Full-wave seismic illumination and resolution analyses: A Poynting-vector-based method

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ABSTRACT

Illumination and resolution analysis provides vital information regarding the response of an imaging system to subsurface structures. However, generating the resolution function is often computationally intensive, which prevents it from being widely used in practice. This problem is particularly severe for the time-domain migration method, such as reverse time migration (RTM). To solve this problem, we have developed a fast full-wave-based illumination and resolution analysis method. The source- and receiver-side waves are extrapolated to the subsurface for simulating the imaging process. To create the relations linking the incident and scattering directions to the target wavenumber components, we have adopted an efficient Poynting-vector-based method for wavefield angle decomposition. By taking the approximation that the time-domain wavefield preserves the source spectrum during propagation, massive input/output and trace-by-trace Fourier transform can be avoided and the finite-frequency calculated broadband signal can be directly converted to the wavenumber domain for calculating the point spreading functions (PSFs). Combining these approaches, the resulting method avoids intensive calculations, massive input/output, and huge storage requirements commonly involved in generating the illumination and resolution functions. The method is highly efficient and particularly suitable to team with RTM for resolution analysis. Numerical examples are used to validate this method. We have determined how to calculate the acquisition dip response and PSFs, based on which, the quality of the depth image can be significantly improved.

INTRODUCTION

Illumination analysis has been widely used in optimizing the acquisition system design, evaluating the image quality, and correcting for the distorted depth image caused by uneven illumination. To calculate the illumination measurement, the source and receiver waves should be extrapolated to the subsurface target area. The illumination and resolution calculations are closely related to the propagation angles of incidence and scattering waves and the target structural dip angle. Therefore, the earlier illumination and resolution analyses were built on ray-based methods because they naturally provided directional information (Hubral et al., 1999; Schneider and Winbow, 1999; Bear et al., 2000; Muerdter and Ratcliff, 2001; Gelius et al., 2002). However, the ray-based methods are limited by the high-frequency asymptotic approximation (Hoffmann, 2001); thus, the wave-equation-based methods were adopted for the illumination calculation.

Because the wave-equation-based methods do not explicitly provide the propagation direction, several methods were developed to extract angle information from the wavefield. These methods can be separated under two categories. The first group uses the integral method, e.g., the local-plane wave decomposition and wavelet transform-based methods (Xie and Lay, 1994; Wu and Chen, 2002, 2006; Xie and Wu, 2002; Mao et al., 2010), local Fourier transform-based methods (Xu et al., 2010, 2011; Zhang et al., 2010), and methods that convert the local offset-domain image into local angle-domain image (de Bruin et al., 1990; Prucha et al., 1999; Sava and Fomel, 2003). The second group uses the differential method, e.g., the Poynting-vector- or gradient-vector-based method (Yoon and Marfurt, 2006; Yoon et al., 2011; Zhang and McMechen, 2011). The differential type methods require only the calculation of spatial or temporal derivatives at given locations. Instead, the integral type methods analyze the wavefield within a neighboring region with the dimension of a wavelength. Therefore, the latter
involves more calculations and is much more time consuming. However, compared with the differential method, the integral method is physically sound, more reliable, and can handle cases such as multiply simultaneously arriving waves. Depending on the purposes, trade-offs between the accuracy and efficiency are often required. Yan and Xie (2012) and Jin et al. (2014) review various angle decomposition methods. These methods have been combined with the one-way propagator and successfully used in illumination and resolution analyses (Wu and Chen, 2002, 2006; Jin and Wallraff, 2003; Xie et al., 2003, 2006; Jin et al., 2006; Mao et al., 2010; Mao and Wu, 2011). Recently, along with the increasing applications of the reverse time migration (RTM), there have been demands for the full-wave-based illumination analysis technique. Xie and Yang (2008) propose the illumination and resolution analysis method based on the time-domain local slowness analysis. Cao and Yang (2008) propose the illumination and resolution analysis method based on the frequency-domain angle decomposition. An alternative method, which first calculates the point spreading function (PSF) by directly imaging a point scatter, followed by converting the PSF into the wavenumber domain for illumination analysis, was also proposed (Fletcher et al., 2012; Cao, 2013; Chen and Xie, 2015; Valenciano et al., 2015). Illumination and resolution analysis provides useful information for image correction. Rickett (2003) applies wave-equation-based illumination analysis to normalize the depth image. Based on the ray-tracing method, Gelius et al. (2002), Sjoeborg et al. (2003), and Lecomte (2008) derive the resolution function and use it to correct the image in the wavenumber domain. Based on the one-way propagator, Xie et al. (2005) propose to correct the image using the PSF. Wu et al. (2004, 2006), Cao and Wu (2009b), and Mao and Wu (2010) use the wavelet transform for illumination and resolution analysis. Fletcher et al. (2012) use the resolution for post image inversion. Yang et al. (2013) use an illumination compensation strategy for wavefield tomography in the image domain. Cao (2013) and Yan et al. (2014) test image correction for the full-wave RTM image.

In this paper, we propose an efficient illumination and resolution analysis method under the full-wave frame, in which the angle-domain information is extracted using the Poynting vector. We first briefly review the relationship between the seismic image and the illumination and resolution functions. Then, we present the method to calculate the local illumination matrix (LIM) and convert it to the PSF and the acquisition dip response (ADR). Numerical examples are used to demonstrate the proposed method in illumination and resolution analysis.

**METHODOLOGY**

Using a survey system composed of a source at \( x_s \) and a receiver at \( x_g \) to investigate a small target region \( V(x) \) in the vicinity of location \( x \), the seismic data recorded at \( x_g \) can be expressed as (e.g., Wu et al., 2007)

\[
D(x, x_g; x, \omega) = s(\omega) \int_{V(x)} k_0^2 G(x', x_s; \omega) M(x') G(x'; x_g, \omega) d x',
\]

where \( \omega \) is the frequency, \( x' \) is the local coordinate defined inside the \( V(x) \), \( k_0 = \omega c_0 / c_0(x) \) is the local wavenumber, \( c_0(x) \) is the background velocity, \( M(x') = \delta c(x') / c(x) \) is the relative velocity perturbation, \( \delta c(x') \) is the velocity perturbation, \( s(\omega) \) is the source spectrum, \( G(x', x_s; \omega) \) is the Green’s function propagating the wave from \( x_s \) to the target at \( x' \), and \( G(x'; x_g, \omega) \) is the Green’s function propagating the wave from the target to the receiver at \( x_g \). Both these Green’s functions are calculated in the background model, and the reciprocity \( G(x'; x_g, \omega) = G(x_s; x', \omega) \) is used. For an acquisition system composed of multiple sources and receivers, the depth image at \( x'' \) can be expressed as

\[
I(x, x''; \omega) = s(\omega) \sum_{x_s} \sum_{x_g} G(x''; x_s, \omega)
\times D(x, x_g; x, \omega) G(x', x_s, \omega) G(x', x_g, \omega) G(x', x_g, \omega).
\]

Substituting equation 1 into equation 2, the depth image can be expressed as (e.g., Xie et al., 2005; Yan et al., 2014)

\[
I(x, x''; \omega) = \int_{V(x)} M(x') R(x, x', x'', \omega) d x',
\]

where

\[
R(x, x', x'', \omega) = 2k_0^2 s(\omega) s^*(\omega)
\times \sum_{x_s} \sum_{x_g} G(x''; x_s, \omega) G^*(x'; x_s, \omega) G^*(x', x_g, \omega) G(x', x_g, \omega).
\]

The resolution function \( R(x, x', x'') \), through forward and backward propagations, maps the velocity perturbation at \( x' \) to the image at \( x'' \). Because of the limitations of the system, e.g., the incomplete acquisition aperture or the error in migration velocity model, it may not exactly map the perturbation to its original location, causing the image smearing. Thus, \( R(x, x', x'') \) is also known as the PSF that is the response of the imaging system to a point scatter (Chen and Schuster, 1999; Gelius et al., 2002; Gibson and Tzimas, 2002; Xie et al., 2005). Equation 3 is similar to a convolution, i.e., the PSF convolving with the velocity perturbation to give the image. However, \( R(x, x', x'') \) is localized; i.e., it is defined near location \( x \) and varies in the space due to the variable illumination. Introducing the local Fourier transform over \( V(x) \), the space convolution in equation 3 can be converted into the multiplication in the wavenumber domain

\[
I(x, k_d, \omega) = R(x, k_d, \omega) \cdot M(x, k_d),
\]

where

\[
M(x, k_d) = \int_{V(x)} M(x') e^{j k_d x'} d x',
\]

\[
R(x, k_d, \omega) = 2k_0^2 s(\omega) s^*(\omega) \sum_{x_s} \sum_{x_g} A(x, k_d; x_s, x_g, \omega),
\]

and
where \( k_d = k_i + k_s \) is the wavenumber related to structural dip, \( k_i \) and \( k_s \) are the incident and scattered wavenumbers near the image point, and \( R(x, k_d, \omega) \) is the wavenumber-domain PSF. The mapping in equation 8 converts the response from \((k_i, k_s)\), the wavenumber domain in acquisition coordinate, to \( k_d \), the wavenumber domain in target coordinate (refer to Figure 1).

Given the acquisition system and the overburden structure, methods have been proposed to calculate \( R(x, k_d, \omega) \) or \( R(x, x') \), followed by using it in seismic illumination and resolution analyses (Xie et al., 2006; Lecomte, 2008). It is relatively straightforward to calculate the \( R(x, k_d) \) using the frequency-domain method. For a time-domain method such as the finite difference, theoretically, one can use Fourier transform to convert the entire time-space wavefield into frequency domain. Then, the wavefield can be decomposed into its wavenumber component by using the local slant stacking or local fast Fourier transform (FFT), followed by calculating the PSF using equations 7 and 8 (Yan et al., 2014). However, such a procedure often involves intensive calculations, huge input/output, and storage and is computationally inefficient. Instead, we propose using the Poynting vector to decompose the wavefield into its angle component directly in time domain. Then, by assuming the frequency contents are unchanged during the propagation, the broadband signal can be directly converted into its frequency component. Finally, this frequency-angle-domain information is used in equations 7 and 8 to create the PSF. The resulted method is highly efficient.

Given a broadband source time function, such as a Ricker wavelet, the finite-difference calculated wavefield can be expressed as

\[
A(x, k_d; x_s, x_g, \omega) \leftarrow G^s(x, k_s; x_g, \omega)G^s(x, k_d; x_s, \omega)G(x, k_d; x_g, x_s, \omega),
\]

where \( k_d = k_i + k_s \) is the wavenumber related to structural dip, \( k_i \) and \( k_s \) are the incident and scattered wavenumbers near the image point, and \( R(x, k_d, \omega) \) is the wavenumber-domain PSF. The mapping in equation 8 converts the response from \((k_i, k_s)\), the wavenumber domain in acquisition coordinate, to \( k_d \), the wavenumber domain in target coordinate (refer to Figure 1).

Similarly, for receiver-side wave, we have

\[
G(x, \theta_d; x_g, \omega)G^s(x, \theta_d; x_s, \omega) = \frac{1}{T} \int \left| s(\omega) s^*(\omega) \right| d\omega \int u^2(x, \theta_d; x_s, t) dt,
\]

where \( \theta_d \) is the local scattered angle.

From equations 12 and 13, we can create the LIM for the acquisition system (Xie et al., 2006)

\[
LIM(x, \theta_s, \theta_g, \omega) = 2k_0^2 s(\omega)s^*(\omega) \sum_{x_s} \sum_{x_g} A(x, \theta_s, \theta_g; x_s, x_g, \omega),
\]

where

\[
A(x, \theta_s, \theta_g; x_s, x_g, \omega) = G^s(x, \theta_s; x_s, \omega)G^s(x, \theta_g; x_g, \omega)G(x, \theta_s; x_s, \omega)G(x, \theta_g; x_g, \omega),
\]

\[
= \frac{1}{T^2} \int \left| s(\omega) s^*(\omega) \right| d\omega \int u^2(x, \theta_s; x_s, t) dt \times \int u^2(x, \theta_g; x_g, t) dt.
\]

The LIM \( A(x, \theta_s, \theta_g, x_s, x_g, \omega) \) can be directly calculated from the time-domain wavefield. We convert it to \( R(x, k_d, \omega) \) for resolution analysis. The LIM (equations 14 and 15) is similar to the PSF in equations 7 and 8, except that the former decomposes the illumination into the incidence/scattering angle domain \( \theta_s \) and \( \theta_g \), and the latter decomposes the illumination into dip wavenumber domain \( k_d \). Integrating their components sums up all angle-dependent illumination together, and the two should be equivalent with each other. Therefore, we integrate equation 15 with respect to \( d\theta_s d\theta_g \) and then convert variables from \((\theta_s, \theta_g)\) to \((k_d, \theta_d)\)

\[
\int A(x, \theta_s, \theta_g; x_s, x_g, \omega) \frac{\partial(\theta_s, \theta_g)}{\partial(k_d, \theta_d)} dk_d d\theta_d.
\]
where \( k_d = |k_d| \) and \( \theta_d \) is the dipping angle, \( J[\partial(\theta_d, \theta_d)/\partial(k_d, \theta_d)] \) is the Jacobian. The coordinate transforms involve first rotating the incident/scattering angles \( (\theta_x, \theta_y) \) to dipping/reflection angles \( (\theta_d, \theta_r) \) with \( \theta_d = (\theta_x + \theta_y)/2 \) and \( \theta_r = (\theta_x - \theta_y)/2 \) (refer to Figure 1), followed by converting them to the dipping wavenumber domain \( (k_d, \theta_d) \) with \( k_d = 2k_0 \cos \theta_d \) and \( \theta_d = \theta_d \) (refer to Alkhalifah, 2015; Xie, 2015). The two transforms can be combined into

\[
\begin{aligned}
\theta_x &= \theta_d + \cos^{-1}\left(\frac{k_d}{2k_0}\right), \\
\theta_y &= \theta_d - \cos^{-1}\left(\frac{k_d}{2k_0}\right).
\end{aligned}
\]

The Jacobian is

\[
J \left[ \frac{\partial(\theta_x, \theta_y)}{\partial(k_d, \theta_d)} \right] = \frac{1}{k_0 \sqrt{1 - \left(\frac{k_d}{2k_0}\right)^2}}.
\]

Thus, we have

\[
A(x, k_d; x_0, x_y, \omega) = A \left\{ \left[ \theta_d + \cos^{-1}\left(\frac{k_d}{2k_0}\right), \theta_d - \cos^{-1}\left(\frac{k_d}{2k_0}\right) \right], x_0, x_y, \omega \right\} \times \frac{1}{k_0 \sqrt{1 - \left(\frac{k_d}{2k_0}\right)^2}}.
\]

Substituting equation 15 into equation 19, and then into equation 7, the PSF can be calculated from the LIM

\[
R(x, k_d, \omega) = \frac{2s(\omega)s^*(\omega)k_0^2}{T^2 \left\{ \int [s(\omega)s^*(\omega)]d\omega \right\}^2} \times \frac{1}{k_0 \sqrt{1 - \left(\frac{k_d}{2k_0}\right)^2}} \times \int u^2 \left( x, \left[ \theta_d + \cos^{-1}\left(\frac{k_d}{2k_0}\right) \right], x_0, t \right) dt \times \int u^2 \left( x, \left[ \theta_d - \cos^{-1}\left(\frac{k_d}{2k_0}\right) \right], x_0, t \right) dt \times \frac{1}{k_0 \sqrt{1 - \left(\frac{k_d}{2k_0}\right)^2}}.
\]

Illumination response for a locally planar reflector at target location \( x \) with a dipping angle \( \theta_d \) is defined as the ADR (Wu et al., 2003; Xie et al., 2006), which can be calculated from the LIM

\[
ADR(x, \theta_d) = \int \lim \left( x, \left[ \frac{\theta_x + \theta_y}{2}, \frac{\theta_x - \theta_y}{2}, \omega \right] \right) d\theta_d d\omega.
\]

**THE POYNTING VECTOR METHOD**

Poynting vector gives the wave energy flux direction. Therefore, by calculating the Poynting vector, we can obtain the wave propagation direction at a given time and space location. For scalar wave equation, the Poynting vector can be calculated as (Yoon et al., 2004, 2011; Yoon and Marfurt, 2006)

\[
p = -\frac{\partial u}{\partial t} \cdot \nabla u,
\]

where \( u \) is the finite-difference calculated wavefield and \( \nabla u \) is its spatial gradient. In a staggered-grid finite-difference scheme, the time derivative and spatial gradient are intermediate variables that can be obtained without extra effort. Thus, the resulted method is highly efficient and widely used for several purposes. Yoon et al. (2004) and Pestana et al. (2014) use the Poynting vector method to eliminate the low-wavenumber artifacts in the RTM image. Dickens and Winbow (2011) use this method to generate common angle image gathers in RTM. Wang et al. (2013) test the Poynting vector method in illumination analysis, and Xie (2015) uses this method in the full-waveform inversion.

To compare different angle measurement methods, we calculate the wave propagation directions in the BP salt model (Billiet and Brandsberg-Dahl, 2005) using the local slowness analysis method (Xie and Lay, 1994; Yan and Xie, 2012; Yan et al., 2014) and the Poynting vector method, and the result is shown in Figure 2. The locations of energy peaks from slowness analysis give the slowness vectors, which provide the wave propagation directions. The red arrows give propagation directions calculated using the Poynting vector method. At space-time locations where only one wavefront is encountered, the estimated directions by both methods are consistent. However, if more than one wavefront coexist, slowness analysis method can give propagation directions of individual waves, whereas the Poynting vector method can only provide one single propagation direction, which may be incorrect. Jin et al. (2014) indicate that the wave propagation direction estimated using the Poynting vector method may be unreliable under low amplitudes. To improve its reliability, certain methods were proposed, e.g., averaging the results over multiple time steps (Yoon et al., 2011), smoothing the Poynting vectors in the space domain (Dickens and Winbow, 2011), using a least-squares solution over the time or space (Yan and Ross, 2013), or using the optical flow method to stabilize Poynting vector directions (Vyas et al., 2011; Zhang, 2014). All these methods can improve the accuracy but often with large computation and/or storage cost.

Although the Poynting vector method can encounter certain difficulties in a complicated wavefield, these situations usually only account for a very small portion in the entire space-time domain during the wave propagation process. Unlike in the migration, the illumination calculation does not involve the “data.” To simulate the propagation effect from the receiver side, we use the simple impulse as the “fictitious” data, which is injected from individual receiver locations one after another. Thus, the receiver-side extrapolation is simply calculating a Green’s function from the receiver location, exactly the same as dealing with a source. This largely relaxes the requirement of dealing with complex situations. To further improve the stability, we limit our measurement to within several time windows with most energetic arrivals. This can be easily accomplished by scanning the wavefield twice, locating the time windows in the first scan, and measuring the wave propagation directions and mean square amplitudes in located time windows in the second scan. In RTM imaging, to correlate with the time-reversed receiver wavefield, the source wavefield needs to be stored or regenerated. Therefore, the scanning procedure does not require additional calculation. In addition, this method only saves mean square amplitudes and
directions from a limited number of large-amplitude arrivals for later calculations, rather than save all possible directions at all space-time grids, thus tremendously reducing the input/output and storage requirements. By adopting the above approaches, we largely avoid the weakness of the Poynting vector method.

NUMERICAL EXAMPLES

Local illumination matrix

Equation 14 provides the illumination as a function in the incident/scattering angle domain \((\theta_s, \theta_g)\) called the LIM (Wu and Chen, 2002, 2003; Xie et al., 2006). Figure 3 is a cartoon showing the structure of an illumination matrix in a 2D model. The horizontal and vertical axes denote the incidence angle \(\theta_s\) and scattered angle \(\theta_g\) (refer to Figure 1). As illustrated in the figure, strips parallel to different axes or diagonals indicate different common angle gathers. Note that one-way propagators formulated under the Cartesian coordinate can only handle the incident/scattering angles between \(\pm \pi/2\), the square circled by the dashed line, whereas the full-wave propagator used here can cover incident/scattering angles up to \(\pm \pi\).

As an example, we calculate LIMs in the 2D SEG/EAGE salt model (Aminzadeh et al., 1997). The acquisition system is composed of 325 shots with an interval of 48.8 m. Each shot has 176 left-side receivers, with an interval of 24.4 m. The source time function is a 15 Hz Ricker wavelet. Figure 4 shows LIMs at selected locations, where the illumination is shown with normalized amplitude \((A/A_{\text{max}})^{1/2}\) and \(A_{\text{max}}\) is the maximum illumination in the model. Because an off-end data acquisition system is used in the calculation, the energy occupies the upper-left corner in these illumination matrices. At shallow depth, the illumination covers a wide angle range, but with increasing depth, its coverage becomes narrower. At the subsalt region, the illumination is weak and apparently misses certain dipping and scattering angle components because of the shadowing effect.

The PSF

The PSF can be calculated using equation 20. Figure 5 shows 15 Hz monochromatic PSFs calculated in the SEG/EAGE salt model, at the same locations as in Figure 4. Figure 6 shows the broadband PSFs. Note, the maximum detectable wavenumber is proportional to \(2k_0\). Therefore, in high-velocity region, the resolution is low and vice versa. Combining Figures 4–6, we see that at shallow depth the illumination covers a wide angle range. However, under the salt body, the angle coverage is uneven. In Figure 6g, the PSF shows very limited illumination along upper-right to lower-left direction in wavenumber domain, causing serious image problem to the fault along upper-left to lower-right direction in space domain.

Image correction for 2D SEG/EAGE salt model

The PSF carries the full information regarding the image distortion, including those from the acquisition geometry and overburden.
structures. According to equation 5, the image can be seen as a convolution between the PSF and the velocity perturbation. Ideally, by deconvolving the PSF from the image in space domain or by dividing PSF from the image in wavenumber domain, the uneven illumination can be compensated, and the overall quality of the image can be improved.

As the first example, we apply the illumination and resolution analysis to the 2D SEG/EAGE salt model. To conduct the image correction, we first decompose the source and receiver waves into local beams using equations 12 and 13. Then, use equations 14 and 15 to create LIMs as shown in Figure 3. Using equations 19 and 20, the LIMs are converted to PSFs, which are shown in Figure 6. Finally, by using a 2D sampling function with cosine tapers and the windowed FFT, the depth image is transformed to the local wavenumber domain and corrected by the PSF. The above-mentioned process is demonstrated in Figure 7. Figure 7a shows the conventional RTM image. The acquisition system is the same as that used in generating Figure 4. The area marked by the white square is chosen to demonstrate the procedure of image correction. In the middle, from left to right are the conventional image sampled by the white square (Figure 7b), the wavenumber spectrum of the conventional image (Figure 7c), the wavenumber-domain PSF (Figure 7d), the corrected spectrum computed by dividing panel (c) by panel (d) (Figure 7e), and corrected space-domain image (Figure 7f). In Figure 7b, from upper left to lower right is the image of the fault. Along the upper-right to lower-left direction are several artifacts caused by internal multiples. In Figure 7d, we see strong illumination along the upper-left to the lower-right direction in the wavenumber domain. The uneven illumination weakens the image of the fault and enhances artifacts. In Figure 7e, after correction, the wavenumber spectrum for the fault is enhanced, whereas the spectrum for the artifacts is effectively suppressed. After converting back to the space domain, the image for the fault is largely improved. Using the sampling window to scan the entire image and repeatedly using the correction process mentioned above, we obtain the corrected image shown in Figure 7g. Comparing Figure 7a and 7g, subsalt structures in the corrected image are more balanced, and several steep-dip structures are more emphasized. As a comparison, we apply automatic gain control (AGC) to the conventional RTM, and the result is shown in Figure 7h. By comparing with Figure 7a, the AGC can raise the image amplitude at a deeper depth. However, comparing with Figure 7g, the AGC result cannot match the quality of the illumination corrected image. The image in the
Figure 6. Wavenumber-domain PSF at selected locations in the 2D SEG/EAGE salt model, calculated using the broadband signal.

Figure 7. (a) The conventional RTM image for the 2D SEG/EAGE salt model, in which the area marked by a white square is chosen to demonstrate the correction procedure. (b) The original image, (c) its wavenumber spectrum, (d) wavenumber-domain PSF, (e) corrected spectrum, and (f) corrected image. The entire corrected image is shown in (g). (h) The conventional RTM image with the AGC.
subsalt area is still weak, and some steeply dipping structures are missing.

The image can also be corrected using the ADR maps in the dip angle domain (Wu et al., 2004; Mao and Wu, 2011; Yan et al., 2014). First, we apply the FFT to transform the conventional RTM image shown in Figure 7a to wavenumber domain, in which the image is divided into 24 equal-sized fan-shaped areas, each occupying a 15° interval and using its center angle as the nominal dip angle. Using inverse FFT, the images in individual wedges are transformed back to space domain to form 24 common dip angle images. Figure 8c and 8d shows two examples at \( \theta_d = 45° \) and 45°. Second, we calculate ADR maps using equation 21, and two corresponding examples are shown in Figure 8a and 8b. The common dip images are corrected by dividing the correspondent ADRs, and the results are shown in Figure 8e and 8f. Finally, sum up all 24 ADR corrected common dip images to obtain the corrected image, which is shown in Figure 8g. Comparing Figures 7g and 8g, image corrected using the ADR is usually less accurate as that using the PSF because the former uses only the information in dip angle \( \theta_d \), but the latter further uses the scale information in \( k_d \).

**Image correction for the Sigsbee 2A velocity model**

In this example, we investigate the Sigsbee 2A salt model (Paffenholz et al., 2002). The acquisition system is composed of 500 shots with a source interval of 45.7 m. Minimum and maximum offsets are 0 and 7932 m, respectively, and the receiver interval is 22.9 m. The source is a 20 Hz Ricker wavelet. The conventional RTM image is shown in Figure 9a. Because subsalt structures are nearly horizontal, shown in Figure 9b is the 0° ADR map, which is generated by half of the sources and receivers used for the RTM. From the ADR map, we see that near-horizontal structures below the overhang part of the salt body are poorly illuminated. These areas are responsible for the missing image in the RTM result (circled by ellipses). Similar to the previous example, we use the PSF to correct the RTM image, and the result is shown in Figure 10a. Compared with the conventional RTM image in Figure 9a, the image quality is significantly improved. In general, the amplitudes are more balanced, particularly in the subsalt region. By magnification in the areas labeled by white squares, Figure 10b and 10d shows the enlarged details in the corrected image. Compared with the conventional images shown in Figure 10c and 10e, the corrected images have consistent layered structures that can be traced to closer to the salt flank, improved focusing to point like scatters, and generally sharper images. Comparing Figure 10a with Figure 9a, the correction also removes long-wavelength artifacts in the image (can also be shown in Figure 7). At shallow depth, the PSFs have very strong near-zero-wavenumber components generated by diving waves, the same source causing the low-wavenumber artifacts. After dividing

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**Figure 8.** (a and b) The −45° and 45° ADR maps, (c and d) correspondent common-dip images, (e and f) common-dip images after ADR correction, and (g) the final image after summing up partial images for all dipping angles.
the PSF in wavenumber domain, low-wavenumber artifacts are effectively eliminated from the image.

COMPUTATIONAL ISSUES AND THE EFFICIENCY OF THE METHOD

From the formulations of the illumination and resolution analysis, we see that intensive computations are required to calculate Green’s functions and decompose the wavefield into angle components. In RTM imaging, the wavefield needs to be extrapolated from sources to the subsurface. If the illumination and resolution analysis is accompanying the imaging (e.g., for related quality control or image correction purpose), we actually use the existing source wavefields as the Green’s functions without having to recalculate them.

In the resolution calculation, one of the important steps is converting the measurement from the incident/scattering angle domain to dipping wavenumber domain (equation 20). Because \((\theta_s, \theta_g)\) and \((k_d, \theta_d)\) are defined in discretized grids, proper interpolations should be adopted.

The illumination and resolution related functions, the LIM, PSF, and ADR, are all slowly varying functions. We can use a coarser grid to calculate these functions and use interpolation to extend them to the entire model. In addition because the illumination calculation is intrinsically a modeling, there is no noise being involved. Their calculations usually do not need as much of sources and receivers used in a RTM. Taking the 2D SEG/EAGE salt model as an example, we calculate the 45° ADR map using different grid sizes and numbers of sources and receivers. The results are shown in Figure 11, in which Figure 11a shows the ADR calculated for all grid points using all sources and receivers as used in the RTM. Figure 11b shows the ADR calculated for a \(2 \times 2\) interval, and interpolated to every point. Figure 11c shows the ADR calculated for a \(3 \times 3\) interval, and interpolated to every point. We see even using the \(3 \times 3\) interval, the result is still acceptable. Note that in a 3D case, using the \(2 \times 2 \times 2\) interval can cut the calculation and storage to one-eighth, and using the \(3 \times 3 \times 3\) interval can cut the cost to \(1/27\). Figure 11d and 11e shows ADRs calculated using one-half and one-quarter of the sources

Figure 9. (a) The conventional RTM image of the Sigsbee 2A model and (b) the ADR map for horizontal (zero dipping angle) reflector.

Figure 10. (a) Corrected RTM image by deconvolving with the PSF. To illustrate the details, (b and d) are enlarged corrected images indicated by white squares in (a), and (c and e) are original images at the same area.
and receivers. The results show that, even with one quarter of the sources and receivers, the generated ADR maps are still reasonably accurate, except at very shallow depth. In correcting the RTM image using the PSF, we actually sample the image using a 21 × 21 space window. After correction, we use 5 × 5 corrected pixels at the center of the sampling window to reconstruct the corrected image. Then, move the sampling window according to a 5 × 5 interval to scan the model and repeat the calculation. In this way, the PSFs are only calculated in a 5 × 5 interval. All the above-mentioned parameters can be adjusted by trade-off between the quality and efficiency and the resulted method can be highly flexible.

The illumination and resolution analysis is a useful tool as a supplement to seismic imaging, but it usually demands intensive calculations and huge storage, which prevent it from being used in practice. This problem is particularly severe for the time-domain migration, such as the RTM. We introduce several techniques to mitigate the efficiency and storage problem encountered in the broadband analysis, i.e., (1) measuring the wave propagation direction using the Poynting vector method, (2) selecting several most energetic phases to calculate and store, instead of measuring the entire wave train, and (3) assuming the wavefield preserves the source spectrum instead of actually calculating Fourier transforms of waveforms. For a typical 2D case, if the illumination analysis is conducted along with the RTM, compared with the time spend by the RTM itself, the angle decomposition using the Poynting vector at a 1 × 1 interval takes approximately 10%–20% of extra time. Creating the LIM and ADR at a 1 × 1 interval using all sources and receivers spends approximately 10% of extra time. Creating the PSF and conduct the image correction at a 5 × 5 interval will take approximately 20% additional time. Although current numerical examples are all calculated in 2D models, extending the method to 3D is straightforward. Under the 3D case, the integral type methods for angle decomposition are extremely slow. Roughly speaking, to extend the problem from 2D to 3D, the differential type methods (e.g., the Poynting vector method) will increase computations by a factor of \( N_y \), but the integral type methods will increase computations by a factor of \( N_x \times L \), where \( N_y \) is the model grid size in the third dimension and \( L \) is the grid size for a dominant wavelength. Even worse, integral type methods often require to output local space-time wavefield for processing. The massive input/output and the storage can cause further problem. Therefore, the method proposed here will be even more attractive under the 3D case.

**DISCUSSION**

In the proposed method, the Poynting vector is adopted to decompose the wavefield into its angle components. Compared with local slant stacking or local Fourier transform, the Poynting vector method is much more efficient, although less accurate when encountered multiple incoming waves simultaneously. We use several approaches to mitigate its disadvantages. However, the effectiveness of these approaches under more complex environment may need further testing. To conduct the resolution analysis, the frequency information is required to convert the angle-domain information into the wavenumber-domain information. To avoid saving the entire space-time wavefield and formally carry out Fourier transforms at all space locations, we assume that the wavefield keeps the spectrum of the source wavelet and directly convert the broadband signal into its spectrum. In an actual wave propagation process, frequency-dependent phenomena such as the attenuation, scattering, and diffraction may modify the original source spectrum. Thus, this approximation may generate certain errors in resolution analysis. However, compared with the huge savings in computations and storage, it is worth to adopt this approximation. If the analysis is conducted along with the RTM imaging, the source wavefield calculated for the image can be used as the Green’s functions for the sources and receivers in the resolution analysis. Because the resolution analysis uses much less sources and receivers than the actual imaging, we usually do not need calculate additional Green’s functions, and this can save a lot of computational effort. However, under certain cases, such as the 3D wide azimuth acquisition, calculating additional Green’s functions may be required.

**CONCLUSION**

A full-wave-equation-based broadband method is proposed for illumination and resolution analysis. By using the Poynting vector method to calculate the wave propagation direction, the proposed method is highly efficient and flexible. If the source wavefield generated in the RTM is used as Green’s functions, the illumination and resolutions analysis only takes an extra time that is a fraction of the time used for the migration imaging. It is particularly suitable for illumination and resolution analysis, when teamed with the RTM.
imaging. We present several numerical examples to demonstrate how to calculate the LIM, ADR, and PSF, as well as correcting depth images using the PSF or ADR.

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