Effects of Laterally Varying Mantle Lid Velocity Gradient and Crustal Thickness on $Pn$ Geometric Spreading with Application to the North Korean Test Site

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Abstract The $Pn$ phase for crustal earthquakes refracts through the uppermost mantle to become the first seismic-wave arrival at distances of $\sim 200$ to $\sim 1500$ km. The amplitude of $Pn$ is particularly sensitive to the mantle lid radial velocity gradient, and the elastic geometric spreading is frequency dependent. Lateral variations in the radial velocity gradient and in the crustal thickness encountered by $Pn$ phases that traverse boundaries of geological provinces further complicate the frequency-dependent geometric spreading behavior. We use 2D finite-difference (FD) calculations to explore the effects of smoothly varying lateral structures in the lid and crust on $Pn$ phases in the 0.4–10 Hz frequency range. $Pn$ geometric spreading for 1D structures with mantle lid gradients is calculated first to ensure that the FD calculations correctly capture the expected frequency dependence. Then, 2D models with smooth laterally varying radial lid gradients are computed to explore how the phase samples the structures to give the overall geometric spreading. The direction of propagation through the laterally varying structure affects the frequency dependence and the transition from source-side to receiver-side dominance in the spreading. Laterally varying crustal thickness is explored in the context of the North Korean nuclear test site, for which many observations of $Pn$ are made by stations with paths either through continental structure or along paths with variation from continental to oceanic to arc structure (in Japan). Frequency-dependent amplitude effects are observed on the paths through oceanic structure. 2D FD calculations for structures with a central region of thin oceanic crust reproduce the observed first-order frequency dependence of $Pn$ along corresponding paths, including rapid amplitude decay for frequencies above 6 Hz. These analyses demonstrate that even though $Pn$ propagation is very complex, modeling approaches can guide interpretations of the behavior when constraints on the structure are available.

Electronic Supplement: Figures of $Pn$-wave snapshots, first $Pn$ arrival times, and frequency dependence of the $Pn$ amplitudes.

Introduction

Regional seismic phases (e.g., $Pn$, $Sn$, $Pg$, $Lg$) play central roles in global monitoring of low-yield underground nuclear tests. Given the typical paucity of teleseismic observations for a small event, regional phase arrival times are often critical for locating the source. Numerous empirical analyses have also demonstrated that regional phase amplitudes are important for small event magnitude and yield estimation and for discrimination between small explosions and earthquakes (e.g., Taylor et al., 1989; Walter et al., 1995; Fisk et al., 1996; Taylor, 1996; Hartse et al., 1997; Kim et al., 1997; Taylor and Hartse, 1997; Patton, 2001; Xie, 2002; Richards and Kim, 2007; Zhao et al., 2008). Use of travel times and amplitudes of regional phases for these location, discrimination, and yield estimation procedures requires that the regional velocity and attenuation structures be sufficiently well approximated (e.g., Phillips et al., 2007; Myers et al., 2010; Yang, 2011). As a result, there have been extensive efforts to determine regional crustal and upper-mantle velocity models and tomographic models for attenuation of regional phases. These efforts commonly proceed independ-
ently, with geometric spreading models being constructed for simple or generic structures, even though specific detailed regional velocity models are determined for travel times. This situation is evolving, with an increasing emphasis on unified model-based procedures rather than on empirical data trends for operational systems. This presents both challenges and opportunities for developing self-consistent model-based procedures. Our focus is on the important regional phase \( Pn \), which traverses the uppermost mantle and commonly is assumed to have very simple wave behavior controlling its amplitudes and travel times despite very large variation in actual observations (e.g., Fisk et al., 2005). In reality, \( Pn \) has complex, nonlinear sensitivity to Earth structure that is not well represented by standard geometric spreading assumptions (particularly when used to determine attenuation models needed for the model-based procedures). This motivates development and use of regionally appropriate velocity structures for computing amplitude effects, similar to what is now done for precise event location.

**\( Pn \) Geometric Spreading**

Accurately accounting for geometric spreading is critical for the development of meaningful regional-phase attenuation models. This is particularly true for \( Pn \) and \( Sn \) waves because the nature of their wave propagation renders them acutely sensitive to uppermost mantle velocity structure and Earth’s sphericity. Even simple 1D spherical velocity models produce geometric spreading of \( Pn \) and \( Sn \) that is strongly dependent on frequency (e.g., Sereno and Given, 1990; Yang et al., 2007; Avants et al., 2011). If frequency dependence of the geometric spreading is neglected, any attenuation model obtained using that geometric spreading intrinsically acquires incorrect frequency dependence.

In the previous modeling efforts (Yang et al., 2007), we used the reflectivity method to generate broadband synthetic seismograms with frequencies up to 10 Hz for a reference spherical 1D structure (Table 1) from Sereno and Given (1990), determining \( Pn \) and \( Sn \) geometric spreading. After extracting the \( Pn \) and \( Sn \) portions of the synthetic seismograms using fixed-velocity windows, the spectral amplitude variations with distance were computed. The calculated 10-Hz \( Pn \) amplitude decay for the reference model is shown in Figure 1, along with the amplitude decay for a conical headwave in a plane one-layer-over-half-space model (Aki and Richards, 2002, their equation 6.26) and the amplitude decay of an infinite-frequency direct turning wave in the same Earth model from raytracing. At distances close to the critical distance \( Pn \) geometric spreading is similar to that for a conical headwave. As distance increases, \( Pn \) spreading departs from that of a headwave and, near 5°, 10-Hz \( Pn \) amplitudes begin to increase for this model. This is because the phase behaves like an interference headwave, which is a superposition of multiple waves reflected from the Moho that constructively enhance the signal at larger distances. At teleseismic distances, \( Pn \) spreading is dominated by the direct wave and approaches the behavior of the infinite-frequency diving \( P \) wave from raytracing. This amplitude variation with distance is frequency dependent; all frequencies have concave upward spreading curves, with minima that increase in distance as frequency decreases (e.g., Yang et al., 2007). Amplitudes at higher frequencies are affected more by sphericity than are lower frequencies. Models with 1D velocity gradients in the mantle lid have similar behavior, but more pronounced frequency dependence of geometric spreading (Avants et al., 2011). Introduction of small-scale heterogeneities in the Moho discontinuity or in the mantle lid tends to reduce the curvature in the amplitude decay trends, but further enhance the frequency dependence (e.g., Avants et al., 2011; Xie and Lay, 2017). \( Sn \) behaves the same as \( Pn \) in general.

Although details of the small-scale structure in the lithosphere are not well known for any path, large-scale variations in mantle lid radial velocity gradients can be inferred from curvature of regional travel-time curves (e.g., Myers et al., 2010), and crustal thickness variations are fairly well known at low resolution in many places. Building on our earlier \( Pn \) modeling with 1D background structures, we forward model \( Pn \) geometric spreading for velocity structures with regional scale lateral variations in mantle lid gradients and crustal thickness, with specific consideration of \( Pn \) signals for events at the North Korean test site that traverse paths with oceanic path segments.

**\( Pn \) Frequency Dependence for 1D Velocity Models**

We use a finite-difference (FD) code to compute \( Pn \), so we first establish that the FD computations match the frequency-dependent behavior established by prior reflectivity calculations for 1D models. We consider the suite of models in Figure 2. The model velocities shown are functions of radius before earth-flattening transformation (EFT). The constant velocity lid model (Const-Lid) is listed in Table 1. We also consider \( Pn \) propagation in the constant velocity lid model without applying the EFT (Const-Lid w/o EFT). Models with mantle lid radial gradients of \( 0 \times 10^{-3} \text{ s}^{-1} \) (Gradient-0.000), \( 1 \times 10^{-3} \text{ s}^{-1} \) (Gradient-0.001), and \( 2 \times 10^{-3} \text{ s}^{-1} \) (Gradient-0.002) all connect to the IASP91 model gradient at varying depths below the 40-km-deep Moho.

Using improved computational capabilities, we significantly extend the bandwidth of our fourth-order FD code calculations of large-distance \( Pn \) synthetics (Xie and Lay, 1994;
structure. This enables examination of models with a flat free surface and laterally varying internal for ranges of up to 1000 km and frequencies up to 10 Hz for et al. Avants 2007).

**Figure 1.** 10-Hz *Pn* amplitude decay from reflectivity synthetics in the constant velocity lid (Const-Lid) model (Table 1) with earth-flattening transformation (EFT) is shown by the circles. Quality factor *Q* is infinite throughout the model. The solid line depicts the theoretical amplitude decay of a conical headwave in a plane one-layer-over-half-space Earth model. The dashed line is the amplitude decay of infinite-frequency direct wave in a spherical homogeneous Earth model from raytracing calculations (Yang et al., 2007).

Avants et al., 2011). Stable *P*-SV calculations were achieved for ranges of up to 1000 km and frequencies up to 10 Hz for models with a flat free surface and laterally varying internal structure. This enables examination of *Pn* geometric spreading behavior in laterally varying 2D structure over much broader bandwidth than before, allowing us to determine any frequency dependence. The velocity model used for FD calculation extends 1000 km in horizontal distance and 400 km in depth, with a grid spacing of 0.05 km in both horizontal and vertical directions, and a time step of 3.5 ms. An isotropic source located at 15 km depth is used for *Pn*-wave excitation.

**Figure 2.** 1D velocity models used for *Pn* computation. The structures are spherical models prior to EFT.

Figure 3 shows wavefield snapshots at 120 s for four 1D structures from Figure 2 out to a range of about 900 km. The amplitude normalization is the same for these models so that amplitudes are directly comparable. For Figure 3a (Const-Lid w/o EFT), there is no induced positive velocity gradient from earth-flattening, so the model is a classic constant velocity layered structure that gives rise to a true headwave case. The upper-mantle refracted energy is quickly depleted, resulting in rapid decay of the *Pn* amplitude and a frequency-independent power law decay as for the headwave case in Figure 1. In Figure 3a, the *Pn* wave is too weak to be seen in the crust, due to the common amplitude normalization. The wavefield behavior is quite different when the EFT is included, as seen in Figure 3b. The *Pn* amplitudes are significantly higher at large range. This corresponds to the turning ray behavior of the multiples in the lid modifying the geometric spreading of the first arrival. Increasing the positive gradients beyond the mild gradient from EFT further increases the *Pn* amplitudes overall, as shown in the snapshots for models (Gradient-0.001) and (Gradient-0.002) in Figure 3c and 3d, respectively. The strengthening of the *Pn* amplitudes is proportional to radial lid velocity gradient and the development of underside reflections from the Moho as the gradient increases. The corresponding wavefronts are apparent with variable incident angles immediately below the Moho. These calculations extend the frequency band of the FD calculations from ~1 (Avants et al., 2011) to ~10 Hz at 1000 km, comparable to what we had previously explored only with frequency–wavenumber integration. This enables exploration of high-frequency *Pn* geometric spreading behavior in 2D models below.

Figure 4 shows waveforms of the *Pn* arrivals for the same four models in reduced-velocity (8.1 km/s) distance profiles. The amplitudes of synthetics for the two (Const-Lid) models are amplified by a factor of 3 relative to those for gradient models. The clear increase in *Pn* amplitude with increasing range and earlier *Pn* arrivals for the (Gradient-0.001) and (Gradient-0.002) models (see also Fig. S5, available in the electronic supplement to this article) demonstrates the concave upward curvature of the geometric spreading curve seen in Figure 1.

For the same four 1D models, we computed frequency-dependent geometric spreading amplitude decay out to 1000 km, using the FD calculations, correcting for excitation of a point source versus a line source. The synthetic seismograms are band-pass filtered between 0.25–0.5, 0.5–1.0, 1.0–2.0, 1.5–3.0, 2.5–5.0, 4.0–6.0, 6.0–8.0, and 8.0–10.0 Hz. The root mean square (rms) amplitudes are measured in a 0.3 km/s group velocity window around the picked first arrival times from the broadband synthetics. Then, the amplitudes are normalized by their values at 300 km, which
eliminates the effect of the source spectrum. This yields the amplitude–distance curves shown in Figure 5. These patterns are very similar to those from wavenumber-integration calculations for similar 1D velocity models computed in Avants et al. (2011). The complex frequency dependence of $Pn$, with minima in the amplitude–distance curves and increasing amplitudes at larger distances, particularly for higher frequencies, is evident for all structures with a positive gradient in the 1D flat Earth structure. For lid gradients in excess of 0.002 s$^{-1}$, the distance and frequency dependence become quite dramatic, with complex plateauing and oscillation of the high-frequency components, but we do not show those calculations here as such strong lid gradients appear to be rare (e.g., Myers et al., 2010). Overall, these calculations and comparisons with wavenumber-integration results provide a good level of confidence in the broadband $Pn$ synthetics, and we proceed to consider results for 2D models.

$Pn$ Geometric Spreading for 2D Velocity Models with Varying Lid Radial Gradient

Velocity models with laterally varying mantle lid radial velocity gradients are used for testing $Pn$-wave propagation across distinct provinces. Our models are comprised of two sections with different gradients. One section extends from 0 to 450 km, another from 550 to 1000 km. Between them, from 450 to 550 km, linear interpolation is used to smoothly connect the different end members. We consider four cases connecting regions with models shown in Figure 2:

Figure 3. Comparison of snapshots at time 120 s for models with different velocity gradients. The models correspond to the structures in Figure 2. For easy comparison, the same normalization factors are used for all models. Note the different $Pn$ amplitude and different mantle $P$-wave wavefront expansion. In models with large mantle lid gradient, the wavefront reflecting from the bottom of the Moho can be seen. Snapshots for each model for different times are shown in Figures S1–S4, available in the electronic supplement to this article.
Figure 4. Comparison of $Pn$ waves calculated in the 1D velocity models in Figure 2, with amplification factors used in (a) and (b) being three times those used for (c) and (d). The solid lines are reference group velocities of 8.3, 8.2, 8.1, 8.0, and 7.9 km/s. The central tick marks indicate the picked arrival time, and the two short marks indicate the interval window for amplitude calculation. A reduced time $t = r/8.1$ is used for the horizontal coordinate.

- Gradient-0.000–0.001 (velocity gradient varies from $0 \times 10^{-3}$ s$^{-1}$ to $1 \times 10^{-3}$ s$^{-1}$)
- Gradient-0.001–0.000 (velocity gradient varies from $1 \times 10^{-3}$ s$^{-1}$ to $0 \times 10^{-3}$ s$^{-1}$)
- Gradient-0.000–0.002 (velocity gradient varies from $0 \times 10^{-3}$ s$^{-1}$ to $2 \times 10^{-3}$ s$^{-1}$)
- Gradient-0.002–0.000 (velocity gradient varies from $2 \times 10^{-3}$ s$^{-1}$ to $0 \times 10^{-3}$ s$^{-1}$).

Figure 6 shows displays of the velocity differences of these models relative to model Gradient-0.000, which is always one of the end-member structures.
The \( Pn \) amplitude–distance curves for the 2D models in Figure 6 are shown in Figures 7–9. Figure 7 shows the behavior for structures with end-member 1D models (Gradient-0.000) and (Gradient-0.001), with the near-source and near-receiver environments reversing between these two models. All models have positive gradients from the EFT. In the source region, the spreading will match that for the corresponding end-member case shown in Figure 7a and 7d, but beyond 450 km, the effect of the other end-member structure modifies the behavior for the 2D cases in Figure 7b and 7c. Note the difference between geometric spreading when the source position and velocity structure is interchanged in Figure 7b and 7c (the amplitudes are normalized at 300 km for which there is only a mild difference between the model predictions). This directional dependence indicates that it is not straightforward to use a single average model to represent the frequency-dependent geometric spreading in a laterally varying model. Similar comparisons are shown for a case involving stronger variation in radial gradients in the lid in Figure 8, ranging from end-member models (Gradient-0.000) to (Gradient-0.002). The lateral averaging behavior is similar to that for the models with (Gradient-0.001), although the stronger gradients cause more pronounced differences for the reversed paths in Figure 8b and 8c.

The frequency dependence at large ranges is strong for all cases, but the distance variation in the amplitudes is distinct for each model, as emphasized in Figure 9, which superimposes the 1D and mixed 2D amplitude–distance curves for the 2.5–5.0 Hz band. Models (Gradient-0.000–0.002) and (Gradient-0.002–0.000) show quite different behavior, and neither of them can be approximated by an average model (Gradient-0.001). The complexity of the frequency-dependent amplitudes for the laterally varying models and the strong direction dependence of the amplitude behavior would be incorrectly mapped into complex frequency dependence and possible anisotropic attenuation if the effects of the elastic velocity structure are not correctly accounted for.

**Pn Geometric Spreading for Laterally Varying Crustal Thickness**

Numerous factors may contribute to observed \( Pn \) amplitudes in addition to the variation of upper-mantle lid velocity gradients explored above. These include crustal thickness variations, \( P \)-wave attenuation in the uppermost mantle, and scattering along the propagation path. When the source and station are located in different environments, the transition zone between the environments can play an important role in \( Pn \)-wave propagation efficiency. We extend our investigation of 2D-wave propagation effects to consider smooth large-scale crustal variations.

**Figure 7.** Frequency dependence of \( Pn \) amplitude versus distance relations on log–log scale. (a)–(d) are for different velocity models in Figure 2. The narrowband band-pass frequency filters are indicated by the symbols. Root mean square (rms) amplitudes are used in this figure, and all curves are normalized at 300 km. For the Const-lid w/o EFT model (a), the amplitude–distance relation is close to frequency independent as expected for a true headwave.

**Figure 6.** Laterally varying mantle lid velocity gradient models. 2D models after subtracting the structure in model (Gradient-0.000) are shown. From (a) to (d) are for Gradient-0.000–0.001, Gradient-0.001–0.000, Gradient-0.000–0.002, and Gradient-0.002–0.000, respectively.
Effects of Laterally Varying Mantle Lid Velocity Gradient and Crustal Thickness

Figure 7. Amplitude–distance relations in different 1D and 2D laterally varying velocity models and frequency bands. The models are (a) Gradient-0.000, (b) Gradient-0.000–0.001, (c) Gradient-0.001–0.000, and (d) Gradient-0.001.

Figure 8. Amplitude–distance relations in different laterally varying velocity models and frequency bands. The models are (a) Gradient-0.000, (b) Gradient-0.000–0.002, (c) Gradient-0.002–0.000, and (d) Gradient-0.002.

extensively studied. Zhao et al. (2015) found that the Pn paths from the North Korean test site include a range of continental and oceanic paths characterized by very different crust and upper-mantle structures, which appears to affect Pn-wave propagation. Although there is almost no information to constrain mantle lid gradients or lateral variations of lid gradients along these paths, we do have information about the large-scale crustal variations. We simulate Pn-wave propagation along paths with crustal thickness variations for nuclear tests located in North Korea and compare the model calculations with observed data.

Figure 10 shows locations of the North Korea nuclear test site (NKTS) and stations providing Pn observations. Pn waves observed in mainland China primarily traverse continental paths, whereas Pn observations in Japan traverse the Japan Sea, with predominantly oceanic paths. Given that explosion sources are virtually isotropic and the observations span a large azimuth range, Zhao et al. (2015) compared the Pn spectral amplitudes along the combined continental and oceanic paths at 1000 km epicentral distance (recorded by those stations near the large circle in Fig. 10). After removing empirical estimates of the Pn-wave source excitation functions for three presumed nuclear explosions in 2006, 2009, and 2013, the observed spectral amplitudes are shown in Figure 11 as a function of azimuth from the source region, with different symbols indicating normalized spectral amplitudes from the three NKT explosions. The black, blue, and red colors indicate 0.8, 7.0, and 10.0 Hz data, respectively. Solid circles with error bars indicate the mean values and standard deviations for each frequency obtained within 30° azimuth windows. Prominent differences can be observed for Pn waves along different paths. For azimuths from 60° to 180°, the Pn signals traversing oceanic paths are strongly frequency dependent, with very low high-frequency amplitudes. For azimuths between 230° and 280°, the wavepaths are along continental paths, and the lower-frequency content (0.8 and 7 Hz) is similar to that along the oceanic paths, but the high-frequency (10 Hz) spectral amplitudes are much higher than for the oceanic paths. The geometric spreading function tends to raise the high-frequency signal, whereas intrinsic attenuation tends to reduce the high-frequency content. The detailed structure of transition zones may also affect the frequency dependence.

To simulate the propagation of Pn waves in mixed oceanic and continental structures, we designed a Japan Sea model as shown in Figure 12a. The depth of the Moho discontinuity is obtained from CRUST1.0 (Laske et al., 2013) for a typical Pn path. In this model, the crustal thickness beneath the NKTS is ~33 km, and under the Japan Sea it is
Figure 10. Map depicting locations of the North Korean test site (red star labeled NKTS) and seismic stations at which \( Pn \) phase measurements were made. The stations located near the large circle have an epicentral distance of \( \sim 1000 \) km from the NKTS, with the pink and blue colors indicating the \( Pn \) waves that traverse primarily continental or oceanic paths, respectively (Zhao et al., 2015).

Figure 11. \( Pn \) spectral amplitudes measured from stations near a distance of 1000 km versus azimuth (Fig. 10). Different symbols are amplitudes from the three North Korea nuclear tests. Black, blue, and red colors indicate 0.8, 7.0, and 10.0 Hz measurements. Solid circles with error bars represent their mean values and standard deviations, obtained within 30° azimuth windows. The \( Pn \) source excitation functions are removed from the data (Zhao et al., 2015).

\( \sim 11 \) to 15 km. On the NKTS side, the crustal thickness quickly decreases from 33 to about 10–12 km over \( \sim 100 \) km distance, giving the rapid reduction in depth of the Moho discontinuity. On the side of the Japan Islands, the crustal thickness increases relatively slowly, forming a broader dip in the Moho discontinuity. For the continental path, we use a simple 1D model with a 30-km-flat Moho (Fig. 12b). In both models, the upper-mantle velocity gradient is set to be zero, mainly because constraints on the local lid structure are very limited, and the water column is not included in the model. As these are spherical models, we still expect the \( Pn \) spreading to be frequency dependent as discussed in the previous sections.

To calculate the synthetic seismograms, we use the same source time function as used in the previous calculations. However, we set the source depth at 1 km to simulate a shallow explosion. Figure 13 displays wavefield snapshots on the source side near the NKTS to within the oceanic segment (from 50 to 350 km) of the Japan Sea model. Figure 14 shows wavefield snapshots on the Japan Island side (from 650 to 950 km). Similar snapshots for \( Pn \) waves in the continental model with uniform 30-km-thick crust have much simpler character (Figs. S7 and S8). A shallowing Moho tends to allow more energy from the source to penetrate into the upper mantle, whereas a deepening Moho allows more energy to come back from the upper mantle into the crust. Aside from any intrinsic attenuation, these conditions favor enhanced \( Pn \) wave traversing the Japan Sea model compared to transmission along a flat continental path. This can also be understood by comparing the velocity models in Figure 12, for which the varying dip of the Japan Sea Moho is equivalent to an additional curvature relative to the 1D model. Figure 15 compares the \( Pn \) waves in both models as they approach 1000 km distance. It is evident that in the Japan Sea model \( Pn \) is stronger than in the continental model. We found that introducing a zone of thickened crust along the path tends to have opposite effects to the thinned crust of the Japan Sea model, weakening \( Pn \) energy that traverses the thickened zone.

Figure 16 shows the synthetic seismograms for both the Japan Sea model and the flat continental model, along with the Japan Sea velocity structure. The \( Pn \)-wave amplitude is much stronger in the Japan Sea model. Because the \( Pn \) wave penetrates the low-velocity crust with laterally varying thickness, its travel-time curve deviates from a straight line. A similar procedure as used in the previous sections on mantle lid velocity gradients is used to calculate the frequency-dependent rms amplitudes. However, to be consistent with
the treatment of the observed data, a fixed group-velocity window between 8.2 and 7.6 km/s is used (as indicated in Fig. 16 by vertical markers on the seismograms). The $Pn$ amplitudes versus distance at nominal center frequencies of 0.4, 0.8, 1.5, 3.0, 6.0, and 10.0 Hz are illustrated in Figure 17, where the diamonds are for the Japan Sea model, crosses are for a 30-km-thick flat crust model, and plus signs are for a 10-km-thick flat crust model. All results are normalized with the 30-km crust model at 300 km. It is evident that the uniform crustal thickness alone does not much affect the $Pn$ amplitude. However, the $Pn$ amplitudes in the Japan Sea model are stronger than those in either flat Moho model. These differences are frequency dependent. At lower frequencies, the difference is less than a half order of magnitude, but it increases to about an order of magnitude at higher frequencies. It appears that the deepening Moho on the Japan Island side also plays an important role in the $Pn$ wave amplitudes. At lower frequencies, it raises the $Pn$ amplitudes. For higher frequencies, its effect is complicated. As can be seen in Figure 16, due to the slope in the Japan Sea model, certain crustal multiples may enter the measurement window at longer distances. This complicates $Pn$ amplitude measurement using group velocity windows.

Given that the dataset in Japan spans a narrow distance range around 1000 km distance, we cannot evaluate $Pn$ amplitude variations versus distance with our simulations. Instead, we investigate the observed frequency dependence of $Pn$-wave amplitudes at 1000 km for the NKTS explosion dataset. Figure 18a shows the observed average spectral amplitudes for both the oceanic paths through the Japan Sea and the continental paths through mainland China. At lower frequencies, $Pn$ waves traversing both paths have similar amplitudes. However, at higher frequencies, $Pn$ waves traversing the Japan Sea path are much weaker than on the continental paths. In general, $Pn$ geometric spreading tends to raise the high-frequency signal, whereas intrinsic attenuation tends to decrease the high-frequency signal. By comparing the numerical simulations of $Pn$-wave geometrical spreading (Fig. 17) and the observed $Pn$ amplitudes from the NKTS data (Fig. 18a), we see that the observed drop off of amplitudes at high frequency for the Japan Sea is likely due to stronger intrinsic attenuation than for the continental paths. To try to match the observed $Pn$ spectra, we combine $Pn$ $Q$ models with the calculated geometrical spreading functions. For $Pn$ waves from the NKTS to continental stations, Zhao et al. (2015) obtained an attenuation...
model with $Q_{Pn} = 237f^{0.36}$. For the Japan Sea oceanic paths, using a trial-and-error method, we find a lower $Q_0$ and a slightly higher $\eta$, giving a $Pn$ $Q$ model of $Q_{Pn} = 150f^{0.40}$. By combining these two $Q_{Pn}$ models with the numerically calculated geometrical spreading, the resulting $Pn$-wave amplitudes are shown in Figure 18b. The predictions are quite consistent with the average observed data shown in Figure 18a. As discussed above, many factors affect the $Pn$-wave amplitude and its frequency dependence, including data processing details. Thus, this result is not unique. However, this provides some constraint on the $P$-wave attenuation beneath the oceanic crust.

Figure 14. Wavefield snapshots of $Pn$ waves propagating in the receiver-side section (650–950 km) of the Japan Sea model, approaching the stations.

Figure 15. Comparison between $Pn$ waves propagating in the continental and Japan Sea models between 650 and 950 km. Note that the $Pn$-wave amplitude is stronger in the Japan Sea model.
Discussion and Conclusions

Computations of $Pn$ geometric spreading for 2D models with lateral variations in mantle lid velocity gradients have established that the amplitude behavior is dependent on...
direction of the path and that complex frequency dependence is expected to involve a mixing of contributions from the near-source and near-receiver structures. This further weakens the rationale for using average frequency-independent power-law representations of $Pn$ (and $Sn$) geometric spreading.

Effects of crustal thinning and Moho dip on $Pn$ from North Korean nuclear tests to stations in Japan account for observed relatively enhanced low-frequency North Korean nuclear tests to stations in Japan account for spreading. The lateral transitions from continent to ocean and ocean to island arc favor transfer of $P$ energy into the $Pn$ wavefield near the source region and into the crust near the receivers. Above 6 Hz, the observations in Japan show rapid relative amplitude decrease, which we attribute to intrinsic attenuation effects. When large-scale crustal structure variations are independently known, modeling can provide guidance on basic behavior of the geometric spreading, allowing improved models of intrinsic attenuation to be obtained. This can ultimately improve event identification and yield estimation for underground nuclear tests.

Data and Resources
No new data were collected for this study; all data from the North Korean site are from the publication by Zhao et al. (2015). The original measurements are available from the first author upon request.

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