methods can be useful in reservoir modeling and imaging, they are currently excluded from the applications for strong-contrast media, such as salt or basalt inclusions. In order to take its full advantages, it is pressing to extend the one-way elastic method to the case of strong-contrast media.

To handle the strong heterogeneous media with sharp boundary, we propose to solve the boundary problem by applying a local reflection/transmission operator and combine it with thin-slab propagator for weak perturbations in the framework of one-way marching algorithm. In this study, we first briefly summarize the theory of the elastic one-way propagator realized by sequential thin-slab for weak perturbations. Secondly, we present the formulations for the new theory on hybrid elastic one-way propagator in strong contrast medium with sharp boundary and describe its three essential components in detail. Finally, we conduct two numerical tests to verify the accuracy and efficiency of the hybrid elastic one-way propagator and also use the Subsalt model to demonstrate its application to seismic imaging.

**ELASTIC THIN-SLAB PROPAGATOR FOR WEAKLY HETEROGENEOUS MEDIA**

For an arbitrary heterogeneous medium, we slice the medium into numerous thin-slabs transversal to the propagation direction (preferred direction). Shown in Figure 1 is an example of an individual thin-slab for elastic media. Assume each thin-slab is thin enough so that the Born approximation can be used for the volume scattering calculation. In the perturbation theory-based method, the weak heterogeneities in a thin slab are treated as parameter perturbations from the background medium. (See Aki and Richards, 1980; Wu and Aki, 1985)
\[ \rho(x) = \rho_0 + \delta \rho(x) \]
\[ c(x) = c_0 + \delta c(x) \]
\[ u(x) = u_0 + u_v(x) \]  \hspace{1cm} (1)

where \( \rho_0 \) and \( c_0 \) are the density and elastic constants of the background medium, \( \delta \rho \) and \( \delta c \) are the corresponding perturbations, \( u_0 \) is the incident displacement field, \( u_v \) is the scattered field by the volume heterogeneities and \( u \) is the total field. See Figure 1.

In the frequency domain, elastic wave equation can be written as

\[ -\omega^2 \rho(x) u(x) = \nabla \cdot \left[ \frac{1}{2} c : (\nabla u + u \nabla) \right], \hspace{1cm} (2) \]

where \( \nabla \) is the spatial gradient operator. From equation (2) and (3) we derive the wave equation for the scattered field

\[ -\omega^2 \rho_0 u_v - \nabla \cdot \left[ \frac{1}{2} c_0 : (\nabla u_v + u_v \nabla) \right] = Q, \hspace{1cm} (3) \]

where

\[ Q = \omega^2 \delta \rho u + \nabla \cdot [\delta c : \varepsilon], \hspace{1cm} (4) \]

is the equivalent body force due to volume scattering.

In a thin slab, local Born approximation is used to calculate the scattered wave

\[ u_v(x, z_j) = \int_{z_{j-1}}^{z_j} dz \int d^2x_r \left\{ Q_0(x_r, z) \cdot G_0(x, z_j; x_r, z) \right\}, \hspace{1cm} (5) \]

where \( u_v(x, z_j) \) is the scattered field in the horizontal wavenumber domain, \( z_{j-1} \) is the entrance of the thin slab, \( z_j \) is the exit of the thin slab, \( G_0(x, z_j; x_r, z) \) is the background elastic Green's function in the current thin slab, where
\[
Q_0(x_T, z) = \delta \rho \omega^2 u_0(x_T, z) + \nabla \cdot \left[ \delta c : \varepsilon_0(x_T, z) \right],
\]

(6)
is the equivalent body force under Born approximation, where \( u_0 \) and \( \varepsilon_0 \) are the incident displacement and strain fields at the thin-slab, respectively.

Following the derivation of this equation, we express the scattered displacement fields for P and S modes within a thin-slab in the horizontal wavenumber domain (Wu, 1994, 1996; Wu et al., 2007):

\[
u_P^I(K_T, z_j) = \int_{iz_{j-1}}^{iz_j} dz \int d^2x_T Q_0(x_T, z) \cdot G_0^P(K_T, z_j; x_T, z),
\]

(7)

\[
u_S^I(K_T, z_j) = \int_{iz_{j-1}}^{iz_j} dz \int d^2x_T Q_0(x_T, z) \cdot G_0^S(K_T, z_j; x_T, z),
\]

(8)

where \( G_0^P(K_T, z_j; x_T, z) \) and \( G_0^S(K_T, z_j; x_T, z) \) are the background Green's function for P and S waves, respectively (see Appendix A).

For isotropic media,

\[
Q_0(x_T, z) = \delta \rho (x_T, z) \omega^2 u_0(x_T, z) + \nabla \cdot \left[ \delta \lambda (x_T, z) \varepsilon_0(x_T, z) \right] I + 2\delta \mu (x_T, z) \varepsilon_0(x_T, z),
\]

(9)

where \( \lambda \) and \( \mu \) are the Lame constants, \( I \) is the unit tensor. In this equation, the space-domain incident field, its divergence and strain field (composed of its gradients) at level \( z \) are calculated in the spectral domain as

\[
u_0^I(x_T, z) = \int u_0^P(K_T', z_{j-1}) e^{i\beta'(z-z_{j-1})} d^2K_T'
\]

\[
i \int u_0^S(K_T', z_{j-1}) e^{i\beta'(z-z_{j-1})} d^2K_T'
\]

\[
\nabla \cdot \left[ \varepsilon_0(x_T, z) \right] I = -\left[ k_{\alpha}^I u_0^P(K_T', z_{j-1}) \right] k_{\alpha}^I e^{i\beta'(z-z_{j-1})} d^2K_T',
\]

(10)

\[
\nabla \cdot \varepsilon_0(x_T, z) = -\left[ k_{\beta}^I u_0^P(K_T', z_{j-1}) + u_0^P(K_T', z_{j-1}) k_{\beta}^I e^{i\beta'(z-z_{j-1})} d^2K_T'
\]

\[
-\int [ k_{\beta}^I u_0^S(K_T', z_{j-1}) + u_0^S(K_T', z_{j-1}) k_{\beta}^I e^{i\beta'(z-z_{j-1})} d^2K_T'.
\]
with \( k_\alpha = (K_T, \gamma_\alpha) \) and \( k_\beta = (K_T, \gamma_\beta) \) representing the P and S wavenumber vectors respectively, with \( K_T \) as the horizontal wavenumber while \( \gamma_\alpha \) and \( \gamma_\beta \) are the vertical wavenumbers for P and S waves, respectively.

The incident waves are propagated with the background reference velocities in the current slab

\[
\mathbf{u}_0^P(K_T, z_j) = \mathbf{u}_0^P(K_T, z_{j-1}) e^{i\gamma_\alpha(z_j - z_{j-1})},
\]

\(11\)

\[
\mathbf{u}_0^S(K_T, z_j) = \mathbf{u}_0^S(K_T, z_{j-1}) e^{i\gamma_\beta(z_j - z_{j-1})}.
\]

\(12\)

At the exit (bottom) of each thin-slab, the scattered field is added to the incident field to obtain the total wave-field which in turn is treated as the updated incident wave-field for the next thin-slab.

\[
\mathbf{u}^P(x_T, z_j) = \frac{1}{4\pi^2} \int d^2 K_T e^{iK_T x_j} \left[ \mathbf{u}_0^P(K_T, z_j) + \mathbf{u}_0^P(K_T, z_{j-1}) \right],
\]

\(13\)

\[
\mathbf{u}^S(x_T, z_j) = \frac{1}{4\pi^2} \int d^2 K_T e^{iK_T x_j} \left[ \mathbf{u}_0^S(K_T, z_j) + \mathbf{u}_0^S(K_T, z_{j-1}) \right],
\]

\(14\)

The incident field updating is realized by a marching algorithm along the forward propagation direction (z-direction) step-by-step (see Figure 1). The marching algorithm is implemented by an operator split method in the dual domain: background propagation in the wavenumber domain and interaction with perturbations in the space domain.

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**HYBRID ELASTIC ONE-WAY PROPAGATOR IN STRONGLY HETEROGENEOUS MEDIA WITH SHARP BOUNDARY**

For weak heterogeneities, the perturbation method is a valid and convenient tool for elastic wave scattering and propagation. For strongly heterogeneous media with sharp boundaries, such as
salt or basalt inclusions, the perturbation approach fails in most cases. To handle this specific case, we divide the model into different domains along the sharp boundary (Figure 2). In each domain, the wave field can be computed with the representation integral:

\[
\mathbf{u}(\mathbf{x}) = \int \mathbf{Q}(\mathbf{x}') \cdot \mathbf{G}(\mathbf{x}; \mathbf{x}') d\Omega(\mathbf{x}') + \int \left[ (\hat{n} \cdot \mathbf{\sigma}(\mathbf{x}')) \cdot \mathbf{G}(\mathbf{x}; \mathbf{x}') - \mathbf{u}(\mathbf{x}') \cdot [\hat{n} \cdot \mathbf{\Sigma}(\mathbf{x}; \mathbf{x}')] \right] dS(\mathbf{x}'),
\]

where \(\mathbf{u}(\mathbf{x})\) is the displacement field at point \(\mathbf{x}\) within the volume \(\Omega\) enclosed by surface \(S\), \(\mathbf{G}(\mathbf{x}; \mathbf{x}')\) is Green’s displacement tensor (dyadic) and \(\mathbf{\Sigma}(\mathbf{x}; \mathbf{x}')\) is Green’s stress tensor (triadic). \(\hat{n}\) is the surface normal as towards to the exterior of \(\Omega\). \(\mathbf{u}(\mathbf{x}')\) and \(\mathbf{\sigma}(\mathbf{x}')\) are the displacement and stress on the surface; \(\mathbf{Q}(\mathbf{x}')\) is body forces or equivalent body forces due to scattering. The volume integral term yields the contribution due to the sources inside \(\Omega\), while the surface integral term, that is, Kirchhoff integral, accounts for the energy communication between different domains. The traditional way to calculate the scattered wave in each domain is to solve the integral equation along the boundary. However, it involves huge computations because the interactions between all the boundary elements are considered. Following the spirit of the thin-slab propagator, we solve the problem iteratively in a one-way fashion. The resultant operator is called the hybrid elastic one-way propagator.

Figure 2 schematically shows the realization of a typical hybrid propagator in a strong heterogeneous medium with a sharp boundary. As the velocity model is sliced into a thin slab, the sharp boundary is discretized into many boundary elements. The thin slab is separated into two domains: a high velocity one and a low velocity one with two boundary elements. Within each domain, the parameter variations (perturbations) are relatively weak. They are treated as volume scatterings and handled by thin-slab propagator. Besides the volume scatterings, each
domain will admit internal reflections and transmissions from adjacent domains at the boundary elements. They together are called boundary scatterings. We assume that the boundary elements within one thin slab are decoupled from each other so that the reverberations between them are neglected. The displacement and traction fields of the boundary scatterings can be approximately calculated by applying a reflection/transmission operator to the wave incident on the boundary element. It corresponds to a tangent plane approximation for smoothly curved boundary (Voronorich, 1989). When an incident wave enters a thin slab, it will interact with the volume heterogeneities as well as the boundary element in the current slab. The representation integral will give the scattered wave which will be added to the incident wave at the exit of the thin slab. Based on the one-way propagation principle, the wavefields are updated iteratively step-by-step in the forward direction with no consideration of the backward scatterings.

In this subsection, we will describe three critical components of migration algorithms: thin-slab propagator for weak volume scatterings, background propagation, and reflection/transmission operator for sharp boundary scatterings.

1. Thin-slab propagator for weak volume scatterings

When several domains exist in a velocity model, the wave propagation in each domain will be handled separately with traditional thin-slab propagator. Different domains have different background velocities, so that the weak perturbations assumptions of thin-slab propagator can be guaranteed in each domain. The interaction with the weak heterogeneities will be calculated with the Green’s tensor $G_0^{P(i)}$ and $G_0^{S(i)}$ for the domain $i$ using the corresponding background velocities:
where $\Omega_i$ is domain $i$, $Q_0(x_r,z)$ is the equivalent body force which is defined in equation (9).

2. **Background propagation**

The free propagation (background propagation) in the hybrid propagator is slightly different from the traditional propagator as below:

\[
\mathbf{u}^{\nu(i)}_0(\mathbf{K}_r,z) = e^{i\rho(z-z_{j-1})} \int dz' d^2x_r e^{-i\mathbf{k}_r x_r} \mathbf{u}^{\nu(i)}_0(x_r,z), \quad (x_r,z) \in \Omega_i, \tag{18}
\]

\[
\mathbf{u}^{\sigma(i)}_0(\mathbf{K}_r,z) = e^{i\rho(z-z_{j-1})} \int dz' d^2x_r e^{-i\mathbf{k}_r x_r} \mathbf{u}^{\sigma(i)}_0(x_r,z), \quad (x_r,z) \in \Omega_i, \tag{19}
\]

where $\Omega_i$ is domain $i$.

3. **Transmission/reflection operators on the boundary elements**

Boundary scatterings are formulated in different way from volume scatterings in the thin slab. To compute the boundary scattering, we make a tangent plane approximation (Voronovich, 1999) which assumes the boundary surface is smoothly curved so that the reflection/transmission coefficients defined for an infinite plane surface can be applied locally at each surface element. In this sense the theory is a high-frequency asymptotic solution. It is nearly accurate for smoothly varying interface but has limited use for rough surface.

We focus our attention to one boundary element which separates the medium into upper and lower ones. Regardless of their real domain numbers, we set the index for the upper medium as 1 and the lower medium as 2. The outer normal of the upper and lower medium are
\begin{align}
\hat{n}_1 &= -\hat{n} \\
\hat{n}_2 &= \hat{n}
\end{align}

(20)

where \( \hat{n} \) is the normal vector of the boundary element (see Figure 3).

The incident waves of the upper and lower medium are given from the output of previous thin slab as \( U_{0,1} \) and \( U_{0,2} \) in wavenumber domain. For simplicity, the wave type is ignored if the same operation is applied to both P- and S-waves. These incident waves include the scatterings of the previous boundary elements and the contributions from underneath are ignored. To accurately calculate the energy partition at the boundary element, we transform both the wavenumber vector and the displacement of the incident waves at the upper and lower mediums from the Cartesian coordinate to the local boundary coordinate

\[
\begin{align}
\tilde{U}_{0,I} &= M^f U_{0,I} \, , \\
\tilde{k} &= M^f k ,
\end{align}
\]

(21) (22)

where

\[
M^f = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} ,
\]

(23)

is the transform matrix from the Cartesian coordinate to local boundary coordinate in 2D case. \( \theta \) is the transform angle from the original coordinate to the local boundary coordinate. See Figure 3. Note that \( \theta \) varies with location. \( U_{0,I} \) and \( \tilde{U}_{0,I} \) are the incident waves in the Cartesian coordinate and in the local boundary coordinate, respectively. \( k \) and \( \tilde{k} \) are the wavenumber vectors in the Cartesian coordinate and in the local boundary coordinate, respectively.
In the local boundary coordinate, reflection/transmission coefficients are applied to the incident wave:

\[
\begin{bmatrix}
\tilde{U}_1^P \\
\tilde{U}_1^S \\
\tilde{U}_2^P \\
\tilde{U}_2^S
\end{bmatrix}
= \begin{bmatrix}
R_{11}^{PP} & R_{11}^{SP} & T_{11}^{PP} & T_{11}^{SP} \\
R_{12}^{PS} & R_{12}^{SS} & T_{21}^{PS} & T_{21}^{SS} \\
T_{12}^{PP} & T_{12}^{SP} & R_{22}^{PP} & R_{22}^{SP} \\
T_{12}^{PS} & T_{12}^{SS} & R_{22}^{PS} & R_{22}^{SS}
\end{bmatrix}
\begin{bmatrix}
\tilde{U}_0^P \\
\tilde{U}_0^S \\
\tilde{U}_0^P \\
\tilde{U}_0^S
\end{bmatrix},
\]

where \(R\) and \(T\) are reflection and transmission coefficients calculated by Zoeppritz’s equation (Aki and Richards, 1980). The coefficients are dependent on the horizontal slowness in the local boundary coordinate. The first and second subscripts of the coefficients are the medium indexes. The superscripts of the coefficients specify the wave types in the upper and lower medium, respectively. \(\tilde{U}_i, i = 1, 2\) is the displacement of the boundary scattering which is the sum of the internal reflected wave and the transmitted wave from the other domain. For the sake of simplicity, the wave type is ignored in the next few equations.

The traction of the boundary scattering can be calculated from the displacement by the constitutive equation in wavenumber domain. So in isotropic media, the traction corresponding to the displacement \(\tilde{U}_i\) is

\[
\tilde{T}_i(\tilde{n}_j) = i \left[ \lambda_i \tilde{n}_j (\tilde{k} \cdot \tilde{U}_j) + \mu_i \tilde{n}_j \cdot (\tilde{k} \tilde{U}_j + \tilde{U}_j \tilde{k}) \right], \quad i = 1, 2
\]

where \(\lambda_i\) and \(\mu_i\) are the Lame constants; \(\tilde{n}_j\) is the unit normal vector of the boundary in the local boundary coordinate, and it is equivalent to the unit vector of positive or negative \(z'\)-direction in the local coordinate

\[
\tilde{n}_1 = z', \\
\tilde{n}_2 = -z'.
\]
Wavenumber-domain displacement $\vec{U}_I$ and traction $\vec{T}_I$ are still global. They become localized in space when we transform them into space domain and pick the values right at the boundary location. In this way, we get displacement and traction at the boundary element, but defined in the local boundary coordinate. In the space domain, we transform them back to the Cartesian coordinate:

$$U_I = M^b\vec{U}_I, \quad I = 1,2, \quad (27)$$

$$T_I = M^b\vec{T}_I, \quad I = 1,2, \quad (28)$$

where

$$M^b = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, \quad (29)$$

is the transform matrix from the local boundary coordinate to the Cartesian coordinate. $U_I$ and $\vec{U}_I$ are the space-domain displacements of boundary scattering in the Cartesian and local boundary coordinate, respectively. $T_I$ and $\vec{T}_I$ are the space-domain tractions of the boundary scattering in the Cartesian and local boundary coordinate, respectively.

For a given boundary element in a thin slab, we have determined the total displacement and traction on the element by the reflection/transmission operator. The scattered waves due to the boundary element at the slab exit can be calculated by Kirchhoff integral in the horizontal wavenumber domain (Wu, 1989)

$$U^p_{b,I}(K_T, z_j) = \int_1^L [T^p_I(x_T, z, \vec{n}_j) \cdot G^p_I(K_T, z_j; x_T, z) - U^p_I(x_T, z) \cdot \Gamma^p_I(K_T, z_j; x_T, z, \vec{n}_j)]dS, \quad I = 1,2, (30)$$

$$U^s_{b,I}(K_T, z_j) = \int_1^L [T^s_I(x_T, z, \vec{n}_j) \cdot G^s_I(K_T, z_j; x_T, z) - U^s_I(x_T, z) \cdot \Gamma^s_I(K_T, z_j; x_T, z, \vec{n}_j)]dS, \quad I = 1,2, (31)$$
where \( L \) is the boundary segment which separates the upper and lower mediums. \( \mathbf{G} \) and \( \mathbf{\Gamma} \) are the background Green’s tensor of displacement and traction (see Appendix A). The scattered wave will be added to the domains on both sides of the boundary element as \( u_{b}^{(i)}(K_T, z_j) \), where \( i \) is the domain number.

At the exit of each thin-slab, the total field is composed of three parts: the incident (free-propagated) wave \( u_0^{(i)} \), the scattered field by volume heterogeneities \( u_p^{(i)} \) and the boundary scattered field \( u_b^{(i)} \).

\[
\begin{align*}
u^{(i)}_p(x_T, z_j) &= \frac{1}{4\pi^2} \int d^2K_T e^{iK_Tx_0} \left[ u_0^{(i)}(K_T, z_j) + u_p^{(i)}(K_T, z_j) + u_b^{(i)}(K_T, z_j) \right], \quad (x_T, z_j) \in \Omega_j, \quad (32) \\
u^{(i)}_s(x_T, z_j) &= \frac{1}{4\pi^2} \int d^2K_T e^{iK_Tx_0} \left[ u_0^{(i)}(K_T, z_j) + u_p^{(i)}(K_T, z_j) + u_b^{(i)}(K_T, z_j) \right], \quad (x_T, z_j) \in \Omega_j, \quad (33)
\end{align*}
\]

where \( \Omega_j \) is domain \( j \).

**NUMERICAL TESTS FOR THE HYBRID ELASTIC ONE-WAY PROPAGATOR**

First we test the elastic one-way propagator using a salt with a wedge shape embedded in a homogeneous media. Figure 4 illustrates the velocity model with elastic parameters. The source is a 15 Hz Ricker wavelet and located in the center of the model. In this simple case, the tangent plane approximation is expected to work the best. Illustrated in the upper panel of Figure 5 are the horizontal and vertical displacements calculated by the elastic one-way propagator. Compared with the snapshots calculated from full-wave finite difference (FD) (the lower panel of Figure 5), the transmitted wavefront of the elastic one-way propagator matches with them.
very well. The energy partition at the interface is almost the same as full-wave FD. In addition, to demonstrate the flexibility of the propagator, we show different wave paths (including PPP, PPS, PSP and PSS) by switching on/off wave mode at the interface in Figure 6. In terms of efficiency, the full-wave FD takes 1 hour with grid interval of 5 m while the hybrid one-way propagator takes 5 minutes with grid interval of 10 m.

We continue to compute the wave propagation for an elliptical salt model shown in Figure 7. The model is comprised of a sedimentary background and a salt inclusion with an elliptical shape. The elastic parameters are also provided in Figure 7. An explosive source is initiated on the center of the model surface. The source is a 15 Hz Ricker wavelet. Shown in Figure 8 are the comparison of the snapshots generated by the hybrid one-way propagator and full-wave FD. Notice that the major phases are very clear, even though some diffraction noises appear at very large angles. Different wave paths are depicted in Figure 9 by switching on/off wave mode at the boundary of the salt.

Finally, we move on to a more complex model – a 2D velocity profile simulating the Subsalt model (Figure 10). The outline of the sharp boundary is shown in Figure 11a and its dip angle is shown in Figure 11b. On top of the model is a water layer. FD cannot solve the fluid-solid problem very well. On the contrary, the hybrid propagator can do a good job to handle the interface. The model is divided into three different domains (Figure 11c) by the sharp boundary. Each domain has its own background and perturbation parameters. We design an observation system and conduct a seismic experiment. The acquisition system was comprised of 301 shots from 7000 m to 37000 m with an interval of 100 m. Each shot was recorded by a line of
receivers with double-spread configuration. The number of receivers was 561 and the maximal offset is 7000 m. Both sources and receivers are located on the water surface. The recorded data are pressure-only simulating a hydrophone response. The source is a 15 Hz Ricker wavelet and the total recording time is 12 s with a time interval of 0.01 s. The synthetic data were modeled using the Tesseral 2D application package with a FD approach. Shown in Figure 12 are sample records for shot numbers 100, 150, 200 and 250. In each shot, the area for computation is 25.6 km $\times$ 13.5 km. We plot the horizontal and vertical components of 71st-shot snapshots in Figure 13. The converted waves penetrating through the salt base still carry considerable energy, which offers great potential to improve the subsalt imaging. We migrate the seismic data with the hybrid one-way propagator and obtain the PP, PS, SP and SS images. Here we only show the PP image in Figure 14 because it has the highest signal-to-noise ratio. From the migration image, a large portion of subsalt reflectors can be clearly identified, except the section with very large dips. There are some migration artifacts near the true images due to multiples and cross-talk. Preprocessing the seismic data will help to improve the quality of the migration image.

**CONCLUSIONS AND DISCUSSIONS**

We developed the theory and method of a hybrid elastic one-way operator which combines the elastic thin-slab propagator in weakly heterogeneous media and reflection/transmission operators at the sharp boundaries. Two approximations are made. One is the local Born approximation for weak heterogeneities. The other is the tangent plane approximation at the sharp boundaries. Overall, it works well for strong contrast media with a smoothly curved boundary, such as salt intrusions, but may not be suitable for random heterogeneities with strong perturbations. Both the accuracy and efficiency of the dual-domain, depth-marching propagator are validated by two simple numerical examples. In terms of the computational efficiency, the hybrid propagator is at
least one to two orders faster than full-wave FD in 2D case. When applied to 3D velocity model, it may save more computational time. The application of the propagator to seismic imaging is demonstrated on the Subsalt model and the three subsalt reflectors are shown clearly in the resulting migration image. The propagator has the flexibility to switch on/off wave modes at the sharp boundary, so that it is capable to produce multiple subsalt images by selecting different converted-paths during migration. It may offer great potential to identify the true reflection from the artifacts and further enhance the true image, especially the ones with steep dip. But this will be the target of future work.

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Appendix A. Weyl integrals of the elastic Green’s tensors

The background Green’s displacement tensor in the horizontal wavenumber domain is

$$G(x_T, z; x'_T, z') = G^P(x_T, z; x'_T, z') + G^S(x_T, z; x'_T, z')$$

$$= \frac{1}{4\pi^2} \int d^2K_T e^{iK_T x_T} \left[ G^P(K_T, z; x'_T, z') + G^S(K_T, z; x'_T, z') \right]$$  \hspace{1cm} (A1)

where

$$G^P(K_T, z; x'_T, z') = \frac{ik_a^2}{2\rho_0\omega^2} \hat{k}_a \cdot \hat{k}_a \frac{1}{\gamma'_{\alpha}} \frac{1}{\gamma_{\alpha}} e^{-iK_{Ta}(z-z')}$$  \hspace{1cm} (A2)
\[ G^s \left( \mathbf{K}_r, z; \mathbf{x}_r, z' \right) = \frac{ik_{\beta}^2}{2\rho_0\omega^2} \left( \mathbf{I} - \hat{k}_{\beta} \hat{k}_{\beta} \right) \frac{1}{\gamma_{\beta}} e^{-i\mathbf{k}_\beta \cdot \mathbf{x}_r + i\gamma_{\beta}(z-z')} , \]  

(A3)

are wavenumber-domain Green’s function for P and S waves, in which \( \mathbf{I} \) is the identity matrix, and

\[ \mathbf{k}_\alpha = (\mathbf{K}_r, \gamma_{\alpha}) , \]  

(A4)

\[ \mathbf{k}_\beta = (\mathbf{K}_r, \gamma_{\beta}) , \]  

(A5)

are the P and S wavenumber vectors respectively, with \( \mathbf{K}_r \) as the horizontal wavenumber and \( \gamma_{\alpha} \) and \( \gamma_{\beta} \) as vertical wavenumbers for P and S waves, respectively, defined by

\[ \gamma_{\alpha} = \sqrt{k_{\alpha}^2 - \mathbf{K}_r^2} , \]  

(A6)

\[ \gamma_{\beta} = \sqrt{k_{\beta}^2 - \mathbf{K}_r^2} , \]  

(A7)

where \( k_{\alpha} = \omega/\alpha_0 \) and \( k_{\beta} = \omega/\beta_0 \) with \( \alpha_0 \) and \( \beta_0 \) as the P and S wave background velocities in the thin-slab, respectively. The unit direction vectors of P and S plane wave propagation are

\[ \hat{k}_{\alpha} = \frac{\mathbf{k}_{\alpha}}{k_{\alpha}} , \]  

(A8)

\[ \hat{k}_{\beta} = \frac{\mathbf{k}_{\beta}}{k_{\beta}} , \]  

(A9)

where the source location is \( (\mathbf{x}_r', z') \) and the observation location is \( (\mathbf{x}_r, z) \).

Correspondingly, the Weyl integral of the Green’s traction tensor can be obtained

\[ \Gamma \left( \mathbf{x}_r, z; \mathbf{x}_r', z', \hat{n} \right) = \Gamma^p \left( \mathbf{x}_r, z; \mathbf{x}_r', z', \hat{n} \right) + \Gamma^s \left( \mathbf{x}_r, z; \mathbf{x}_r', z', \hat{n} \right) \]

\[ = \frac{1}{4\pi^3} \int d^2 \mathbf{K}_r e^{i\mathbf{K}_r \cdot \mathbf{x}_r} \left[ \Gamma^p \left( \mathbf{K}_r, z; \mathbf{x}_r', z', \hat{n} \right) + \Gamma^s \left( \mathbf{K}_r, z; \mathbf{x}_r', z', \hat{n} \right) \right] \]  

(A10)

\[ \Gamma^p \left( \mathbf{K}_r, z; \mathbf{x}_r', z', \hat{n} \right) = -\frac{k_{\alpha}^3}{2\rho_0\omega^2} \left[ \lambda \hat{k}_{\alpha} \hat{n} + 2\mu (\hat{n} \cdot \hat{k}_{\alpha}) \hat{k}_{\alpha} \hat{k}_{\alpha} \right] \frac{1}{\gamma_{\alpha}} e^{-i\mathbf{k}_\alpha \cdot \mathbf{x}_r + i\gamma_{\alpha}(z-z')} , \]  

(A11)
\[
\Gamma^S \left( \mathbf{K}, z; x', z'; \hat{n} \right) = \frac{-k_0^2}{2\rho_0 c_0^2} \mu \left[ \left( \hat{n} \cdot \hat{k}_\beta \right) I + \hat{k}_\beta \hat{n} - 2 \left( \hat{n} \cdot \hat{k}_\beta \right) \hat{k}_\beta \hat{k}_\beta \right] e^{i\mathbf{K} \cdot \mathbf{r} + i\gamma \left( z - z' \right)} , \quad (A12)
\]
where \( \hat{n} \) is the outer normal of the surface.

References


FIGURES

Figure 1. Schematic illustration of the thin-slab propagator.
Figure 2. Schematic illustration of the hybrid elastic one-way propagator.

Figure 3: The coordinate transform from Cartesian coordinate to local boundary coordinate. $X - Z$ is the Cartesian coordinate and $X' - Z'$ is the local boundary coordinate. $\theta$ is the rotation angle from Cartesian coordinate to the local boundary coordinate. The red curve is the boundary and $\hat{n}$ is the normal vector to the boundary.

Figure 4. An elastic salt model. The elastic parameters and shot location are indicated in the figure.
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Figure 7. An elliptical salt model. (a) is the P-wave velocity. (b) is the S-wave velocity. (c) is the density.
Figure 8. The snapshots generated in the elliptical model by hybrid elastic propagator (upper panel) and finite difference (lower panel). The horizontal components are shown on the left and the vertical components are shown on the right.
Figure 9. The snapshots related to different wave paths generated by hybrid elastic one-way propagator. (a) and (b) are horizontal and vertical component of wave path PPP. (c) and (d) are horizontal and vertical component of wave path PPS. (e) and (f) are horizontal and vertical component of wave path PSP. (g) and (h) are horizontal and vertical component of wave path PSS.
Figure 10. The simplified 2D Subsalt model: (a) P-wave velocity; (b) S-wave velocity and (c) density.
Figure 11. Model parameters of simplified 2D Subsalt related to hybrid one-way propagator: (a) the outline of sharp boundary; (b) three domains separated by the sharp boundary and (c) the dip angle of the boundary.
Figure 12. Synthetic records for subsalt model: Shown here are samples for shot numbers 100, 150, 200 and 250.
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Figure 14. The PP image of 2D Subsalt model generated by elastic one-way propagator.