Acquisition aperture correction in the angle domain toward true-reflection reverse time migration

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ABSTRACT

Due to incomplete aperture coverage and complex overburden structures, the migration process cannot provide a true-amplitude image even though a true-amplitude propagator is used. Amplitude compensation based on source-side illumination ignores the aperture effects on the receiver side, and it may fail to recover the true-reflection/scattering strength of a geologic structure from the image. The structural dip largely controls if the wave incident on the structure can be reflected back and received by the acquisition aperture, so it should be taken into account in removing the acquisition effects from the migration image. We derived a dip-angle domain amplitude correction from the resolution theory. The stacked migration image created by reverse time migration was decomposed into common dip images, which were compensated individually by the corresponding amplitude correction factor. Then, we summed up the corrected images to form a final image. To construct the amplitude correction factor, we generated a monofrequency Green’s function at the shot/geophone location and further decomposed it into incident/scattered plane waves. They were combined based on Snell’s law to construct correction factors for different dips. The final amplitude correction factor was formed by visiting all the shot/geophone pairs in the observation system. We devised efficient algorithms to make the amplitude correction more practical. We evaluated two numerical experiments, a five-layer model and the SEG/EAGE salt model, in which the amplitude correction led to a scalar/pressure image with an amplitude better matching the true impedance contrasts of subsurface structures, especially in areas with steep dips and in subsalt regions.

INTRODUCTION

The ultimate goal of seismic imaging is to provide a reflector map consistent with the real subsurface structure with correct geometric locations of reflectors and the correct image amplitude corresponding to the reflecting/scattering strength. In this context, the basic requirement is that the migration process is of “true amplitude.” Various reasons justify the claim that migration is an adjoint operator, which only undoes the kinematics of the seismic experiment and it is not really amplitude preserving. The limited acquisition aperture combined with complex velocity structures cause irregular illumination to the subsurface regions, leading to biased image amplitude. To tackle this problem, Tarantola (1987) treats the image process as an inverse problem and solves it in an iterative way. These general inverse approaches are typically computation intensive, and they tend toward ill posedness in areas of low illumination. Another approach lies between conventional migration and general inversion. This method, called true-amplitude or true-reflection imaging, retrieves the true reflectivity by applying a deconvolution operator to the migration image. The deconvolution operator can be predicted based upon the migration model and acquisition configuration.

One important ingredient of true-amplitude/reflection imaging is the true-amplitude propagator. By solving the full wave equation, reverse time migration (RTM) honestly preserves the wave amplitude during wave propagation, so it is usually chosen as the propagator in true-amplitude/reflection imaging. However, using the true-amplitude propagator alone cannot guarantee a true-amplitude image. Acquisition geometry is proved to be equally or even more important in recovering the true reflectivity (Wu et al., 2004; Cao and Wu, 2005). If the full aperture is available, geometric spreading and transmission loss can be automatically compensated during back propagation. However,
the limited acquisition aperture will cause irretrievable energy loss in the system and lead to incorrect image amplitude. To distinguish between imaging using true-amplitude propagators alone with those pursuing true reflectivity, we call the latter true-reflection imaging. In true-reflection imaging, we try to correct the amplitude of the conventional migration image and make it proportional to the local reflectivity. There are two different approaches in the literature. One is a Hessian-matrix-based approach, which uses the inverse Hessian derived from the theory of least-squares inversion (Tarantola, 1987) as the filter function applied to the original image. Because the inverse Hessian is difficult, if not impossible, to calculate, various approximations have been proposed for this approach (Hu et al., 2001; Rickett, 2003; Guitton, 2004; Plessix and Mulder, 2004; Valenciano et al., 2006, 2009; Du et al., 2012; Fletcher et al., 2012; Vasconcelos and Rickett, 2013). The second approach is the resolution- or illumination-based approach. Gelius et al. (2002) derive the resolution function, which relates the migration image to the physical properties of the medium, and use it to correct the migration image in the wavenumber domain. This approach has been applied to ray-based migration (Gelius et al., 2002; Lecomte et al., 2003, 2008) and one-way wave migration (Xie et al., 2005; Wu et al., 2006). Wu et al. (2004) propose to compensate the image amplitude in the local image matrix, which is in the reflection and dip-angle domain. Wu and Luo (2005) test different approaches and demonstrate that the correction in the dip-angle domain performs the best in eliminating the acquisition effect. Later, the local exponential frame (Jin et al., 2006; Cao and Wu, 2009; Mao and Wu, 2010) was published to efficiently compute angle-domain image and illumination. Some authors (Gherasim et al., 2012; Cao, 2013) propose to calculate the resolution function in an RTM by placing a grid of point scatterers in the model and normalize the image with resolution function in either the reflection angle or dip-angle domain.

In this paper, we develop a dip-angle domain amplitude correction algorithm to retrieve the true impedance contrast of the subsurface structure. First, we review the resolution theory and present the relationship between the image and model parameters in the dip-angle domain. Second, we provide a method to compute a dip-angle domain image from an RTM stacked image. Third, we derive the dip-angle amplitude correction factor from resolution function. Fourth, we discuss the computational issues of amplitude correction. Finally, we use 2D synthetic examples to demonstrate the benefit of dip-angle domain amplitude correction.

![Figure 1](https://example.com/figure1.png)

Figure 1. The schematic diagram showing the modeling and imaging processes. The $k_d$ and $\partial_n$ are plotted in the figure. The polar axis is defined as the negative $z$-axis.

### THEORY

Consider using a survey system composed of a shot at $x_s$ and a geophone at $x_g$ to investigate subsurface target region $V(x)$ in the vicinity of location $x$ (see Figure 1). The shot sends out a signal, which propagates to the target region, interacts with the reflectors within the target region, and generates reflected/scattered waves. With the single-backscattering approximation, the data received at the geophone can be expressed as (Wu et al., 2007)

$$D(x_g; x_s, \omega) = s(\omega) \int_{V(x)} k_0^2(\omega)G(x'; x_s, \omega)M(x')G(x'; x_g, \omega)dx',$$

(1)

where $\omega$ is the angular frequency, $k_0(\omega) = \omega/c_0(x')$ is the local wavenumber corresponding to the local background velocity $c_0(x')$, $M(x')$ is the model perturbation, $s(\omega)$ is the source spectrum, $G(x'; x_s, \omega)$ is the Green’s function from shot location $x_s$ to the target location $x'$, and $G(x'; x_g, \omega)$ is the Green’s function from geophone location $x_g$ to the target location $x'$. Both of them are calculated in the background model. The reciprocity $G(x'; x_g, \omega) = G(x_s; x'_g)$ is used here.

We assume that the image at $x$ is created by the seismic data, which only considers the reflection from the local region surrounding $x$ (see Figure 1). Therefore, for a system composed of multiple shots and geophones, the image can be formed as

$$I(x, \omega) = \sum_{x_s} s(\omega)G(x; x_s, \omega)\left[\sum_{x_g} D^*(x_g; x_s, \omega)\frac{\partial}{\partial n}G(x; x_g, \omega)\right],$$

(2)

where $G$ is the response due to a unit monopole, $\partial G/\partial n$ is the response of a unit dipole, and $n$ is the normal vector of the recording surface.

Substituting equation 1 into equation 2 yields

$$I(x, \omega) = \int_{V(x)} M(x')R(x, x', \omega)dx',$$

(3)

with resolution function

$$R(x, x', \omega) = k_0^2(\omega)s(\omega)G^*(\omega)$$

$$\times \sum_{x_s} \sum_{x_g} G(x; x_s, \omega)G^*(x'; x_s, \omega)G^*(x'; x_g, \omega)$$

$$\times \frac{\partial}{\partial n}G(x; x_g, \omega),$$

(4)

where two Green’s functions are responsible for the propagation effects on the source side and the other two accounts for the propagation and acquisition aperture effects on the receiver side. The image can be considered as a generalized convolution of the resolution function and the model parameter. In other words, the resolution function maps a point scatterer from the model space to the image space. It is also called the point spread function.

The generalized convolution in space domain corresponds to the multiplication in wavenumber domain. Thus, equation 3 can be transformed to (Xie et al., 2005, 2006; also, see Appendix B).
\( \hat{I}(k_d, x, \omega) = \hat{M}(k_d, x)\hat{R}(-k_d, x, \omega), \)

(5)

where \( \hat{I}(k_d, x, \omega) \), \( \hat{M}(k_d, x) \), and \( \hat{R}(k_d, x, \omega) \) are the local spatial spectra of image, model perturbation, and resolution function. Here, \( k_d \) is the wavenumber related to structural dip \( \theta_d \) and \( \theta_d \) is defined as the polar angle of \( k_d \) in Figure 1.

The purpose of amplitude correction is to retrieve \( M \) from \( I \). Equation 5 implies that it can be done in the \( k_d \)-domain. The \( k_d \) is composed of two parts: characteristic scale \( k_d \) and dip angle \( \theta_d \). The dip angle of a target controls the direction of the reflected/scattered wave when a wave is incident on it. The image amplitude changes with the dip angle because it determines how much reflection/scattering energy can be detected by the geophones in a velocity model. On the contrary, the characteristic scale does not influence the image amplitude as much as the dip angle so it can be neglected in amplitude correction. Therefore, we will convert equation 5 from the \( k_d \)-domain to the \( \theta_d \)-domain using the equations given by Appendix A. For convenience, only 2D derivations are provided, and 3D expressions can be similarly obtained.

A monofrequency wave (frequency \( \omega \)) can detect the local structure whose spectrum is from zero (parallel reflection) up to \( 2k_0(\omega) \) (normal reflection). For the 2D case, the dip component of the model perturbation can be written as (refer to equation A-4)

\[
\hat{M}(\theta_d, x) = \int_0^{2k_0(\omega)} \hat{M}(k_d, x)k_ddk_d.
\]

(6)

For a point scatterer, i.e., a spatial delta function, its spectrum \( \hat{M}(k_d, x) \) is a constant. For a locally planar reflector, its spectrum \( \hat{M}(k_d, x) \) is a constant over \( k_d \) along its dip direction. For a real model, as long as the structure is sharp enough, its local spectrum is nearly constant up to the detecting power of the source wavelet. In other words, \( \hat{M}(k_d, x) \) is independent of \( k_d \), at least up to \( 2k_0(\omega) \). Thus, it can be taken out of the integral in equation 6; i.e.,

\[
\hat{M}(k_d, x) = \frac{\hat{M}(\theta_d, x)}{2k_0(\omega)}.
\]

(7)

Substituting equation 7 into equation 5, we have

\[
\hat{I}(k_d, x, \omega) = \frac{1}{2k_0(\omega)}\hat{M}(\theta_d, x)\hat{R}(-k_d, x, \omega).
\]

(8)

Integrating both sides of equation 8 over \( k_d \) in the polar coordinate yields (refer to equation A-4)

\[
\hat{I}(\theta_d, x, \omega) = \frac{1}{2k_0(\omega)}\hat{M}(\theta_d, x)\hat{R}(\theta_d + \pi, x, \omega).
\]

(9)

As part of the imaging condition, we integrate equation 9 over \( \omega \):

\[
\hat{I}(\theta_d, x) = \hat{M}(\theta_d, x)\hat{A}(\theta_d, x),
\]

(10)

where \( \hat{I}(\theta_d, x) \) is the common dip image (CDI) and

\[
\hat{A}(\theta_d, x) = \int \frac{1}{2k_0(\omega)}\hat{R}(\theta_d + \pi, x, \omega)d\omega
\]

(11)

is the corresponding amplitude correction factor. It is the dip-angle domain illumination energy provided by the acquisition system, so we call it the acquisition dip response (ADR). Note that the conversion from the \( k_d \)-domain to the \( \theta_d \)-domain will introduce a factor of \((1/2k_0^2(\omega))\). For the 3D case, it will be \((3/8k_0^2(\omega))\) \( \sin \theta_j \).

The space-domain model perturbation can be obtained by

\[
M(x) = \int \frac{\hat{I}(\theta_d, x)}{\hat{A}(\theta_d, x)}d\theta_d,
\]

(12)

which represents the amplitude correction in the dip-angle domain. Each CDI is normalized by the corresponding ADR and then is summed up to form the corrected image. In the following sections, we will describe how to decompose the image and resolution function into the dip-angle domain.

**IMAGE DECOMPOSITION**

CDIs are computed after we obtain the full-stacked image. In the 2D case, CDI can be expressed as (refer to equation A-4)

\[
\hat{I}(\theta_d, x) = \int \hat{I}(k_d, x)k_ddk_d,
\]

(13)

where \( \hat{I}(k_d, x) \) is calculated by applying local slant stacking to the stacked image (refer to equation A-1):

\[
\hat{I}(k_d, x) = \int W(x' - x)I(x')e^{ik_d(x' - x)}dx',
\]

(14)

where \( W(x' - x) \) is a local spatial window function centered at \( x \) and \( I(x) \) is the stacked migration image. It is a postimage process, so it is quite efficient.

**ACQUISITION DIP RESPONSE**

A structure characterized by spatial wavenumber \( k_d \) can be detected by a pair of incident-scattered waves (i.e., incident wave is coming from the shot and the scattered wave is coming from the geophone) meeting the following condition:

\[
-k_d = k_s + k_g,
\]

(15)

where \( k_s \) and \( k_g \) are the spatial frequencies of the incident and scattered waves approaching the image point \( x \) and \( k_s \) and \( k_g \) are the scales of \( k_s \) and \( k_g \), and they satisfy the dispersion relation:

\[
k_s = k_g = k_0(\omega).
\]

(16)

In the 2D case, incident angle \( \theta_s \) and scattering angle \( \theta_g \) are defined as the polar angles of \( k_s \) and \( k_g \) (see Figure 2). They are related to dip angle \( \theta_d \) and reflection angle \( \theta_r \) by Snell’s law:

\[
\pi + \theta_d = \frac{\theta_s + \theta_g}{2}
\]

(17)

and

\[
\theta_r = \frac{\theta_s - \theta_g}{2}.
\]

(18)

Because \( k_s \) is closely related to \( k_s \) and \( k_g \), we start from equation 4 and Fourier transform it to the local \( (k_s \) and \( k_g \) domain:
\[
\hat{R}(\mathbf{k}_x, \mathbf{k}_y, \mathbf{x}, \omega) = k_0^2(\omega)s(\omega) s^*(\omega) \\
\times \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_g} \hat{G}(\mathbf{k}_x, \mathbf{x}; \mathbf{x}_s, \omega) \hat{G}^*(\mathbf{k}_y, \mathbf{x}; \mathbf{x}_g, \omega) \hat{G}(\mathbf{k}_y, \mathbf{x}; \mathbf{x}_g, \omega) \hat{G}^*(\mathbf{k}_x, \mathbf{x}; \mathbf{x}_s, \omega), \\
\times \frac{\partial}{\partial \theta_g} \hat{G}(\mathbf{k}_y, \mathbf{x}; \mathbf{x}_g, \omega), \tag{19}
\]

where \(\hat{G}(\mathbf{k}_x, \mathbf{x}; \mathbf{x}_s, \omega)\) and \(\hat{G}(\mathbf{k}_y, \mathbf{x}; \mathbf{x}_g, \omega)\) are the local wavenumber components of incident and scattered Green’s functions.

Integrating equation 19 over \(k_x\) and \(k_y\) in the polar coordinate results in a resolution matrix as a function of the incident and scattering angle (refer to equation A-4):

\[
\hat{R}(\theta_x, \theta_y, \mathbf{x}, \omega) = \int \hat{R}(\mathbf{k}_x, \mathbf{k}_y, \mathbf{x}, \omega) k_x dk_x k_y dk_y. \tag{20}
\]

The integrand in equation 20 merely has contributions at which the dispersion relation (equation 17) is satisfied. It is equivalent to introducing \(\delta(k_x - k_0)\delta(k_y - k_0)\) into the integral:

\[
\hat{R}(\theta_x, \theta_y, \mathbf{x}, \omega) = \int \delta(k_x - k_0) \delta(k_y - k_0) \hat{R}(\mathbf{k}_x, \mathbf{k}_y, \mathbf{x}, \omega) k_x dk_x k_y dk_y, \tag{21}
\]

where \(k_x\) and \(k_y\) are eliminated. Hence, the resolution matrix will be reduced to \(\hat{R}(\theta_x, \theta_y, \mathbf{x}, \omega) = k_0^2(\omega)s(\omega) s^*(\omega)\):

\[
\sum_{\mathbf{x}_s} \sum_{\mathbf{x}_g} \hat{G}(\theta_x, \mathbf{x}; \mathbf{x}_s, \omega) \hat{G}^*(\theta_x, \mathbf{x}; \mathbf{x}_g, \omega) \hat{G}(\theta_y, \mathbf{x}; \mathbf{x}_g, \omega) \hat{G}^*(\theta_y, \mathbf{x}; \mathbf{x}_s, \omega) \\
\times \frac{\partial}{\partial \theta_g} \hat{G}(\theta_y, \mathbf{x}; \mathbf{x}_g, \omega), \tag{22}
\]

where \(\hat{G}(\theta_x, \mathbf{x}; \mathbf{x}_s, \omega)\) and \(\hat{G}(\theta_y, \mathbf{x}; \mathbf{x}_g, \omega)\) are the local incident and scattered plane waves, respectively. They can be obtained by applying local slant stacking to the Green’s function (Xie and Wu, 2002; Xie and Yang, 2008; Yan and Xie, 2012; refer to equation A-1):

\[
\hat{G}(\theta, \mathbf{x}, \omega) = \int W(x' - x) G(x', \omega) e^{i k_0(\omega)\mathbf{k}(x' - x)} dx', \tag{23}
\]

where \(W(x' - x)\) is a spatial window function centered at location \(x, G(x', \omega)\) is the Green’s function initiated at a shot/geophone location, \(\theta\) is the propagation direction of the plane wave, and \(\mathbf{e}\) is the unit vector associated with \(\theta\).

Figure 3 shows the schematic illustration of the resolution matrix. It covers all the possible scenarios of incident-/scattering-angle combinations. Each element represents the directional illumination provided by a pair of incident-scattered plane waves. In the matrix, the horizontal and vertical axes are the incident and scattering angles. The elements along the diagonal direction share the same dip angle but have different reflection angles. Summing up all the elements along the common dip line will produce:

\[
\hat{R}(\theta_d + \pi, \mathbf{x}, \omega) = \int \hat{R}(\theta_d, \theta_g, \mathbf{x}, \omega) d\theta_g. \tag{24}
\]

For commonly used seismic source functions such as the Ricker wavelet, the major energy is carried by waves near the dominant frequency. Therefore, ADR in equation 11 can be approximately computed with the resolution matrix at the dominant frequency \(\omega_0\):

\[
\hat{A}(\theta_d, \mathbf{x}) = \frac{1}{2k_0^2(\omega_0)} \int \hat{R}(\theta_d, \theta_g, \mathbf{x}, \omega_0) d\theta_g. \tag{25}
\]

**COMPUTATIONAL ISSUES**

The amplitude correction requires computing CDI and ADR. The dip components of an image are calculated by applying slant stacking to the local image selected by a window function. The same operation is repeated for every image point until the entire model is scanned.
ADR is formulated with the angle components of Green’s functions. As mentioned in the previous sections, ADR is calculated at the dominant frequency. Thus, we generate the Green’s function by running full-wave forward modeling at the shot/geophone location and extract the dominant-frequency component. The monofrequency Green’s function is decomposed into angle components in a similar fashion as image decomposition. Though the decomposition is efficient, it still involves a lot of computation to repeat the procedure for every possible shot/geophone location in the observation system.

To speed up the algorithm, we need to carefully consider the factors affecting the computational complexity of ADR. They include the number of shots/geophones, the size of the velocity model, among others. First, in a seismic survey, many shot/geophone locations are visited more than once. To avoid repetitive computation, we calculate a set of Green’s functions, no matter if they are from the shot or geophone, further decompose them, and output them to disk. They will be matched later to construct ADR. Second, in the image process, if the seismic array is not sufficiently dense, the image may lose resolution and/or it may be affected by noises. However, ADR is less sensitive to data spatial sampling. If we decimate the shots/geophones to a certain degree, reasonable accuracy can still be achieved. Third, ADR is a slowly varying function compared with the migration image. This fact allows us to compute ADR at a sparse mesh grid and then interpolate it into the image grid.

### NUMERICAL EXAMPLES

In the first example, we carry out a seismic experiment on a simple five-layer model (shown in Figure 4a) to test the accuracy of the amplitude correction. In the observation system, 201 surface shots are used and the shot interval is 0.05 km. Each shot is matched with 201 double-sided receivers with a maximum offset of 2 km. The source time function is a 20-Hz Ricker wavelet. The synthetic seismic data are modeled by the finite-difference method and then migrated with RTM. Figure 4b is the conventional RTM image. For comparison, shown in Figure 5 is the RTM image compensated by source illumination, which is the total energy of the source wavefields from all of the shots. The image is decomposed into 24 dip angles. Superimposed on the image are the comparisons of the image amplitude and the theoretical normal reflectivity for the five layers.

![Figure 4](image1.png)  
**Figure 4.** (a) The five-layer velocity model and (b) the regular RTM image.

![Figure 5](image2.png)  
**Figure 5.** The migration image compensated by source illumination (the total energy of the source-side wavefields from all of the shots). The vertical profiles are the velocity contrasts (red) and the image amplitudes (blue).

![Figure 6](image3.png)  
**Figure 6.** The migration image compensated by the ADR. Overlapped are the theoretical normal reflectivity (red) and the image amplitudes (blue).
selected vertical lines. Although the image amplitudes of near-horizontal reflectors agree well with the true reflectivity, the amplitudes of steep reflectors show apparent discrepancies from the true reflectivity. Figure 6 compares the RTM image compensated by ADR with the theoretical normal reflectivity. The image amplitudes of the steep reflectors are boosted up. For a reflector with changing dips with position, the image amplitudes are almost constant. This shows the great advantage of ADR correction over source illumination correction.

Next, we conduct the amplitude correction on the 2D SEG/EAGE salt model (Figure 7a). The observation system for the model is composed of 325 shots with an interval of 48.768 m. Each shot is matched with 176 left-side receivers separated by 24.384 m. The shot records are modeled with the finite-difference method with a 15-Hz Ricker wavelet. The migration image and ADR are calculated based on the true-velocity model. The image is computed at a mesh grid of 24.384 × 24.384 m. The shot and receiver increments used to calculate ADR are 48.768 m. ADR is calculated at a mesh grid of 48.768 × 48.768 m. The window size for wave decomposition is 195.072 × 195.072 m. With this set of parameters, source illumination correction does not introduce almost any extra cost, whereas imaging with ADR correction takes 2.5 times the cost of regular RTM. Figure 7b shows the image resulting from regular RTM. Figure 8 shows the source illumination and the image corrected by it. Though the amplitudes of the images are more balanced than the regular image, the aperture effects and propagation effects from the target to geophones are ignored. To conduct the image correction in the dip-angle domain, we decompose the stacked migration image into 24 CDIs, with four samples shown in Figure 9. Figure 10 displays the corresponding ADRs for the selected dips. By comparing the CDIs and ADRs, we notice that the amplitude of CDI is consistent with the strength of ADR. We correct individual CDIs with the ADRs, and the results are shown in Figure 11. Then, all the corrected images are summed up to form the final image, which is shown in Figure 12. After the correction, the image amplitudes of reflectors with different dips become more balanced and the
Figure 9. CDIs for selected dip angles with (a) $-15^\circ$, (b) $15^\circ$, (c) $-45^\circ$, and (d) $45^\circ$.

Figure 10. ADRs for selected dip angles with (a) $-15^\circ$, (b) $15^\circ$, (c) $-45^\circ$, and (d) $45^\circ$.

Figure 11. The corrected CDIs for selected dip angles with (a) $-15^\circ$, (b) $15^\circ$, (c) $-45^\circ$, and (d) $45^\circ$. 
structures appear more continuous. The subsalt structures are greatly enhanced, particularly for steep structures. Figure 13 compares the images corrected by source illumination (Figure 13a) and ADR (Figure 13b). Superimposed on the images are the image amplitudes versus the theoretical normal reflectivity at certain well logs. In Figure 13a, the image amplitudes in the shallow sedimentary layer are generally proportional to the theoretical values. However, the amplitudes are much smaller in the deeper area, particularly in the subsalt region, in which serious illumination problems exist due to the limited effective aperture. In Figure 13b, over the entire model, the image amplitudes match with the true impedance contrast very well.

The two 2D examples demonstrate that the extra cost introduced by ADR correction has the same order of magnitude as the cost of regular RTM. However, when ADR correction is applied to 3D velocity volume, its cost will overpowers regular RTM. In practice, we have to make stronger numerical approximations to make it computationally affordable, e.g., doubling the shot space of forward modeling and/or adopting zero-window overlapping for wave decomposition.

CONCLUSIONS

We formulate true-reflection imaging as a process of regular RTM plus a postimage amplitude correction in the dip-angle domain. ADR, as the amplitude correction factor, strongly depends on the overburden velocity structure, the acquisition geometry, and the target dip. We conduct full-wave forward modeling at the shot/geophone location and decompose the resultant wavefield into local incident/scattered plane waves using local slant stacking. ADR for one shot-geophone pair is formed as the energy contributed from local incident and scattered plane waves satisfying Snell’s law for a given dip. This process is repeated for many shot-geophone pairs in the observation system until the final ADR is constructed. The full-stacked RTM image is decomposed by local slant stacking into CDIs, which are normalized individually with the corresponding ADRs and then are stacked to form the final image. Compared with source illumination compensation, ADR correction shows significant improvement in relating the image amplitude to the true impedance contrasts.

The image amplitude after ADR correction represents the average reflecting/scattering strength over the illuminated reflection angle. In fact, the reflecting/scattering strength is reflection-angle dependent. By decomposing the migration and the resolution function into the reflection-angle domain, we are able to output true-amplitude angle gathers, which are quite useful in amplitude versus angle analysis.

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APPENDIX A

CONVENTION OF SPACE-WAVENUMBER OR SPACE-ANGLE REPRESENTATION

Assuming \( f(x) \) to be an arbitrary space-varying function, we define its wavenumber component localized at \( x \) such that

\[
\hat{f}(k, x) = \int W(x' - x)f(x')e^{ik(x' - x)}dx',
\]

where \( W(x' - x) \) is the space sampling window centered at \( x \) and it is defined such that
True-reflection RTM

\[ W(x' - x) = \begin{cases} 1, & |x' - x| \leq C, \\ 0, & |x' - x| > C, \end{cases} \quad \text{(A-2)} \]

in which \( C \) is a constant.

The original function can be reconstructed with the wavenumber components

\[ f(x) = \int \hat{f}(k, x) dk, \quad \text{(A-3)} \]

We expand this equation in the 2D polar coordinate:

\[ f(x) = \int \hat{f}(k, x) k dk d\Omega, \quad \text{(A-4)} \]

or in the 3D spherical coordinate:

\[ f(x) = \int \hat{f}(k, x) k^2 \sin \theta dk d\Omega, \quad \text{(A-5)} \]

where \( k \) is the absolute value of \( k; \theta \) is the polar angle in the 2D polar coordinate; \( \Omega = \Omega(\theta, \phi) \) is the solid angle in the 3D spherical coordinate, in which \( \theta \) is the polar angle; and \( \phi \) is the azimuth angle.

We define the angle components in the following way so that the original function \( f(x) \) can be reconstructed by summing up all the angle components. For the 2D case,

\[ \hat{f}(\theta, x) = \int \hat{f}(k, x) k dk, \quad \text{(A-6)} \]

and for the 3D case,

\[ \hat{f}(\Omega, x) = \int \hat{f}(k, x) k^2 \sin \theta dk. \quad \text{(A-7)} \]

**APPENDIX B**

**THE REPRESENTATION OF RESOLUTION THEORY IN THE SPACE-WAVENUMBER DOMAIN**

Equation 3 can be rewritten in a different form:

\[ I(x, \omega) = \int W(x' - x) M(x') R(x, x', \omega) dx'. \quad \text{(B-1)} \]

in which a window function \( W \) is introduced to represent the integration in the vicinity of \( x \) and its definition is the same as equation A-2; \( I, M, \) and \( R \) are all space-varying functions and their localized wavenumber components are defined in equation A-1. Integrating \( \hat{I}(k, x, \omega) \) in the \( k \)-domain produces

\[ \int \hat{I}(k, x, \omega) dk = \int dk \int W(x' - x) M(x') e^{ik(x' - x)} dx', \]

\[ = I(x, \omega). \quad \text{(B-2)} \]

Integrating \( \hat{M}(k, x) \hat{R}(-k, x, \omega) \) in the \( k \)-domain generates

\[ \int \hat{M}(k, x) \hat{R}(-k, x, \omega) dk = \int dk \int W(x' - x) M(x') e^{ik(x' - x)} dx', \]

\[ \int W(x' - x) R(x, x', \omega) e^{-ik(x' - x)} dx' \]

\[ = \int W(x' - x) M(x') R(x, x', \omega) dx'. \quad \text{(B-3)} \]

Note that the property \( W^2(x' - x) = W(x' - x) \) is used in the above derivation.

Equation B-1 links the right sides of equations B-2 and B-3; thus,

\[ \int \hat{I}(k, x, \omega) dk = \int \hat{M}(k, x) \hat{R}(k, x, \omega) dk. \quad \text{(B-4)} \]

And it further leads to

\[ I(k, x, \omega) = \hat{M}(k, x) \hat{R}(k, x, \omega). \quad \text{(B-5)} \]

**REFERENCES**


