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A Generalized Reflection-Transmission Coefficient Matrix Method To Calculate Static Displacement Field Of A Dislocation Source In A Stratified Half-Space

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The generalized reflection-transmission coefficient matrix method for calculating synthetic seismograms is extended to deal with the static problems. A method of calculating the static displacement field for a dislocation source in a stratified half space is developed. The advantages of the original reflection-transmission coefficient matrix method are retained. By comparing numerical results with analytical solutions, it is shown that this is an effective method for investigating the static deformation produced by seismic faulting process.

Key words: Stratified half-space, Dislocation source, Static displacement field.

I. INTRODUCTION

During the occurrence of strong earthquakes the faulting process will produce, apart from the radiation of elastic waves, large scales of permanent deformations near the epicentral area. These deformations are near source seismic data with zero frequency. Detailed analysis of deformation data can provide a large amount of information on the faulting process. To study the permanent deformations due to faulting process one must have a method of solving the static response of the media to a dislocation source. For this purpose Maruyama (1964) presented the analytical expressions for the displacement field of a point dislocation in infinite or semi-infinite media. Press (1965) derived the displacement and strain field on the free surface of a semi-infinite homogeneous elastic half-space produced by a vertical finite fault. Savage and Hastie (1966, 1969), Mansinha and Smylie (1971) gave the compact analytic expressions for displacement field in semi-infinite elastic half-space due to an arbitrarily oriented rectangular fault. Chen et al (1975) extended the solution to the case in which two Lame's constants were not necessarily equal to each other.

Normally, the earth's crust is composed of different materials. If the relative differences between the elastic constants for different layers are large, the displacement field on the surface will be seriously affected by the layered structure (e.g. Kasahara, 1964; Sato and Matsu'ura, 1973; Xu and Su, 1988), and the stratified model has to be considered. Ben Menaham and Singh (1968), Singh (1970), Sato (1971), Sato and Matsu'ura (1973) and Xu and Zhou (1982) presented methods based on the Haskell's matrix method for calculating the static displacement field of a point dislocation in a stratified half-space. For calculating the synthetic seismograms, Kennett (1974, 1980), Kennett and Kerry (1979) developed a generalized reflection-transmission coefficient matrix method, which deal with the 2×2 matrix instead of the 4×4 matrix used in Haskell's matrix method. Bouchon (1981) developed a discrete wave number integration method to efficiently process the wave number integration. Yao and Harkrider (1983), Yao and Zheng (1984) took advantages of the above-mentioned methods and developed a method of generalized reflection-transmission coefficient matrix and discrete wave number (RTDW) method for calculating near source Green's functions. Compared with the method based on Haskell's matrix, this is a faster and more accurate method. In this paper the RTDW method for calculating synthetic seismograms is extended to deal with the static problems and a method of calculating the static displacement field of a point dislocation in a stratified half-space is developed. Accordingly, we call it the static reflection-transmission coefficient matrix and discrete wave number integration (SRTDW) method. The advantages of the original RTDW method are preserved. By comparing numerical results with analytical solutions, it is shown that this is an effective method for investigating static deformations produced by seismic faulting process. From careful analysis, we find that there are both similarities and differences between the wave propagation and static problems.

II. ELASTIC STATIC EQUATION AND ITS SOLUTION

Let the time dependent inertial term in the elastodynamic equation be zero, we can obtain the static equilibrium equation

$$(\lambda + \mu)\nabla\nabla \cdot \boldsymbol{u} + \mu\nabla^2\boldsymbol{u} = 0, \tag{1}$$

Introduce the (B_m, C_m, P_m) coordinates

$$\begin{aligned} B_m &= \left(e_r \frac{\partial}{\partial\zeta} + e_\theta \frac{1}{\zeta}\frac{\partial}{\partial\theta}\right) J_m(\zeta)e^{im\theta}, \\ C_m &= \left(e_r \frac{1}{\zeta}\frac{\partial}{\partial\theta} - e_\theta \frac{\partial}{\partial\zeta}\right) J_m(\zeta)e^{im\theta}, \\ P_m &= e_z J_m(\zeta)e^{im\theta}. \end{aligned} \tag{2}$$

into the cylindrical coordinates e_r, $e_\theta e_z$, the solution of Equation (1) can be expressed in the form of integrals

$$u = \sum_{m=0}^{\infty} \int_0^{\infty} u_m k\,dk, \tag{3}$$

$$\sigma = \sum_{m=0}^{\infty} \int_0^{\infty} \sigma_m k\,dk. \tag{4}$$

In Equations (3) and (4), **u** is the displacement field, σ is the e_z component of the stress,

$$u_m = q_m B_m + w_m P_m + v_m C_m, \tag{5}$$

$$\sigma_m = \tau R_m B_m + \sigma R_m P_m + \tau L_m C_m. \tag{6}$$

For solving the vector equation, displacement potentials

$$\begin{cases} \phi = \sum_{m=0}^{\infty} \int_0^{\infty} \phi_m J_m(kr)e^{im\theta} k\ dk, \\ \psi = \sum_{m=0}^{\infty} \int_0^{\infty} \psi_m J_m(kr)e^{im\theta} k\ dk, \\ \chi = \sum_{m=0}^{\infty} \int_0^{\infty} \chi_m J_m(kr)e^{im\theta} k\ dk. \end{cases} \tag{7}$$

are introduced, which satisfy the scalar Laplace equation. Borrowing the representation in the field of seismic wave propagation, the potentials can be written as

$$\begin{cases} \phi_m = \phi_m^-(z) + \phi_m^+(z) = A_m^- e^{k(z-z_0)} + A_m^+ e^{-k(z-z_0)}, \\ \psi_m = \psi_m^-(z) + \psi_m^+(z) = B_m^- e^{k(z-z_0)} + B_m^+ e^{-k(z-z_0)}, \\ \chi_m = \chi_m^-(z) + \chi_m^+(z) = C_m^- e^{k(z-z_0)} + C_m^+ e^{-k(z-z_0)}, \end{cases} \tag{8}$$

From Ben Menaham and Singh (1968), Singh (1970), the relations between the displacements and displacement potentials in (B_m, C_m, P_m) coordinates can be expressed as

$$
\begin{cases}
q_m = \phi_m - \left[1 + 2\Delta(z - z_0)\dfrac{\partial}{\partial z}\right]\phi_m, \\
w_m = \dfrac{1}{k}\dfrac{\partial \phi_m}{\partial z} + \dfrac{1}{k}\left[\dfrac{\partial}{\partial z} - 2\Delta k^2(z - z_0)\right]\phi_m, \\
v_m = \chi_m.
\end{cases}
\tag{9}
$$

Similarly, for the e_z component of the stress field we have

$$
\begin{cases}
\tau_{Rm} = 2\mu\left\{\dfrac{\partial \phi_m}{\partial z} - \Delta\left[\dfrac{\partial}{\partial z} + 2k^2(z - z_0)\right]\phi_m\right\}, \\
\sigma_{Rm} = 2\mu k\left\{\phi_m + \Delta\left[1 - 2(z - z_0)\dfrac{\partial}{\partial z}\right]\phi_m\right\}, \\
\tau_{Lm} = \mu\dfrac{\partial \chi_m}{\partial z}.
\end{cases}
\tag{10}
$$

in Equations (9) and (10) $\Delta = (\lambda + \mu)/(\lambda + 3\mu)$. Hence we have obtained the general solution of the elastodynamic equation in terms of the displacement potentials. Next we shall determine the coefficients in these expressions.

III. THE SOURCE COEFFICIENTS OF A STATIC POINT DISLOCATION

The source coefficients of a static point dislocation will be expressed in terms of the coefficients of displacement potentials, which are similar to that used in dealing with the wave radiation problems, and are therefore convenient to combine the wave motion data in the investigating of faulting process.

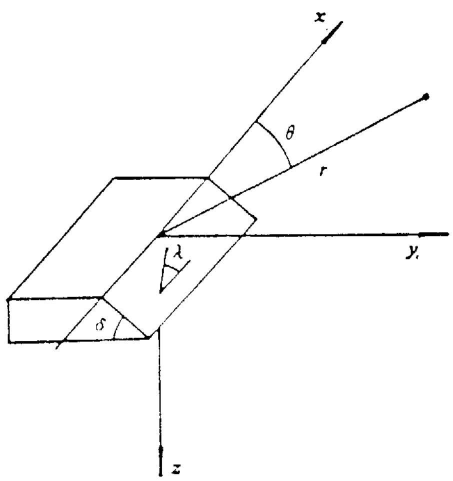

Fig. 1. The geometry of the fault and the receiver. In which δ and λ are dip and rake of the fault, θ is the angle from the fault strike to direction of the receiver.

Consider a fault with dip δ and rake λ shown in Figure 1. The angle from the strike to the direction of receiver is θ. In a homogeneous infinite space, the static displacement field due to a point dislocation is

$$\boldsymbol{u} = \frac{M_0}{\mu} \nu_i T_{ij} n_j , \tag{11}$$

in which

$$\boldsymbol{\nu} = (\cos\lambda,\ -\sin\lambda\cos\delta,\ -\sin\lambda\sin\delta) \tag{12}$$

$$\boldsymbol{n} = (0,\ \sin\delta,\ -\cos\delta) \tag{13}$$

are unit vector along the slip direction and normal vector of the fault surface, respectively. T_{ij} is the displacement field produced by a unit dislocation. Their detailed expressions are referred to Ben Manaham and Singh (1968). $M_0 = \mu \Delta D ds$ is the seismic moment of the point source, ΔD is the scalar dislocation, ds is the area of the fault element. Replacing the left-hand side of Equation (11) with the general solution given in section II and the right-hand side by its detailed expressions, the coefficients in displacement potentials can be determined by comparing the coefficients of both side in B_m, C_m, P_m coordinates. Finally, the displacement potentials for a static point dislocation can be obtained as

$$\left\{\begin{aligned}
\phi &= \frac{M_0}{4\pi} \sum_{m=0}^{2} A_m(\lambda,\delta,\theta) \int_0^\infty P_m e^{-k|z-h|} J_m(kr) k\,dk, \\
\psi &= \frac{M_0}{4\pi} \sum_{m=0}^{2} A_m(\lambda,\delta,\theta) \int_0^\infty SV_m e^{-k|z-h|} J_m(kr) k\,dk, \\
\chi &= \frac{M_0}{4\pi} \sum_{m=1}^{2} A_{m+3}(\lambda,\delta,\theta) \int_0^\infty SH_m e^{-k|z-h|} J_m(kr) k\,dk.
\end{aligned}\right. \tag{14}$$

where h is the z-coordinate of the source,

$$\left\{\begin{aligned}
A_0 &= \frac{1}{2}\sin\lambda\sin 2\delta, \\
A_1 &= \cos\lambda\cos\delta\cos\theta - \sin\lambda\cos 2\delta\sin\theta, \\
A_2 &= \frac{1}{2}\sin\lambda\sin 2\delta\cos 2\theta + \cos\lambda\sin\delta\sin 2\theta, \\
A_4 &= -\cos\lambda\cos\delta\sin\theta - \sin\lambda\cos 2\delta\cos\theta, \\
A_5 &= \cos\lambda\sin\delta\cos 2\theta - \frac{1}{2}\sin\lambda\sin 2\delta\sin 2\theta,
\end{aligned}\right. \tag{15}$$

are radiation patterns, the source coefficients in Eq.(14) are

$$\begin{cases} P_0 = \dfrac{-1+4\Delta}{2\mu(1+\Delta)}, & SV_0 = -\dfrac{3}{2\mu(1+\Delta)}, & SH_0 = 0, \\ P_1 = -\dfrac{\varepsilon\Delta}{\mu(1+\Delta)}, & SV_1 = \dfrac{\varepsilon}{\mu(1+\Delta)}, & SH_1 = \dfrac{\varepsilon}{\mu}, \\ P_2 = \dfrac{1}{2\mu(1+\Delta)}, & SV_2 = -\dfrac{1}{2\mu(1+\Delta)}, & SH_2 = -\dfrac{1}{\mu}. \end{cases} \tag{16}$$

$$\varepsilon = \begin{cases} -1, & \text{for - superscript;} \\ 1, & \text{for + superscript.} \end{cases}$$

The terms with subscripts 0, 1 and 2 are contributions from 45 degree dip-slip fault, dip-slip fault and strike-slip fault, respectively. Contribution from fault with arbitrary orientations can be expressed as a linear combination of the three basic faults. After obtaining the source coefficients for a point dislocation, we can discuss the response for a layered media.

IV. STATIC REFLECTION-TRANSMISSION COEFFICIENTS AND STATIC DISPLACEMENT SOLUTIONS IN HORIZONTALLY LAYERED MEDIA

For multilayered media, the existence of the interfaces between different materials will cause the reflection and transmission of elastic waves on these interfaces. For static problems there are no reflection or transmission in their original sense, but the existence of the interface will still affects the displacement field in both sides. Therefore we can extend the original concepts of reflection and transmission to static problem, and correspondingly call them the static reflection and transmission coefficients.

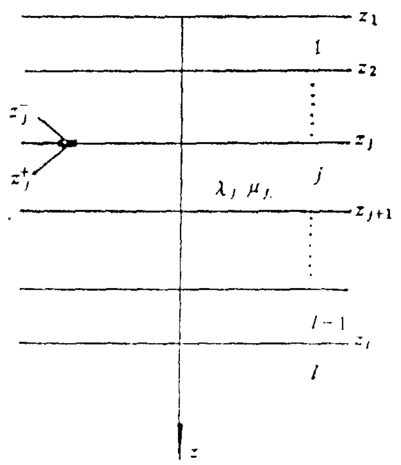

Fig. 2. Horizontally layered media model.

The geometry of the multilayered media is shown in Figure 2. The jth layer is between the interfaces z_j and z_{j+1}, with the thickness of $d_j = z_{j+1} - z_j$, the medium is homogeneous inside one layer and characterized by two Lame's constants λ_j and μ_j. $z_j^- = z_j + 0^-$, $z_j^+ = z_j + 0^+$ are points approaching the interface z_j from both the upper and lower sides.

In (B_m, C_m, P_m) coordinates, the B and P components of the displacement field could be decoupled with the C component, consequently, the two parts can be dealt separately. The former is similar to *P-SV* wave in wave propagation problems and the latter is equivalent to *SH* wave. We shall call them R problem and L problem according to Singh (1970), and introduce two kinds of displacement-stress vectors

$$s_{Rm}(z) = (q_m, w_m, \sigma_{Rm}, \tau_{Rm})^{\mathrm{T}} = \begin{bmatrix} w_m \\ \sigma_m \end{bmatrix}, \tag{17}$$

$$s_{Lm}(z) = (v_m, \tau_{Lm})^{\mathrm{T}}, \tag{18}$$

and displacement potential vectors

$$\phi_m(z) = (\phi_m^-, \psi_m^-, \phi_m^+, \psi_m^+)^{\mathrm{T}} = \begin{bmatrix} \phi_m^- \\ \phi_m^+ \end{bmatrix}, \tag{19}$$

$$\chi_m(z) = (\chi_m^-, \chi_m^+)^{\mathrm{T}}. \tag{20}$$

respectively. From Eqs.(9) and (10), the relations between displacement-stress vector and displacement potential vector in the jth layer are (we temporally dropped the subscripts m for simplicity)

$$s_R(z) = \mathbf{D}_j(z)\phi(z), \tag{21}$$

$$s_L(z) = \mathbf{T}_j(z)\chi(z). \tag{22}$$

where

$$\mathbf{D}_j(z) = \begin{bmatrix} 1 & -[1 + 2k\Delta_j(z - z_j)] & 1 & -[1 - 2k\Delta_j(z - z_j)] \\ 1 & [1 - 2k\Delta_j(z - z_j)] & -1 & -[1 + 2k\Delta_j(z - z_j)] \\ \xi_j & \xi_j\Delta_j[1 - 2k(z - z_j)] & \xi_j & \xi_j\Delta_j[1 + 2k(z - z_j)] \\ \xi_j & -\xi_j\Delta_j[1 + 2k(z - z_j)] & -\xi_j & \xi_j\Delta_j[1 - 2k(z - z_j)] \end{bmatrix}, \tag{23}$$

$$\mathbf{T}_j(z) = \begin{bmatrix} 1 & 1 \\ \xi_j/2 & -\xi_j/2 \end{bmatrix}, \tag{24}$$

and

$$\xi_j = 2\mu_j k.$$

We shall discuss the R problem first. Using the concept of Q matrix (see e.g. Yao and Zheng 1984), the displacement potentials at any two interfaces z_i and z_j (assuming there is no source interface between them) has the relation

$$\phi(z_j^+) = \mathrm{Q}(z_j^+, z_i^+)\phi(z_i^+), \tag{25}$$

where

$$\begin{aligned}
&\mathrm{Q}(z_j^+, z_i^+) = \mathrm{Q}(z_j^+, z_{j+1}^+)\mathrm{Q}(z_{j+1}^+, z_{j+2}^+)\cdots\mathrm{Q}(z_{i-1}^+, z_i^+),\\
&\mathrm{Q}(z_{k-1}^+, z_k^+) = \exp(-\Lambda_{k-1}d_{k-1})\mathrm{D}_{k-1}^{-1}\mathrm{D}_k,\\
&\exp(-\Lambda_k d_k) = \begin{bmatrix}\mathrm{E}_k^{-1} & 0\\ 0 & \mathrm{E}_k\end{bmatrix},\\
&\mathrm{E}_k = e^{kd_k}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.
\end{aligned}$$

Similar to wave propagation problems, the elements of matrix $\mathrm{Q}(z_k^-, z_k^+) = \mathrm{D}_{k-1}^{-1}(z_k^-)\mathrm{D}_k(z_k^+)$ can be expressed as

$$\mathrm{Q}(z_k^-, z_k^+) = \begin{bmatrix}\mathrm{T}_U - \mathrm{R}_D\mathrm{T}_D^{-1}\mathrm{R}_U & \mathrm{R}_D\mathrm{T}_D^{-1}\\ -\mathrm{T}_D^{-1}\mathrm{R}_U & \mathrm{T}_D^{-1}\end{bmatrix}, \tag{26}$$

where T_D, R_D, T_U and R_U are downward and upward static reflection-transmission coefficient matrices on interface z_k. Their derivation and detailed expressions are given in the appendix. Matrix $\mathrm{Q}(z_{k-1}^+, z_k^+) = \exp\ (-\Lambda_{k-1}d_{k-1})\ \mathrm{D}_{k-1}^{-1}\mathrm{D}_k$ can be expressed in the form similar to Eq.(26)

$$\mathrm{Q}(z_{k-1}^+, z_k^+) = \begin{bmatrix}\tilde{\mathrm{T}}_U - \tilde{\mathrm{R}}_D\tilde{\mathrm{T}}_D^{-1}\tilde{\mathrm{R}}_U & \tilde{\mathrm{R}}_D\tilde{\mathrm{T}}_D^{-1}\\ -\tilde{\mathrm{T}}_D^{-1}\tilde{\mathrm{R}}_U & \tilde{\mathrm{T}}_D^{-1}\end{bmatrix}, \tag{27}$$

where

$$\begin{cases}\tilde{\mathrm{R}}_D = \mathrm{E}^{-1}\mathrm{R}_D\mathrm{E}^{-1},\\ \tilde{\mathrm{T}}_D = \mathrm{T}_D\mathrm{E}^{-1},\\ \tilde{\mathrm{R}}_U = \mathrm{R}_U,\\ \tilde{\mathrm{T}}_U = \mathrm{E}^{-1}\mathrm{T}_U.\end{cases} \tag{28}$$

For multilayered media, the iterative formula

$$\begin{cases}\mathrm{T}_D^{12} = \mathrm{T}_D^2(\mathrm{I} - \mathrm{R}_U^1\mathrm{R}_D^2)^{-1}\mathrm{T}_D^1,\\ \mathrm{T}_U^{12} = \mathrm{T}_U^1(\mathrm{I} - \mathrm{R}_D^2\mathrm{R}_U^1)^{-1}\mathrm{T}_U^2,\\ \mathrm{R}_D^{12} = \mathrm{R}_D^1 + \mathrm{T}_U^1\mathrm{R}_D^2(\mathrm{I} - \mathrm{R}_U^1\mathrm{R}_D^2)^{-1}\mathrm{T}_D^1,\\ \mathrm{R}_U^{12} = \mathrm{R}_U^2 + \mathrm{T}_D^2\mathrm{R}_U^1(\mathrm{I} - \mathrm{R}_D^2\mathrm{R}_U^1)^{-1}\mathrm{T}_U^2.\end{cases} \tag{29}$$

can be used, where the superscripts 1, 2 and 12 denote the static reflection-transmission matrices between z_1^+ and z_2^+, z_2^+ and z_3^- as well as z_1^+ and z_3^-, respectively.

With the transmit relation for the displacement potential vectors and the expressions for static reflection-transmission matrices in hand and considering the boundary conditions on the free surface z_1 and at the top of the half space z_L

$$s(z_1) = \begin{bmatrix} w(z_1) \\ 0 \end{bmatrix}, \quad \phi(z_L^+) = \begin{bmatrix} 0 \\ \phi^+(z_L^+) \end{bmatrix},$$

and the discontinuity conditions of the displacement potential at z_s to simulate the source

$$\phi(z_s^+) - \phi(z_s^-) = \begin{bmatrix} \Sigma^- \\ \Sigma^+ \end{bmatrix} \tag{30}$$

the static displacement solution for multilayered media can be obtained as (recovering the subscripts m)

$$\begin{bmatrix} q_m \\ w_m \end{bmatrix} = \mathbf{R}_{EV}(\mathbf{I} - \mathbf{R}_D^{RS}\tilde{\mathbf{R}})^{-1}\mathbf{T}_U^{RS}(\mathbf{I} - \mathbf{R}_D^{SL}\mathbf{R}_U^{FS})^{-1}\left[\mathbf{R}_D^{SL}\begin{bmatrix} P_m^+ \\ SV_m^+ \end{bmatrix} + \begin{bmatrix} P_m^- \\ SV_m^- \end{bmatrix}\right]. \tag{31}$$

In deriving (31), we used the similar method in dealing with the wave propagation problems (see Yao and Zheng 1984). Equation (31) is similar to the form in the problems of wave motion fora buried source. In Eq. (31) $\mathbf{R}_{EV}$ is the static receiver function matrix, $\tilde{\mathbf{R}}$ is the static reflection matrix on the free surface (the detailed derivation is given in the appendix), P_m and SV_m are given in Eq.(16), $\mathbf{R}_D^{SL}$ is the static reflection coefficient matrix between z_s^+ and z_L^+. $\mathbf{R}_U^{FS}$, $\mathbf{R}_D^{RS}$ and $\mathbf{T}_U^{RS}$ are the static reflection-transmission coefficient matrices between z_1^+ and z_s^+, which can be calculated by Eqs.(28), (29) and formulas in the appendix. The definitions of $\mathbf{R}_D^{RS}$, $\mathbf{T}_U^{RS}$ and $\mathbf{R}_U^{FS}$ are slightly different from that for the wave motion problems. The latter uses the reflection-transmission coefficient matrices defined between z_1^+ and z_s^-. This difference results from the fact that the matrix $\mathbf{D}_j(z)$ is a function of local coordinate $z - z_j$, which we have mentioned before.

The displacement integrand for L problem can be derived in the similar way, except that the static reflection-transmission coefficient matrix is replaced accordingly

$$v_m = 2(1 - R_{D,L}^{RS})^{-1}T_{U,L}^{RS}(1 - R_{D,L}^{SL}R_{U,L}^{FS})^{-1}(R_{D,L}^{SL}SH_m^+ + SH_m^-), \tag{32}$$

Using Eqs.(2) and (5), and transform the q_m, w_m and v_m back into the cylindrical coordinates, the static displacement field in cylindrical coordinates can be written as

$$\begin{cases} w_z = \dfrac{M_0}{4\pi} \displaystyle\sum_{m=0}^{2} \left\{A_m(\lambda, \delta, \theta)\int_0^\infty w_m J_m(kr)kdk\right\}, \\ q_r = \dfrac{M_0}{4\pi} \displaystyle\sum_{m=0}^{2} \left\{A_m(\lambda, \delta, \theta)\int_0^\infty \left[q_m J_m'(kr) - v_m \dfrac{m}{kr} J_m(kr)\right] kdk\right\}, \\ v_\theta = \dfrac{M_0}{4\pi} \displaystyle\sum_{m=1}^{2} \left\{A_{m+3}(\lambda, \delta, \theta)\int_0^\infty \left[q_m \dfrac{m}{kr} J_m(kr) - v_m J_m'(kr)\right] kdk\right\}. \end{cases} \tag{33}$$

where w_m, q_m, v_m are given by Eqs.(31) and (32).

V. NUMERICAL RESULTS AND DISCUSSIONS

First, we give a brief discussion on the similarity and difference between the static and wave motion problems . In section IV, equations (17) to (33) are similar to their counterparts for wave motion problems, though all the reflection-transmission coefficient matrices are for static problems in this case. Also the detailed expression of source coefficients are different. In equation (23), matrix $\mathbf{D}_j(z)$ is not only characterized by the elastic constants of the jth layer but also dependent on the local coordinate $z - z_j$. This is the basic difference between the static and the wave motion problem. This difference results from the fact that there is no concept of wave speed for static problem, the potentials ϕ and ψ are no longer linearly independent. To obtain another linearly independent solution a function like $z\psi$ is constructed, which produces the terms dependent on $z - z_j$ in $\mathbf{D}_j(z)$. This effect also appears in some other places. For example, in the appendix it can be seen that for a presumed interface between two layers made by the same material, the transmission matrix is not a unit matrix, though the reflection matrix is a null matrix. Also this effect will cause some differences in the definitions of reflection-transmission coefficient matrices in equations (31) and (32).

To verify the validity of the formulas and the accuracy of the numerical algorithm, the homogeneous elastic half-space is simulated by a series of horizontal layers made by the same material, and the static displacement field on the free surface due to a point dislocation is calculated using our numerical algorithm. The results are compared with the analytical solution given by Maruyama (1964). Table 1 lists the analytical solution and integral solution at various epicenter distances. The distances are given by the normalized source depth h. From the table we can see that all the numerical deviations are at or beyond the fourth significant digit, so the numerical accuracy is quite satisfactory.

Table 1

Comparison between the analytical solution and integrated solution

		$r = h$	$2h$	$3h$	$4h$	$5h$
u_θ, $\lambda = 0°$, $\delta = 90°$, $\theta = 90°$	A	0.965437×10^{-2}	0.679673×10^{-2}	0.435762×10^{-2}	0.294141×10^{-2}	0.209775×10^{-2}
	B	0.965439×10^{-2}	0.679674×10^{-2}	0.435762×10^{-2}	0.294143×10^{-2}	0.209774×10^{-2}
u_z, $\lambda = 90°$, $\delta = 90°$, $\theta = 90°$	A	0.844047×10^{-1}	0.170823×10^{-1}	0.452963×10^{-1}	0.160280×10^{-2}	0.692593×10^{-3}
	B	0.844050×10^{-1}	0.170823×10^{-1}	0.452960×10^{-1}	0.160278×10^{-2}	0.692578×10^{-3}
u_r, $\lambda = 90°$, $\delta = 90°$, $\theta = 90°$	A	0.844047×10^{-1}	0.341646×10^{-1}	0.135889×10^{-1}	0.641120×10^{-2}	0.346297×10^{-2}
	B	0.843990×10^{-1}	0.341588×10^{-1}	0.135830×10^{-1}	0.640537×10^{-2}	0.345714×10^{-2}

A analytic B integrated

The above results show that the RTDW method, which is originally used to calculate the synthetic seismograms, can be extended to deal with the static problem. Its efficiency and accuracy can satisfactorily meet the purpose of investigating the deformation data due to seismic faulting process. For a major earthquake, the fault length may extend to several tens or even several hundreds kilometers. In such situations we can divide the fault into a large quantity of elements and calculate the response of each element, then sum up these responses. When the response from the sources at different depths are needed, the fast algorithm suggested by Yao and Zheng (1984) can be adapted. By introducing the complex mudulus into equation (1) and using Fourier transforms, the above formulation can be modified to calculate the qusi-static creep movement of a dislocation in stratified viscoelastic materials, which can be used to investigate the problems such as the fault creep before or after the earthquake.

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APPENDIX

STATIC REFLECTION-TRANSMISSION COEFFICIENT MATRIX AND RECEIVER FUNCTION MATRIX

1. Static reflection-transmission coefficient matrix

Without losing the generality, let the interface be at z_2. The relation between displacement potentials on both sides of the interface is

$$\phi(z_2^+) = \mathbf{D}_2^{-1}(z_2^+)\mathbf{D}_1(z_2^-)\phi(z_2^-). \tag{A1}$$

Let $\mathbf{Q}^{-1} = \mathbf{D}_2^{-1}(z_2^+)\mathbf{D}_1(z_2^-)$. Substituting the detailed expressions of $\mathbf{D}_1(z_2^-)$ and $\mathbf{D}_2^{-1}(z_2^+)$ into this formula, we obtain

$$\mathbf{Q}^{-1} = \left[\begin{array}{cc|cc} a & b & 0 & c \\ 0 & d & e & f \\ \hline 0 & c & a & -b \\ e & -f & 0 & d \end{array}\right] = \begin{bmatrix} \mathbf{Q}_{11}^{-1} & \mathbf{Q}_{12}^{-1} \\ \mathbf{Q}_{21}^{-1} & \mathbf{Q}_{22}^{-1} \end{bmatrix}, \tag{A2}$$

where

$$\begin{cases} a = \dfrac{\mu_1/\mu_2 + \Delta_2}{1 + \Delta_1}, & b = -\dfrac{2\Delta_1 k d_1(\mu_1/\mu_2 + \Delta_2)}{1 + \Delta_2}, \\ c = \dfrac{(\mu_1/\mu_2)\Delta_1 - \Delta_2}{1 + \Delta_2}, & d = \dfrac{(\mu_1/\mu_2)\Delta_1 + 1}{1 + \Delta_2}, \\ e = \dfrac{\mu_1/\mu_2 - 1}{1 + \Delta_2}, & f = \dfrac{2\Delta_1 k d_1(\mu_1/\mu_2 - 1)}{1 + \Delta_2}. \end{cases} \tag{A3}$$

When the disturbance comes from above the z_2 interface, $\phi^{+}(z_2^{-})$, $\phi^{-}(z_2^{-})$ and $\phi+(z_2^{+})$ will not be zero, but $\phi^{-}(z_2^{+}) = 0$. Then we have

$$\begin{bmatrix} 0 \\ \phi^{+}(z_2^{+}) \end{bmatrix} = \begin{bmatrix} Q_{11}^{-1} & Q_{12}^{-1} \\ Q_{21}^{-1} & Q_{22}^{-1} \end{bmatrix} \begin{bmatrix} \phi^{-}(z_2^{-}) \\ \phi^{+}(z_2^{-}) \end{bmatrix}. \tag{A4}$$

For R problem, the downward static reflection and transmission coefficient matrix is defined as

$$\phi^{-}(z_2^{-}) = R_D\phi^{+}(z_2^{-}),$$
$$\phi^{+}(z_2^{+}) = T_D\phi^{+}(z_2^{-}).$$

From (A4) we obtain

$$R_D = \begin{bmatrix} r_{pp}^{+} & r_{sp}^{+} \\ r_{ps}^{+} & r_{ss}^{+} \end{bmatrix} = -Q_{11}Q_{12}^{-1}, \tag{A5}$$

$$T_D = \begin{bmatrix} t_{pp}^{+} & t_{sp}^{+} \\ t_{ps}^{+} & t_{ss}^{+} \end{bmatrix} = Q_{22}^{-1} - Q_{21}^{-1}Q_{11}Q_{12}^{-1}. \tag{A6}$$

Accordingly, when the disturbance comes from below the z_2 interface, we have

$$\begin{bmatrix} \phi^{-}(z_2^{+}) \\ \phi^{+}(z_2^{+}) \end{bmatrix} = \begin{bmatrix} Q_{11}^{-1} & Q_{12}^{-1} \\ Q_{21}^{-1} & Q_{22}^{-1} \end{bmatrix} \begin{bmatrix} \phi^{-}(z_2^{-}) \\ 0 \end{bmatrix}. \tag{A7}$$

For R problem the upward static reflection and transmission coefficient matrix are defined as

$$\phi^{+}(z_2^{+}) = R_U\phi^{-}(z_2^{+}),$$
$$\phi^{-}(z_2^{-}) = T_U\phi^{-}(z_2^{+}).$$

From (A7) we obtain

$$R_U = \begin{bmatrix} r_{pp}^{-} & r_{sp}^{-} \\ r_{ps}^{-} & r_{ss}^{-} \end{bmatrix} = Q_{21}^{-1}Q_{11}, \tag{A8}$$

$$\mathbf{T}_U = \begin{bmatrix} t_{pp}^- & t_{sp}^- \\ t_{ps}^- & t_{ss}^- \end{bmatrix} = \mathbf{Q}_{11}. \tag{A9}$$

Based on equations (A2) to (A9), the detailed expressions for these reflection and transmission coefficient matrices can be obtained as

$$\begin{cases} r_{pp}^+ = be/\Delta_R, & r_{sp}^+ = -(dc - bf)/\Delta_R, \\ r_{ps}^+ = -ae/\Delta_R, & r_{ss}^+ = -af/\Delta_R, \\ t_{pp}^+ = a(ad - ce)/\Delta_R, & t_{sp}^+ = -a(bd + cf)/\Delta_R, \\ t_{ps}^+ = (be + af)e/\Delta_R, & t_{ss}^+ = [(ad - ce)d + (be + af)f]/\Delta_R, \\ r_{pp}^- = 0, & r_{sp}^- = ac/\Delta_R, \\ r_{ps}^- = de/\Delta_R, & r_{ss}^- = (-be + af)/\Delta_R, \\ t_{pp}^- = d/\Delta_R, & t_{sp}^- = -b/\Delta_R, \\ t_{ps}^- = 0, & t_{ss}^- = a/\Delta_R, \end{cases} \tag{A10}$$

where $\Delta_R = ad$.

The L problem can be processed in the similar way, except that all the 2×2 matrices will degenerate into scalars. The result is

$$\begin{aligned} R_{D,L} &= (\mu_1 - \mu_2)/\Delta_L, \\ T_{D,L} &= 2\mu_1/\Delta_L, \\ R_{U,L} &= (\mu_2 - \mu_1)/\Delta_L, \\ T_{U,L} &= 2\mu_2/\Delta_L, \end{aligned}$$

where $\Delta_L = \mu_1 + \mu_2$.

2. The static reflection coefficient matrix and receiver function matrix on the free surface

Near the free surface, for R problem we have

$$\begin{bmatrix} \mathbf{w}(z_1) \\ \mathbf{0} \end{bmatrix} = \mathbf{D}_1(z_1^+) \begin{bmatrix} \boldsymbol{\phi}^-(z_1^+) \\ \boldsymbol{\phi}^+(z_1^+) \end{bmatrix}. \tag{A11}$$

The upward static reflection coefficient matrix $\tilde{\mathbf{R}}$ and receiver function matrix $\mathbf{R}_{EV}$ on the free surface can be defined as

$$\begin{aligned} \boldsymbol{\phi}^+(z_1^+) &= \tilde{\mathbf{R}}\boldsymbol{\phi}^-(z_1^+), \\ \mathbf{w}(z_1) &= \mathbf{R}_{EV}\boldsymbol{\phi}^-(z_1^+). \end{aligned}$$

Through the process similar to the above derivation, we obtain

$$\tilde{R} = -\begin{bmatrix} 0 & \triangle_1 \\ 1/\triangle_1 & 0 \end{bmatrix},$$

$$R_{EV} = (1 + \triangle_1)\begin{bmatrix} 1/\triangle_1 & -1 \\ 1/\triangle_1 & 1 \end{bmatrix}.$$

For L problem, the reflection coefficient on the free surface is 1, and the receiver function is 2.

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