A hybrid elastic one-way propagator for strong-contrast media and its application to subsalt imaging

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ABSTRACT

For elastic-wave propagation in strong-contrast media, we developed the theory and method of a new one-way operator, which can overcome the weak perturbation assumption of a traditional elastic thin-slab propagator. In the framework of the new propagator, the elastic model is separated into several domains along the strong-contrast boundaries. Different background velocities are chosen for different domains so that each domain can be considered as a weakly heterogeneous media with a sharp boundary. The new propagator adopts a hybrid approach to extrapolate the wavefields in each domain: a traditional thin-slab propagator for weak heterogeneities combined with a reflection/transmission operator for wave exchanging at the sharp boundary. It inherits the dual domain (wavenumber-space domain), depth-marching implementation of an elastic thin-slab propagator. The boundary is discretized into a collection of boundary segments based on the marching depth, and the reflection/transmission operators are estimated locally at the boundary segment by making a tangent plane approximation. By neglecting the reverberation between the boundary segments, the reflected/transmitted wave from the boundary can be gradually incorporated into wave propagation in a one-way fashion. We validated the accuracy and efficiency of the hybrid propagator by showing numerical simulation results on two simple salt models. Finally, we applied the hybrid propagator to subsalt imaging using a synthetic data set generated from a 2D subsalt model.

INTRODUCTION

Subsalt seismic imaging can be challenging due to the complex geometry of salt bodies and the large impedance contrasts between salt and surrounding sediment deposits. Traditional imaging uses acoustic fluid approximation for the medium, which uses only P-waves. However, the large velocity contrast across the sedimentary/salt interfaces together with the frequently rugose character of these interfaces prevents P-waves from penetrating through the salt body with sufficient energy. The elastic-wave mode conversion is quite efficient at salt interfaces. Converted S-waves can carry considerable amounts of energy to illuminate the subsalt structure (Purcell, 1992; Wu et al., 2001, 2010). Previous attempts at elastic wave imaging adopted scalar wave propagators for the P- and S-waves (e.g., Zhe and Greenhalgh, 1997; Sun and McMechan, 2001; Hou and Marfurt, 2002), which is dynamically incorrect. Full-wave elastic reverse time migration (RTM) (e.g., Sun and McMechan, 1986; Chang and McMechan, 1987, 1994) extrapolates the wavefield based on the elastic wave equation, but P- and S-modes are mixed together and hard to separate. The ability to identify and select proper converted paths for imaging is critical for elastic wave imaging. Elastic thin-slab and elastic complex screen methods (Wu, 1994; Wu and Xie, 1994; Xie and Wu, 1995, 2001, 2005; Wu and Wu, 2005; for a review, see Wu et al., 2007) can present us such flexibility. The methods have special features and advantages when applied to seismic imaging. First of all, similar to elastic RTM, these one-way migrations use a vector wavefield extrapolator so the dynamic feature of P-S and S-P conversions are well-handled. Second, mode types are book-kept during propagation, which is especially useful for elastic wave imaging in terms of controlling migration...
crosstalks and parameter inversion. Third, the one-way wave method is much more efficient in computation, often orders of magnitude faster than full-wave methods.

However, there is one issue with the elastic thin-slab or elastic complex screen methods when they are based on perturbation theory. They can handle elastic perturbations only up to 30% (Wu and Wu, 2005). The algorithms may become unstable beyond this limit. Although these methods can be useful in many reservoir modeling and imaging cases, they are currently not used in imaging for strong-contrast media, such as salt or basalt inclusions. In this paper, we extend the one-way elastic method to the case of strong-contrast media.

In this paper, we first briefly summarize the theory of the elastic one-way propagator realized by sequential thin slab for weak perturbations. Second, we present the formulations for the new theory on the hybrid elastic one-way propagator in strong contrast medium with a sharp boundary and describe its three essential components in detail. Finally, we conduct two numerical tests to verify the accuracy and efficiency of the hybrid elastic one-way propagator and also use the subsalt model to demonstrate its application to seismic imaging.

ELASTIC THIN-SLAB PROPAGATOR FOR WEAKLY HETEROGENEOUS MEDIA

For a heterogeneous medium, we define the vertical z-direction as the preferred propagation direction and realize the wave propagation in a depth-marching algorithm. Based on the marching step, we slice the medium into a number of horizontal thin slabs. Shown in Figure 1 is an example of an individual thin slab for elastic media. In each thin slab, the elastic heterogeneities are treated as volume perturbations relative to the background medium. For the isotropic elastic case, it can be simplified as P-P and S-S conversion. The background elastic Green’s function (Appendix B), which is derived to implement the interaction with perturbations in the space domain but propagate to the observation point in the wavenumber domain.

By adopting different Green’s functions for P- and S-modes, we can decouple P- and S-modes in an elastic thin-slab propagator. Thus, the scattered displacement fields for P- and S-modes can be expressed as

\[
\mathbf{u}_p^T(\mathbf{K}_T, z_j) = \int_{z_{j-1}}^{z_j} \int d^2 \mathbf{x}_T \mathbf{F}(\mathbf{x}_T, z) \cdot \mathbf{G}_0^P(\mathbf{K}_T, z_j; \mathbf{x}_T, z),
\]

\[
\mathbf{u}_s^T(\mathbf{K}_T, z_j) = \int_{z_{j-1}}^{z_j} \int d^2 \mathbf{x}_T \mathbf{F}(\mathbf{x}_T, z) \cdot \mathbf{G}_0^S(\mathbf{K}_T, z_j; \mathbf{x}_T, z),
\]

where \( \mathbf{G}_0^P(\mathbf{K}_T, z_j; \mathbf{x}_T, z) \) and \( \mathbf{G}_0^S(\mathbf{K}_T, z_j; \mathbf{x}_T, z) \) are the background dual-domain Green’s function for P- and S-waves, respectively (see Appendix B). Note that \( \mathbf{u}_p^T \) includes P-P and S-P conversion and \( \mathbf{u}_s^T \) includes P-S and S-S conversion.

Equation A-5 gives the equivalent body force for general elastic media. For the isotropic elastic case, it can be simplified as

\[
\mathbf{F}(\mathbf{x}_T, z) = \delta \rho \beta^2 \mathbf{u}_0(\mathbf{x}_T, z) + \nabla \cdot [\delta \beta \mathbf{e}_0(\mathbf{x}_T, z) + 2 \delta \mu \mathbf{e}_0(\mathbf{x}_T, z)],
\]

where \( \mathbf{u}_0 \) and \( \mathbf{e}_0 = 1/2 (\nabla \mathbf{u}_0 + \mathbf{u}_0 \nabla) \) are the background displacement and strain fields within the thin slab, respectively; \( \lambda \) and \( \mu \) are the Lamé constants and \( \mathbf{I} \) is the identity tensor. In this equation, the incident field, its divergence, and strain field (composed of its gradient) at level \( z \) are calculated in the spectral domain and then are transformed to the space domain as follows:

\[
\mathbf{u}_0(\mathbf{x}_T, z) = \int d^2 \mathbf{K}_T e^{\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^P(\mathbf{K}_T, z)
\]

\[
+ \int d^2 \mathbf{K}_T e^{\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^S(\mathbf{K}_T, z),
\]

\[
\nabla \cdot \{\mathbf{e}_0(\mathbf{x}_T, z)\} = - \int d^2 \mathbf{K}_T e^{\mathbf{K}_T \cdot \mathbf{x}_T} [\mathbf{k}_\alpha \cdot \mathbf{u}_0^P(\mathbf{K}_T, z)] \mathbf{k}_\alpha,
\]

\[
\nabla \cdot \mathbf{e}_0(\mathbf{x}_T, z) = - \int d^2 \mathbf{K}_T e^{\mathbf{K}_T \cdot \mathbf{x}_T} [\mathbf{k}_\alpha \cdot \mathbf{u}_0^P(\mathbf{K}_T, z)] \mathbf{k}_\alpha
\]

\[
+ \mathbf{u}_0^P(\mathbf{K}_T, z) \mathbf{k}_\alpha \cdot \mathbf{k}_\alpha
\]

\[
- \int d^2 \mathbf{K}_T e^{\mathbf{K}_T \cdot \mathbf{x}_T} [\mathbf{k}_\beta \cdot \mathbf{u}_0^S(\mathbf{K}_T, z)] \mathbf{k}_\beta
\]

\[
+ \mathbf{u}_0^S(\mathbf{K}_T, z) \mathbf{k}_\beta \cdot \mathbf{k}_\beta.
\]

Figure 1. Schematic illustration of a marching step of the elastic thin-slab propagator.
where \( \mathbf{k}_p = (\mathbf{k}_T, \gamma_a) \) and \( \mathbf{k}_s = (\mathbf{k}_T, \gamma_p) \) represent the P- and S-wavenumber vectors, respectively, in which \( \gamma_a \) and \( \gamma_p \) are the vertical wavenumbers for P- and S-waves, respectively.

The background wavefields are efficiently extrapolated in the wavenumber domain. The extrapolation operator is a simple phase shift determined by the background P-wave or S-wave velocity in the current slab

\[
\mathbf{u}_0^P(\mathbf{k}_T, z) = \mathbf{u}^p(\mathbf{k}_T, z_{j-1}) e^{i k_a (z-z_{j-1})},
\]

where \( \mathbf{u}^p(z_{j-1}) \) and \( \mathbf{u}^s(z_{j-1}) \) are the incident waves at the entrance of the thin slab. At the exit of each thin slab, the scattered field is added to the background wavefield to form the total wavefield, which in turn is treated as the updated incident wavefield for the next thin slab:

\[
\mathbf{u}_0^P(x_T, z) = \frac{1}{4 \pi^2} \int d^2 \mathbf{k}_T e^{i \mathbf{k}_T \cdot \mathbf{x}_T} \left[ \mathbf{u}^p_{\delta}(\mathbf{k}_T, z) + \mathbf{u}^p_{\gamma}(\mathbf{k}_T, z) \right],
\]

\[
\mathbf{u}_0^S(x_T, z) = \frac{1}{4 \pi^2} \int d^2 \mathbf{k}_T e^{i \mathbf{k}_T \cdot \mathbf{x}_T} \left[ \mathbf{u}^s_{\delta}(\mathbf{k}_T, z) + \mathbf{u}^s_{\gamma}(\mathbf{k}_T, z) \right].
\]

To summarize, the elastic thin-slab propagator is implemented by an operator split in the dual domain: background propagation in the wavenumber domain and interaction with perturbations in the space domain.

HYBRID ELASTIC ONE-WAY PROPAGATOR IN STRONGLY HETEROGENEOUS MEDIA WITH A SHARP BOUNDARY

The perturbation method is a valid and convenient tool for modeling elastic wave scattering and propagation in weak heterogeneous media, but it fails in strong-contrast media with sharp boundaries, such as salt or basalt inclusions. To handle this specific case, we divide the model into different domains along the sharp boundary (Figure 2). The wavefield in each domain can be computed by the representation integral, which is composed of a volume integral and a boundary integral (Aki and Richards, 1980):

\[
\begin{aligned}
\mathbf{u}(\mathbf{x}') &= \int \mathbf{f}(\mathbf{x}) \cdot \mathbf{G}(\mathbf{x'}, \mathbf{x}) d\Omega(\mathbf{x}) \\
&\quad + \int \left[ \left\{ \mathbf{N} \cdot \mathbf{G}(\mathbf{x'}, \mathbf{x}) - \mathbf{u}(\mathbf{x}) \cdot [\mathbf{N} \cdot \mathbf{G}(\mathbf{x'}, \mathbf{x})] \right\} \right] d\mathbf{S}(\mathbf{x}) \\
&\quad \mathbf{x}, \mathbf{x}' \in \Omega; \quad S = \partial \Omega.
\end{aligned}
\]

where \( \mathbf{u}(\mathbf{x}') \) is the displacement field at point \( \mathbf{x}' \) within the volume \( \Omega \) enclosed by surface \( S, \mathbf{f}(\mathbf{x}) \) is the body force or equivalent body force inside \( \Omega; \mathbf{G}(\mathbf{x'}, \mathbf{x}) \) is the Green’s displacement tensor (dyadic); \( \mathbf{\Sigma}(\mathbf{x'}, \mathbf{x}) \) is the Green’s stress tensor (triadic); \( \mathbf{n} \) is the surface normal as toward to the exterior of \( \Omega \); and \( \mathbf{u}(\mathbf{x}) \) and \( \mathbf{\sigma}(\mathbf{x}) \) are the displacement and stress on the surface. The volume integral term yields the contribution due to the sources inside \( \Omega \), whereas the boundary integral term, that is, the Kirchhoff integral, takes account of the contributions from the source outside \( \Omega \). The boundary values, i.e., displacement and traction on the boundary, reflect the collective effects of external sources. The internal and external sources could be body force or equivalent body force. A traditional elastic one-way propagator only deals with the internal sources. If the sharp boundary exists, the external sources could be incorporated into the wave propagation in a similar way as internal sources but in the form of boundary integral. Because it is an iterative approach, the Green’s function in representation integral (equation 11) can be approximated by the background Green’s function. The resultant operator combines the volume and boundary integrals, so we call it the hybrid elastic one-way propagator.

Figure 2 shows the realization of a typical hybrid propagator using a two-domain model (Figure 2a) as an example. Following the framework of a thin-slab propagator, the sharp boundary is separated into a group of boundary segments based on the marching step. Shown in Figure 2b is a thin slab composed of two domains: a high-velocity region and a low-velocity region. The wave entering the thin slab interacts with the internal volume scatterings, and it also exchanges energy at the boundary segment (Figure 2c) by admitting the transmitted wave from the other domain and the reflected wave from its own domain (Figure 2d). It acts like the boundary segment is radiating waves to both domains. To distinguish it from volume scattering, we call it boundary scattering. The wavefields at the exit of the thin slab are updated by the volume-scattered wave and boundary-scattered wave (Figure 2e). In the following subsection, we will describe three essential components of hybrid propagator algorithms: background propagation, volume scattering for weak heterogeneities, and boundary scattering for strong contrasts.

Figure 2. Schematic illustration of the hybrid elastic one-way propagator: (a) the two-domain model sliced into multiple thin slabs based on the marching step, (b) a thin slab containing a high-velocity region and a low-velocity region, (c) illustrates that the incident wave interacts with the volume perturbation and boundary segment, (d) the wave exchange at the boundary segment, and (e) demonstrates that the wavefields are updated with the volume scattered wave and boundary scattered wave at the exit of the thin slab.
Boundary scattering due to large medium parameter contrast

Background propagation

When several domains coexist in a model, the background propagation in each domain is handled separately. At the entrance of the thin slab, the incident waves are windowed in space based on the domain they belong to, and are propagated separately to the exit of the thin slab with its own background velocity

\[ u^{(i)}_0(K_T, z_j) = e^{i\rho(z-z_{j-1})} \int d^2x_T e^{-iK_T x_T} u^i(x_T, z_{j-1}), \]

(12)

\[ (x_T, z_{j-1}) \in \Omega_i, \]

\[ u^{(i)}_0(K_T, z_j) = e^{i\rho(z-z_j)} \int d^2x_T e^{-iK_T x_T} u^i(x_T, z_j), \]

(13)

\[ (x_T, z_j) \in \Omega_i, \]

where \( u^i(x_T, z_{j-1}) \) and \( u^i(x_T, z_j) \) are the P- and S-waves at the entrance of the thin slab, and \( \Omega_i \) is the domain \( i \).

Volume scattering due to weak heterogeneities

In the thin slab of each marching depth, different background velocities are selected for different domains. The volume scatterings are formulated relative to the background parameters so that the weak perturbation assumption of a traditional thin-slab propagator is guaranteed to work in each domain. They are propagated to the exit of the thin slab using the background Green’s function in the domain which they belong to

\[ u^{(i)}_v(K_T, z_j) = \int_{z_{j-1}}^{z_j} dz \int d^2x_T F(x_T, z) \cdot G^{(i)}_0(K_T, z_j; x_T, z), \]

(14)

\[ (x_T, z_j) \in \Omega_i, \]

\[ u^{(i)}_v(K_T, z_j) = \int_{z_{j-1}}^{z_j} dz \int d^2x_T F(x_T, z) \cdot G^{(i)}_0(K_T, z_j; x_T, z), \]

(15)

\[ (x_T, z_j) \in \Omega_i, \]

where \( \Omega_i \) is the domain \( i \), \( \Omega_i \) is the domain \( i \), and \( F(x_T, z) \) is the equivalent force of volume scattering defined in equation 5.

Boundary scattering due to large medium parameter contrast

Boundary scattering is a new component in the hybrid elastic one-way propagator. To obtain the boundary values, the boundary element method (BEM) (Sánchez-Sesma and Campillo, 1991; Fu and Wu, 2001; Ge et al., 2005) is a classic approach that discretizes the boundary into a set of straight-line elements and sets up a linear matrix by matching the boundary condition on every element. However, it involves intensive computations to solve the matrix because all the boundary elements are coupled. The one-way BEM method (Wu et al., 2011; Wu and Ge, 2014) avoids the huge matrix operations by neglecting the reverberations between the boundary elements. It implies that a boundary element cannot perturb the elements above it or at the same level as it. Under this assumption, the matrix is greatly simplified and can be solved iteratively from top to bottom.

We follow the concept of the one-way BEM, but we propose an alternative operator to compute the strength of the boundary scattering. We make a tangent plane approximation (Voronovich, 1999), which assumes that the boundary is smoothly curved, so that the reflection/transmission coefficients defined for an infinite plane surface can be applied locally at the boundary segment (Figure 3). Numerical tests show that it has adequate accuracy for smoothly varying interfaces. The concept of calculating the boundary scattering can be expressed in this way. The wave incident on the boundary segment is expanded to a superposition of plane waves. Each plane wave is applied with reflection/transmission coefficients and then is summed up again at the exact location of boundary segment to form the strength of the boundary scattering.

We focus our attention on one boundary segment, which separates the medium into the upper and lower ones. We set the index for the upper medium as one and the lower medium as two. The outer normals of the upper and lower medium are

\[ n_1 = -\hat{n}, \]

(16)

and

\[ n_2 = \hat{n}, \]

(17)

where \( \hat{n} \) is the normal vector of the boundary segment (see Figure 3).

The incident waves of media 1 and 2 separated by the boundary segment are given from the output of previous thin slab as \( u^1_0 \) and \( u^2_0 \) in the wavenumber domain. For simplicity, the wave type is ignored as the same operation is applied to P- and S-waves. Each wavenumber component represents a plane wave. The plane-wave component \( u^i_0(k) \) is discarded if \( \mathbf{k} \cdot n_i < 0 \). To accurately calculate the wave perturbation at the boundary segment, we transform the wavenumber vector from the Cartesian coordinate to the local boundary coordinate (with the horizontal axis parallel to the tangent of the local boundary)

\[ \mathbf{k} = M^f \mathbf{k}, \]

(18)

where \( \mathbf{k} \) and \( \mathbf{k} \) are the wavenumber vectors in the Cartesian coordinate and in the local boundary coordinate, respectively, and \( M^f \) is the transform matrix from the Cartesian coordinate to local boundary coordinate. In the 2D case

\[ M^f = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}, \]

(19)

where \( \theta \) is the dip angle of the boundary segment and it varies with location (Figure 3).

Reflection/transmission coefficients are computed based on the horizontal slowness in the local boundary coordinate as well as the elastic parameters of the medium separated by the boundary segment, and then they are applied to the incident wave:

\[
\begin{bmatrix}
\mathbf{u}^{(1)}_0 \\
\mathbf{u}^{(2)}_0 \\
\mathbf{u}^{(1)}_0 \\
\mathbf{u}^{(2)}_0
\end{bmatrix} = \begin{bmatrix}
R_{PP} & R_{PS} & T_{PP} & T_{PS} \\
R_{PS} & R_{SS} & T_{PS} & T_{SS} \\
T_{PP} & T_{PS} & R_{PP} & R_{PS} \\
T_{PS} & T_{SS} & R_{PS} & R_{SS}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}^{(1)}_0 \\
\mathbf{u}^{(2)}_0 \\
\mathbf{u}^{(1)}_0 \\
\mathbf{u}^{(2)}_0
\end{bmatrix},
\]
where $R$ and $T$ are the reflection and transmission coefficients calculated by the Zoeppritz equation (Aki and Richards, 1980). The first and second subscripts of the coefficients are the medium indices. The superscripts of the coefficients specify the wave types in the upper and lower media, respectively. Here, $u^I$ and $u^S$ are the sum of the internal reflected wave and the transmitted wave from the other domain.

We perform a discrete Fourier transform to retrieve the space-domain displacements of the boundary scattering:

$$u^{(I)}(x_T, z) = \int d^2 K' T e^{iK'r x} u^{(I)}(K' T, z), \quad (21)$$

$$u^{(S)}(x_T, z) = \int d^2 K' T e^{iK'r x} u^{(S)}(K' T, z). \quad (22)$$

It is equivalent to summing up all the plane-wave components at the exact location of the boundary segment. The tractions of the boundary scattering can be calculated by the constitutive equation. In isotropic elastic media, it can be expressed as

$$t' = \lambda I n_I \nabla \cdot u' + \mu I [\nabla u' + u' \nabla], \quad (23)$$

where $\lambda$ and $\mu$ are the Lamé constants. Similarly, the tractions can be computed in the spectral domain and then are transformed to the space domain

$$t^{(I)}(x_T, z) = (\lambda I + \mu I) \int d^2 K' T e^{iK'r x} \left[ iK' T' n_I \cdot u^{(I)}(K' T, z) \right] n_I$$

$$+ \mu I \int d^2 K' T e^{iK'r x} [iK' T' n_I \cdot u^{(S)}(K' T, z)] n_I + iu^{(I)}(K' T, z) K' T' n_I, \quad (24)$$

$$t^{(S)}(x_T, z) = \mu I \int d^2 K' T e^{iK'r x} \left[ iK' T' n_I \cdot u^{(S)}(K' T, z) \right] n_I + iu^{(S)}(K' T, z) K' T' n_I. \quad (25)$$

For a given boundary segment in a thin slab, we have determined the total displacement and traction on the segment by the reflection/transmission operator. The boundary scattered waves are propagated to the exit of the thin slab by the Kirchhoff integral in the horizontal wavenumber domain (Wapenaar and Berkhout, 1989; Wu, 1989)

$$u^{(I)}_B(K_T, z_j) = \int_L [t^{(I)}(x_T, z) G^{(I)}_0(K_T, z_j, x_T, z)] dS, \quad I = 1, 2, \quad (26)$$

$$u^{(S)}_B(K_T, z_j) = \int_L [t^{(S)}(x_T, z) G^{(S)}_0(K_T, z_j, x_T, z)] dS, \quad I = 1, 2, \quad (27)$$

where $L$ is the boundary segment within the thin slab, which separates the upper and lower media.

Here, $G_0$ and $\Gamma_0$ are the background Green’s tensor of displacement and traction (see Appendix B). The scattered wave will be added to the domains on both sides of the boundary segment as $u^{(I)}_B(K_T, z_j)$, where $i$ is the domain number.

At the exit of each thin slab, the total field is composed of three parts: the background field $u^{(I)}_B$, the volume-scattered field $u^{(I)}_V$, and the boundary-scattered field $u^{(I)}_B$.
\[ u^{p(i)}(x_T, z_j) = \frac{1}{4\pi^2} \int d^2K_T e^{iK_T \cdot x_f} \left[ u_0^{p(i)}(K_T, z_j) + u_{t}^{p(i)}(K_T, z_j) \right] \]
\[ + u^{p(i)}(K_T, z_j) + u^{p(i)}(K_T, z_j) \right] \], \quad (x_T, z_j) \in \Omega, \quad (28) \]
\[ u^{s(i)}(x_T, z_j) = \frac{1}{4\pi^2} \int d^2K_T e^{iK_T \cdot x_f} \left[ u_0^{s(i)}(K_T, z_j) + u_{t}^{s(i)}(K_T, z_j) \right] \]
\[ + u^{s(i)}(K_T, z_j) + u^{s(i)}(K_T, z_j) \right] \], \quad (x_T, z_j) \in \Omega_i, \quad (29) \]
where \( \Omega_i \) is the domain \( i \).

**NUMERICAL TESTS FOR THE HYBRID ELASTIC ONE-WAY PROPAGATOR**

**Example 1: Wedge model**

First, we test the elastic one-way propagator using a wedge-shaped salt body embedded in a homogeneous medium (Figure 4). The source is in the center of the model, and the wavelet is a 15 Hz Ricker wavelet. We calculated the horizontal and vertical component displacement by an elastic one-way propagator (Figure 5a and 5b). Compared with the snapshots calculated from full-wave elastic finite difference (FD) (Figure 5c and 5d), the transmitted wavefront of the elastic one-way propagator matches with them very well. The energy partition at the interface is almost the same as the full-wave FD. In addition, to demonstrate the flexibility of the propagator, we show different wavepaths (including PPP, PPS, PSP, and PSS) by switching on/off the wave mode at the interface (Figure 6). This has been a great strength and advantage of the propagator. In terms of computational efficiency, the full-wave FD takes 1 h with a grid interval of 5 m, whereas the hybrid one-way propagator takes 5 min with a grid interval of 10 m.

**Example 2: Elliptical salt model**

We continue to compute the wave propagation for an elliptical salt model (Figure 7). The model is comprised of a sedimentary...
background with lateral variations and a salt inclusion in an elliptical shape. So it would be a more challenging model for a hybrid one-way propagator. An explosive source is initiated at the center of the model surface. The source time function is a 15 Hz Ricker wavelet. We compare the wavefield snapshots calculated by the hybrid one-way propagator with full-wave FD (Figure 8). There exist some noticeable amplitude discrepancies between the two methods. This is caused by the errors introduced by the tangent plane approximation, which did not predict the energy partition at the strong-contrast boundary as well as in the wedge model. But in terms of the phase, the hybrid propagator still matches FD very well. On the contrary, the traditional one-way propagator, based on the perturbation theory, usually gives an incorrect phase as a wave passing through the high-velocity body. Different wavepaths are depicted in Figure 9 by switching on/off wave modes at the boundary of the salt.

**Example 3: Subsalt modeling and imaging**

Finally, we move on to a more complex model — a 2D velocity profile simulating the subsalt model (Figure 10). The outline of the sharp boundary is shown in Figure 11a, and its dip angle is shown in Figure 11b. On top of the model is a water layer. FD cannot solve the fluid-solid problem very well (van Vorssen et al., 2002). On the contrary, the hybrid propagator can do a good job handling the interface. The model is divided into three different domains (Figure 11c) by the sharp boundary. Each domain has its own background and perturbation parameters. We design an observation system and conduct a seismic experiment. The acquisition system was comprised of 301 shots from 7000 to 37,000 m with an interval of 100 m. Each shot was recorded by a line of receivers with a doublespread configuration. The number of receivers was 561, and the maximal offset is 7000 m. The sources and receivers are located on the water surface. The recorded data are pressure-only simulating a hydrophone response. The source is a 15 Hz Ricker wavelet, and the total recording time is 12 s with a time interval of 0.01 s. The synthetic data were modeled using the Tesseral 2D application package with an FD approach. Shown in Figure 12 are sample records for shot numbers 100, 150, 200, and 250. In each shot, the area for computation is $25.6 \times 13.5$ km. We plot the horizontal and vertical components of the 71st-shot snapshots in Figure 13. The converted waves
Figure 10. The simplified 2D subsalt model: (a) P-wave velocity, (b) S-wave velocity, and (c) density.

Figure 11. Model parameters of simplified 2D subsalt related to hybrid one-way propagator: (a) the outline of sharp boundary, (b) the dip angle of the boundary, and (c) three domains separated by the sharp boundary.
penetrating through the salt base still carry considerable energy, which offers great potential to improve the subsalt imaging. We migrate the seismic data with the hybrid one-way propagator, and we show the final PP-reflection image in Figure 14. From the migration image, a large portion of subsalt reflectors can be clearly identified, except the section with very large dips. There are some migration artifacts near the true images due to multiples and crosstalk. It is the well-known drawback of elastic wave migration. The propagator has the flexibility to switch on/off wave modes at the sharp boundary so that it is capable of producing several subsalt images by selecting different converted paths during migration. This may offer great potential to identify imaging artifacts and further enhance the true image, especially the ones with a steep dip. But this will be the target of future work.

Figure 12. Synthetic records for subsalt model: shown here are samples for shot numbers 100, 150, 200, and 250.

Figure 13. (a1, b1, and c1) The snapshots of horizontal and (a2, b2, and c2) vertical displacements overlapped on the subsalt model at $t = 2.5$, 3.5, and 4.5 s.
CONCLUSIONS

A hybrid one-way propagator has been developed to handle the elastic-wave propagation and scattering in strong contrast media with sharp boundaries, such as salt or basalt inclusions. The hybrid propagator divides the model into two or more domains along the strong-contrast boundaries and realizes the wave propagation in each domain individually. In each domain, the background wavefields are perturbed by two different kinds of scattered waves: volume scattering and boundary scattering. Volume scatterings are formulated due to the lateral medium variations. They are relatively weak, so the corresponding scattered waves are predicted by perturbation theory and are picked up by the propagating wave at every marching step. The downward propagating wave also exchanges waves with the neighboring domains at the boundary, which is discretized into a group of boundary segments based on the marching step. Boundary scatterings, standing for the consequence of wave exchanging, are calculated under a tangent plane approximation by applying the local reflection/transmission operator to the wave incident on the boundary segment. Similar to the volume scatterings, they are included into the background wavefields step by step but in the form of a Kirchhoff integral. The hybrid propagator shuttles between the wavenumber and space domains: background wave propagation in the wavenumber domain and interaction with volume or boundary scatterings in the space domain. The accuracy of the hybrid propagator is evaluated on two 2D salt models by comparing the results from full-wave elastic FD. The phases match with elastic FD very well but the amplitudes show some discrepancies. In terms of efficiency, the hybrid propagator is at least one order of magnitude faster than elastic FD in these 2D cases. When applied to a 3D velocity model, it is anticipated to save even more computational time. The application of the propagator to seismic imaging is demonstrated on the subsalt model, and the three subsalt reflectors are imaged clearly.

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APPENDIX A

PERTURBATION THEORY

In the perturbation theory, the medium parameters and the wavefield are expressed as background parameters plus perturbations (see Aki and Richards, 1980; Wu and Aki, 1985)

\[
\rho(x) = \rho_0 + \delta \rho(x), \quad c(x) = c_0 + \delta c(x),
\]

\[
u(x) = u_0 + u_v(x),
\]

where \(\rho_0\) and \(c_0\) are the density and elastic constants of the background medium, \(\delta \rho\) and \(\delta c\) are the corresponding perturbations, respectively. Here, \(\nu\) is the total field, \(u_0\) is the background displacement field, and \(u_v\) is the scattered field caused by the volume heterogeneities.

The elastic wave equations corresponding to the true medium parameters and background medium parameters can be expressed in the frequency \(\omega\) domain as

\[
-\omega^2 \rho_0 \nabla \cdot \left( \frac{1}{2} c_0 : (\nabla \nu + \nu \nabla) \right) = f \tag{A-2}
\]

and

\[
-\omega^2 \rho_0 u_0 \nabla \cdot \left( \frac{1}{2} c_0 : (\nabla u_0 + u_0 \nabla) \right) = f, \tag{A-3}
\]

where \(\nabla\) is the spatial gradient operator, \(\cdot\) is the contraction of tensors, and \(f\) is the body force. Subtracting equation A-3 from equation A-2 and in combination of equation A-1, we derive the wave equation for the scattered field \(u_v\)

\[
-\omega^2 \rho_0 \nabla \cdot \left( \frac{1}{2} c_0 : (\nabla u_v + u_v \nabla) \right) = F, \tag{A-4}
\]

where

\[
F = \omega^2 \delta \rho u_0 + \nabla \cdot [\delta c : (\nabla u_0 + u_0 \nabla)] \tag{A-5}
\]
is the equivalent body force due to volume heterogeneities.

The solution of equation A-4 can be formulated as the convolution of the equivalent body force and the Green’s function

\[
u_v(x') = \int F(x) \cdot G(x';x) d\Omega(x), \tag{A-6}
\]

where \(G(x';x)\) is the Green’s displacement tensor (dyadic), which represents the displacement field at \(x'\) due to a point single force at \(x\).
APPENDIX B
WEYL INTEGRALS OF THE ELASTIC GREEN’S TENSORS

The background Green’s displacement tensor is expanded to horizontal wavenumber domain as the Weyl integral

\[
G(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}') = G^P(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}') + G^S(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}'),
\]

where

\[
G^P(\mathbf{K}_T, \mathbf{z}; \mathbf{x}', \mathbf{z}') = \frac{i k^2}{2 \rho_0 \omega^2} \hat{k}_d \hat{k}_a \frac{1}{\gamma_a} e^{-i k^2 x'_d + i \gamma_a z'},
\]

and

\[
G^S(\mathbf{K}_T, \mathbf{z}; \mathbf{x}', \mathbf{z}') = \frac{i k^2}{2 \rho_0 \omega^2} (\mathbf{I} - \hat{k}_d \hat{k}_d) \frac{1}{\gamma_d} e^{-i k^2 x'_d + i \gamma_d z'},
\]

are the wavenumber-domain Green’s function for P- and S-waves, in which \( \mathbf{I} \) is the identity matrix, and

\[
\gamma_d = (\mathbf{K}_T, \gamma_a)
\]

and

\[
\gamma_d = (\mathbf{K}_T, \gamma_d)
\]

are the P and S wavenumber vectors, respectively, with \( \mathbf{K}_T \) as the horizontal wavenumber and \( \gamma_a \) and \( \gamma_d \) as the vertical wavenumbers for P- and S-waves, respectively, defined by

\[
\gamma_a = \sqrt{k^2 - k_t^2},
\]

\[
\gamma_d = \sqrt{k^2 - k_t^2},
\]

where \( k_d = \omega/\omega_0 \) and \( k_d = \omega/\beta_0 \), with \( \omega_0 \) and \( \beta_0 \) as the P- and S-wave background velocities in the thin slab, respectively. The unit-direction vectors of P and S plane-wave propagation are

\[
\hat{k}_d = \frac{k_d}{k_d},
\]

\[
\hat{k}_d = \frac{k_d}{k_d},
\]

where the source location is \((\mathbf{x}', \mathbf{z}')\) and the observation location is \((\mathbf{x}, \mathbf{z})\).

Correspondingly, the Weyl integral of the Green’s traction tensor can be obtained

\[
\Gamma(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}') = \Gamma^P(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}') + \Gamma^S(\mathbf{x}, \mathbf{z}; \mathbf{x}', \mathbf{z}'),
\]

\[
\Gamma^P(\mathbf{K}_T, \mathbf{z}; \mathbf{x}', \mathbf{z}') = \frac{k^2}{2 \rho_0 \omega^2} \hat{k}_d \hat{k}_a \frac{1}{\gamma_a} e^{-i k^2 x'_d + i \gamma_a z'},
\]

and

\[
\Gamma^S(\mathbf{K}_T, \mathbf{z}; \mathbf{x}', \mathbf{z}') = \frac{k^2}{2 \rho_0 \omega^2} (\mathbf{I} - \hat{k}_d \hat{k}_d) \frac{1}{\gamma_d} e^{-i k^2 x'_d + i \gamma_d z'},
\]

where \( \hat{n} \) is the outer normal of the surface.

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