Microseismic moment-tensor inversion and sensitivity analysis in vertically transverse isotropic media

Han Li¹, Xu Chang¹, Xiao-Bi Xie², and Yibo Wang¹

ABSTRACT

Through the study of microseismic focal mechanisms, information such as fracture orientation, event magnitude, and in situ stress status can be quantitatively obtained, thus providing a reliable basis for unconventional oil and gas exploration. Most source inversion methods assume that the medium is isotropic. However, hydraulic fracturing is usually conducted in sedimentary rocks, which often exhibit strong anisotropy. Neglecting this anisotropy may cause errors in focal mechanism inversion results. We have developed a microseismic focal mechanism inversion method that considers velocity anisotropy in a vertically transverse isotropic medium. To generate synthetic data, we adopt the moment-tensor model to represent microearthquake sources. We use a staggered-grid finite-difference method to calculate synthetic seismograms in anisotropic media. We perform seismic moment-tensor (SMT) inversion with only P-waves by matching the synthetic and observed waveforms. The synthetic and field data sets are used to test the inversion method. For the field data set, we investigate the inversion stability using randomly selected partial data sets in the calculation. We pay special attention to analyze the sensitivity of the inversion. We test and evaluate the impact of noise in the data and errors in the model parameters \( V_p_0 \), \( \epsilon \), and \( \delta \) on the SMT inversion using synthetic data sets. The results indicate that, for a surface acquisition system, our method can tolerate moderate noise in the data and deviations in the anisotropy parameters can cause errors in the SMT inversion, especially for dip-slip events and the inverted percentages of non-double-couple (DC) components. According to our study, including anisotropy in the model is important to obtain reliable non-DC components of moment tensors for hydraulic fracturing-induced microearthquakes.

INTRODUCTION

The study of focal mechanisms is important for understanding natural and induced earthquakes (Staněk et al., 2015; Pesicek et al., 2016). During unconventional oil and gas exploration, fluid injection activities may generate fracturing within the reservoir and cause microearthquakes. Hydraulic fracturing can also reactivate faults in adjacent areas to trigger induced earthquakes of moment magnitude 3.0 or higher (Lei et al., 2017; Grigoli et al., 2018). Focal mechanism inversion is of great significance for studying fluid-driven seismicity including injection-induced earthquakes (Tera-kawa et al., 2012; Lei et al., 2019). Focal mechanisms reveal valuable information for understanding stress distributions (Kubo et al., 2002; Vavryčuk, 2011), characterizing fracture activities (Baig and Urbancic, 2010; Taisne et al., 2011), and evaluating hydraulic fracturing effects (Maxwell et al., 2015) throughout the process.

Earthquake sources are dominantly described by a double-couple (DC) source model, composed of shear slips on a fault plane (Aki and Richards, 2002; Minson et al., 2007; Maxwell et al., 2015; Staněk et al., 2015). But recent studies have shown that some natural earthquakes (Miller et al., 1998; Ross et al., 2015) and induced seismic sources (Vavryčuk, 2002; Vavryčuk et al., 2008; Julia et al., 2009; Boettcher et al., 2015) may contain non-DC components. To study the non-DC mechanisms, a general dislocation (GD) source model has been established by adding tensile (or compressive) movement to the DC model (Vavryčuk, 2001, 2011; Minson et al., 2007; Šílený and Horálek, 2016; Li and Yao, 2018; Li et al., 2018a). However, the DC and GD models specify the focal mechanism as...
the slip movement of a fault, which limits their application in seismological studies. Compared with these two models, the seismic moment-tensor (SMT) source model uses body forces to describe an earthquake. Therefore, the SMT model can describe all possible seismic sources (DC and GD sources can be expressed by certain SMTs) and it has been widely applied to microseismic focal mechanism inversion in recent years (Trifu et al., 2000; Song and Toksöz, 2011; Song et al., 2014; Boettcher et al., 2015; Grechka et al., 2016; Willacy et al., 2019). Song et al. (2014) propose a full-waveform-based microseismic moment-tensor inversion method and apply it to microseismic studies in the Barnett Shale. Grechka et al. (2015, 2016) study the feasibility of single-well microseismic moment-tensor inversion, especially for tensile microseismic events. Willacy et al. (2019) develop a full-waveform workflow that can simultaneously locate the induced earthquakes and derive their source moment tensors. 

Focal mechanism inversions often assume an isotropic medium (e.g., Hao and Yao, 2012; Pesicek et al., 2012). However, hydraulic fracturing activities are usually conducted in sedimentary rocks, which often exhibit strong anisotropy, particularly in shale gas reservoirs (Grechka and Yaskевич, 2014; Michel and Tsvankin, 2014; Deng et al., 2015). The observed waveform from a DC source can be represented as $u_p = G_{pk;l} m_{lj}$, where $G_{pk;l}$ is the elastodynamic Green’s function, $m_{lj} = c_{ijkl} S_{d;lj}$ is the moment tensor, $c_{ijkl}$ is the stiffness tensor, $S$ is the fault area, and $d_l$ and $n_l$ are the dislocations of slip vector $d$ and normal vector $n$ on a fault plane (e.g., Aki and Richards, 2002). In the process of inverting the source parameters $d_l$ and $n_l$ from the observed wavefield $u_p$, anisotropy may affect the results in two different ways. First, in an anisotropic medium, the stiffness tensor $c_{ijkl}$ in the near-source region maps the source process $d_l n_l$ to the moment tensor $m_{lj}$ through $m_{lj} = c_{ijkl} S_{d;lj}$. For example, a pure shear faulting $d_l n_l$ (with $d$ perpendicular to $n$) produces a DC mechanism in isotropy, but it may generate non-DC components in anisotropy (Vavryčuk, 2005). This has been investigated for natural earthquake and microseismic sources (see, e.g., Ben-Zion, 2001; Vavryčuk, 2003; Li et al., 2018b; Grechka, 2020). Second, the anisotropy in the model velocity, along with velocity variations, scattering and attenuation, affect wave propagation through the elastodynamic Green’s function $G_{pk;l}$, thus affecting the moment-tensor inversion through $u_p = G_{pk;l} m_{lj}$. These two aspects are independent of each other. Although both of them affect our capability to investigate the faulting process, our investigation focuses only on the effect of velocity anisotropy along the propagation path on moment-tensor inversion.

In most cases, an SMT solution is obtained by finding the best fit between the observed and synthetic data under certain constraints (consistency of their first-arrival P-wave polarities and similarity of their waveforms). Thus, the SMT inversion procedure is heavily dependent on the forward modeling of synthetic records. Anisotropy may affect wave propagation by changing its direction, polarization, arrival time, and amplitude (Li et al., 2019). Therefore, if anisotropy is neglected, interpretations of microearthquakes may lead to incorrect results. Research on borehole microseismic monitoring (King and Talebi, 2007; Warpinski et al., 2009) has demonstrated that failure to include anisotropy in velocity may result in source location errors and that a method should be developed to include anisotropy parameters or reduce the effects of anisotropy. Vavryčuk et al. (2008) find that an SMT inversion without considering velocity anisotropy can lead to artificial non-DC components.

For microseismic SMT studies, it is also very important to evaluate and quantify the uncertainties of the inverted parameters. Du and Warpinski (2011) analyze the uncertainty in fault-plane solutions from moment-tensor inversion and find that the misfit of the rake angle (slip angle) is larger than that of the strike and dip angles. Eaton and Forouhiideh (2011) study the impact of the receiver-array geometry on microseismic moment-tensor inversion and demonstrate that it plays a fundamental role in inversion stability. Staněk et al. (2014) evaluate the stability of source mechanisms inverted from microseismic surface monitoring data and show that strike-slip mechanism inversion is more stable than dip-slip inversion. Eyre and van der Baan (2017) compare the reliability of microseismic moment-tensor solutions in surface and borehole monitoring cases and find that surface array inversion results are closer to the input moment tensors compared with the borehole results.

In this paper, we propose a microseismic focal mechanism inversion method that takes into account the velocity anisotropy in a vertically transverse isotropic (VTI) medium. Synthetic waveforms are generated by solving the 3D elastic wave equation with a staggered-grid finite-difference (SGFD) method. The SMT is then inverted by finding the best fit between the observed and synthetic data. We use synthetic data sets and surface microseismic monitoring field data to validate the proposed method. We quantitatively evaluate the sensitivity and dependence of the SMT inversion on the data and anisotropy parameters. For the microseismic field data, we perform inversion tests using randomly selected partial data to evaluate the reliability of the inverted SMT results.

METHOD

In this section, we provide a detailed introduction of the proposed microseismic focal mechanism inversion method from three aspects, i.e., source modeling, waveform synthesis, and inversion algorithm.

Moment-tensor model

To represent all possible seismic sources including DC and non-DC components in our investigations, we use the moment-tensor model. For the 3D case, the SMT can be expressed by a $3 \times 3$ matrix $M$, with its elements being either dipoles or couples (Jost and Herrmann, 1989). Due to the conservation of the angular momentum, $M$ is a symmetric matrix with only six independent elements (Aki and Richards, 2002) and can be represented by a six-element vector $m$:

$$m = (m_{11}, m_{22}, m_{33}, m_{23}, m_{13}, m_{12})^T$$

(1)

where the subscripts are substituted according to Voigt notation as $11 \rightarrow 1, 22 \rightarrow 2, 33 \rightarrow 3, 23 \rightarrow 4, 13 \rightarrow 5,$ and $12 \rightarrow 6$.

To analyze the SMT, we can decompose it into an isotropic (ISO) and a deviatoric part. The ISO part represents an explosive or implosive component, and the deviatoric part contains the DC and compensated linear vector dipole (CLVD) components (Knopoff and Randall, 1970; Jost and Herrmann, 1989). The fault-plane solutions, including the strike, dip, and rake angles of the fault, can be retrieved from the DC component (Jost and Herrmann, 1989). The percentages of the DC, CLVD, and ISO parts can be calculated.
from the SMT to assess their relative amounts (Vavryčuk, 2005) as follows:

\[
\text{ISO} = \frac{1}{3} \left| \text{Tr}(\mathbf{M}) \right|, \quad 100\%, \\
\text{CLVD} = -2 \lambda \left( 100\% - |\text{ISO}| \right), \\
\text{DC} = 100\% - |\text{ISO}| - |\text{CLVD}|,
\]

where \(\text{Tr}(\mathbf{M})\) represents the trace of the SMT \(\mathbf{M}\). If \(\lambda_i\) (\(i = 1, 2, 3\)) are the eigenvalues of \(\mathbf{M}\), then \(\text{Tr}(\mathbf{M}) = \lambda_1 + \lambda_2 + \lambda_3\). The values \(\lambda_i^\prime\) are the eigenvalues of the deviatoric part of \(\mathbf{M}\), which can be calculated as \(\lambda_i^\prime = \lambda_i - \frac{\text{Tr}(\mathbf{M})}{3}\). The subscripts \(|\max|\) and \(|\min|\) in equation 2 refer to the maximum and minimum of the absolute values of \(\lambda_i\) and \(\lambda_i^\prime\). From equation 2, we can see that the percentage of the DC component is always positive whereas those of the ISO and CLVD can be positive (corresponding to explosive and tensile) or negative (corresponding to implosive and compressive).

**Forward modeling of synthetic seismograms**

To forward model the seismic wave propagation in anisotropic media, forward modeling of the seismic wavefield is based on the 3D elastic wave equation:

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) + \rho F, 
\]

where \(u_i\) represents the displacement, \(x_j\) and \(x_k\) are the space variables, \(t\) is the time, \(c_{ijkl}\) is the stiffness tensor \((i,j,k,l = 1,2,3)\), \(\rho\) is the density, and \(F\) represents the external force density. In a VTI medium, \(c_{ijkl}\) can be represented by

\[
c_{ijkl} = \begin{bmatrix}
    c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
    c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
    c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
    0 & 0 & 0 & c_{44} & 0 & 0 \\
    0 & 0 & 0 & 0 & c_{44} & 0 \\
    0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}.
\]

Then, \(c_{11}, c_{12}, c_{13}, c_{33}, c_{44},\) and \(c_{66}\) can be described by the Thomsen parameters (Thomsen, 1986; Tsvankin et al., 2010) as follows:

\[
c_{11} = \rho \left( 1 + 2\epsilon \right) V_{P0}^2, \\
c_{33} = \rho V_{S0}^2, \\
c_{44} = \rho V_{S0}^2, \\
c_{12} = \rho V_{P0}^2 \left[ 1 + 2\epsilon - 2(1 - \epsilon^2)(1 + 2\gamma) \right], \\
c_{13} = \rho V_{P0}^2 \sqrt{f' \left( f^2 + 2\delta \right) - \rho^2 V_{S0}^2}, \\
c_{66} = \rho \left( 1 + 2\gamma \right) V_{S0}^2 = 0.5 (c_{11} - c_{12}),
\]

where \(f' = 1 - V_{S0}^2 / V_{P0}^2; V_{P0}\) and \(V_{S0}\) are the vertical velocities of the P- and S-waves, respectively; \(\epsilon = V_{P0} / V_{S0}(0^\circ) - 1\) indicates the difference between the vertical and horizontal P-wave velocities, i.e., P-wave anisotropy, \(\gamma = V_{S0} / V_{S0}(0^\circ) - 1\) represents the same measure for SH-waves; and \(\delta\) governs the variation in the P-wave velocity away from the symmetry axis and also influences the SV-wave velocity (Tsvankin et al., 2010).

Equations 3–5 establish the waveform forward-modeling theory in a VTI medium. Given a velocity/density model, the locations of the geophones and microseismic source, and the focal mechanism, the synthetic seismograms can be calculated using the SGFD method (Virieux, 1984, 1986; Graves, 1996; Kristek et al., 2002; Li et al., 2019). The SGFD method used here is proposed by Graves (1996), and it has been proven to be flexible and relatively accurate (Graves, 1996; Kristek et al., 2002).

In this paper, we use the moment-tensor model to represent microseismic sources. From equation 1, \(\mathbf{m}\) has six independent elements and each can be related to an elementary seismogram (i.e., the synthetic waveform of \(M_i\), \(i = 1 - 6\)). Therefore, the \(i\)th-component (north, east, and down) seismogram of the geophone \(n\) can be modeled as (Song and Toksöz, 2011)

\[
d_i(x^n, t) = \sum_{j=1}^{6} A_{ji}(x^n, x_j, t) M_j(x_j),
\]

where \(M_j\) is the \(j\)th element of \(\mathbf{m}\) in equation 1 and \(A_{ji}\) represents the \(i\)th-component elementary seismogram of the \(n\)th geophone at \(x^n\) due to a point-moment-tensor source \(M_j\) at \(x_j\). For example, \(A_{31}(t)\) is the vertical-component synthetic waveform of \(M_1\) based on equations 3–5. In this study, we use a source time function with a central frequency of 30 Hz (obtained by stacking the first-arrival P-waves of the field data) in the SGFD simulation. A 10 m grid spacing with 0.25 ms time sampling is used in all finite-difference calculations. More details of the SGFD forward modeling to calculate synthetic velocity seismograms are given in Appendix A.

**Microseismic SMT inversion**

SMT is retrieved by finding the best fit between the synthetic and observed waveforms (i.e., the waveform matching technique). Due to imperfect modeling or errors in the source location and model parameters, there may be a time shift between the synthetic and observed data (Zhao and Helmberger, 1994; Zhu and Zhou, 2016). The cut-and-paste (CAP) method (Zhao and Helmberger, 1994; Zhu and Helmberger, 1996; Zhu and Zhou, 2016) is a focal mechanism inversion method that introduces time shifts between the observed and synthetic data, and it has the advantage of reducing the inversion dependence on models and source locations. We follow this method and rewrite the matrix equation 6 as

\[
d(t) = A(t - \Delta t) \mathbf{m},
\]

where matrix \(A\) is composed of six columns and the \(i\)th element \((i = 1 - 6)\) column corresponds to the elementary seismograms \(A_i\) of a moment-tensor source \(M_i\) and \(\Delta t\) is the time shift between the observed data vector \(d(t)\) and synthetic seismogram \(A(t)\). For each geophone, the time shift may be different. We calculate \(\Delta t\) for each detector by finding the maximum amplitude of the crosscorrelation between its waveform record \(d^*(t)\) and synthetic signal \(A^*(t)\). These two signals are obtained by

\[
d^*(t) = N(|d(t)|), \quad A^*(t) = N \sum_{l=1}^{6} |A_l(t)|,
\]

where \(N(|d(t)|)\) refers to the normalization of the absolute value of \(d(t)\). The reason for using \(A^*(t)\) and not \(A(t)\) is that the SMT is unknown and we need to adopt a combination of elementary seismograms to fit the observed waveform data. The normalization and the positive operations (e.g., \(N(|d(t)|)\)) can help to obtain an
accurate time shift $\Delta t$. The term $\mathbf{m}$ can then be obtained using the generalized inverse (Song and Toksöz, 2011):

$$
\mathbf{m} = (\hat{\mathbf{A}}^T \hat{\mathbf{A}})^{-1} \hat{\mathbf{A}}^T \mathbf{d},
$$

where $\hat{\mathbf{A}}$ denotes $\mathbf{A}(t - \Delta t)$ and $\hat{\mathbf{A}}^T$ is its transposition.

Based on the moment-tensor source model and the 3D elastic wave equation, we calculate the elementary seismograms in anisotropic VTI media using the SGFD method. SMT is then inverted following waveform matching and the CAP method. The main steps are shown below:

1) Synthesize elementary seismograms $\mathbf{A}_i(t)$, ($i = 1, 2, \ldots, 6$) with the medium model and the locations of geophones and microseismic sources using the SGFD method.

2) For each geophone, calculate two signals $\mathbf{d}^*(t)$ and $\mathbf{A}^*(t)$ using the observed data $\mathbf{d}(t)$ and elementary seismograms $\mathbf{A}_i(t)$. Then, obtain time shift by finding the maximum crosscorrelation coefficient between $\mathbf{d}^*(t)$ and $\mathbf{A}^*(t)$.

3) Perform SMT inversion using observed data $\mathbf{d}(t)$ and synthetic seismograms $\mathbf{A}(t - \Delta t)$.

**NUMERICAL TESTS AND SENSITIVITY ANALYSIS**

We use a synthetic data set to validate the proposed SMT inversion method and analyze the dependence of SMT inversion on the data and model. Through numerical tests, we evaluate the accuracy of the results using data sets with different levels of noise and models with errors in the anisotropy parameters.

**Numerical test parameters**

To generate the synthetic data set, we use a microseismic surface acquisition system and a 3D velocity model, which are the same as those used in processing field data, with 7000, 6000, and 4000 m in the east, north, and depth directions, respectively. Figure 1 shows the surface acquisition system, with 1771 geophones deployed on six crosslines every 20 m. The microseismic source is set at a west–east location equal to 2780 m, a south–north location equal to 2250 m, and a depth of 3880 m.

In actual surface microseismic monitoring cases, only first-arrival P-waves in the vertical component are usually available for processing. Therefore, during numerical tests, synthetic data are calculated for 1.25 s and vertical-component first-arrival P-waves are sampled in a 60 ms time window and used in the inversion. The 3D model parameters, including $V_p$, $\epsilon$, and $\delta$, are marked as model M0 in Figure 2.

During the numerical tests, we adopt a source mechanism inversion analysis similar to the one used by Staněk et al. (2014) to evaluate the sensitivity and stability of the proposed method. Two representative microseismic source mechanisms are chosen in our tests. One is the strike-slip mechanism marked as S1 (100% DC, strike
Influence of noise on SMT inversion

The influence of noise in the data on microseismic SMT inversion is evaluated and analyzed. Based on the velocity model and source parameters, we calculate synthetic data sets for mechanisms S1 and S2 using the SGFD method. We then add different noise levels to the synthetic waveforms. The noise is defined as (Staněk et al., 2014)

\[
\text{Noise} = \frac{d_{\text{noise}}}{\text{MA}(d_{\text{noise}})} \cdot \text{MA}(d_{\text{syn}}) \cdot \text{NoiseLevel},
\]

where \(d_{\text{syn}}\) and \(d_{\text{noise}}\) are the synthetic data and background noise, respectively, \(\text{MA}(d_{\text{noise}})\) refers to the arithmetic mean of all the maximum amplitudes of the noise data, and \(\text{NoiseLevel}\) is the reciprocal value of the signal-to-noise ratio (S/N). Because the acquisition system and velocity model in the numerical tests are from a field data set, for each geophone, we use observed background noise \(d_{\text{noise}}\) (Figure 3) to generate \(\text{Noise}\) for a certain NoiseLevel and add it to the synthetic waveform \(d_{\text{syn}}\) to simulate the recorded data.

We add five different levels of noise to the synthetic waveforms, from NoiseLevel = 0 (no noise with S/N = +∞) to NoiseLevel = 4 (the noise is four times the signal with S/N = 0.25) in equation 10. The SMT inversion test results are shown in Figure 4a–4d, including percentages of the DC/CLVD/ISO components and four angles representing the DC component, i.e., the strike, dip, and rake angles of the nodal planes (fault-plane solution, derived from the DC component, shown as the beach balls) together with the omega angle (\(\omega\)), calculated by the following equation (a modification of equation 67 in Tape and Tape, 2012):

\[
\omega = \text{acos} \left( \frac{M_{\text{DC}} \cdot N_{\text{DC}}}{M_{\text{DC}}^T \cdot N_{\text{DC}}} \right) = \text{acos} \left( \frac{\sum m_{ij}n_{ij}}{\left( \sum n_{ij}^2 \right)^{1/2} \left( \sum m_{ij}^2 \right)^{1/2}} \right),
\]

where \(M_{\text{DC}}\) and \(N_{\text{DC}}\) are the DC components of the input and inverted seismic moment tensors, with \(m_{ij}\) and \(n_{ij}\) as their elements, respectively. The percentage misfit is derived from the percentage of the inverted SMT minus the input value. The misfit in the fault-plane solution is represented by a comparison of the input and output beach-ball diagrams, which contain the information of the strike/dip/rake angles, and the omega angle, which indicates the difference between the input and inverted orientations of the principal axis that correspond to the DC components (Tape and Tape, 2012, 2015; Staněk et al., 2014; Shang and Tkaldić, 2020).

From Figure 4a to 4d, we can see that, for the two mechanisms S1 and S2, misfits in the percentages of the DC/CLVD/ISO components are less than 10%, and the inverted beach balls (only corresponding to the DC component) are all close to the true beach balls of S1 and S2, with the omega angles between the input and output DC components being less than 2°. We further use the normal (100% DC, strike 0°, dip 45°, and rake −90°) and reverse (100% DC, strike 0°, dip 45°, and rake 90°) faults as input sources to test the inversion. The results (Figure 4e–4h, marked as S3 and S4, respectively) show that their percentage misfits are about twice those for S1 and S2, whereas all the DC misfits are very small (\(\omega < 2^\circ\)). The results indicate that the microseismic SMT inversion can tolerate moderate noise under an adequate surface monitoring system.

Influence of \(V_{P0}\) on SMT inversion

In microseismic data processing, an accurate subsurface velocity model is often unavailable. Therefore, we perform numerical tests to assess the sensitivity of the inverted focal mechanism solutions to errors in \(V_{P0}\). Two models (marked as MV1 and MV2) with different vertical velocities from the true model M0 are selected. Because we only use the first-arrival P-waves during SMT inversion, MV1 and MV2 have 5% random \(V_{P0}\) perturbations in each layer compared with M0. Details of these models (M0/MV1/MV2) are shown in Figure 2a.

The synthetic waveforms are computed with the true model M0, and then SMT inversions are conducted using the elementary seismograms generated by the incorrect models MV1 and MV2. The test results for the two mechanisms S1 and S2 are shown in Figure 5. The first two columns S1-MV1 and S1-MV2 indicate the results using models MV1 and MV2 for S1, respectively. The third and fourth columns S2-MV1 and S2-MV2 indicate the results using...
these two models for S2, respectively. From Figure 5, we make the following three observations for the tests with NoiseLevel from 0 to 4:

1) The maximum misfits in the percentages of the DC, CLVD, and ISO components are approximately 15% for S1 (Figure 5b) and 60% for S2 (Figure 5d), when the noise level increases to four times the signal. The influence of velocity on the percentages is more prominent when compared with the results from the synthetic data with noise (Figure 4a–4d). This also means that, for different mechanisms, the impact of errors in $V_{P0}$ on the SMT inversion is different. For a surface array, the dip-slip SMT inversion is more likely to be affected than the strike-slip inversion.

2) The inverted DC components are relatively accurate for S1, with omega angles of less than 4° and beach balls close to the true ones (Figure 5e and 5f), whereas for S2, the omega angles reach 21° and the beach balls are slightly different from the correct ones (Figure 5g and 5h). Comparing the results with the noise in the data (Figure 4a–4d), it is evident that the influence of $V_{P0}$ errors on the inverted fault-plane solutions of S1 is at the same level, whereas its influence on the DC components of S2 is relatively large.

3) The percentages of the DC, CLVD, and ISO components are more easily influenced by errors in the vertical velocity, with a 15% misfit for S1 and a 60% misfit for S2 (Figure 5a–5d), whereas the fault-plane solutions (strike, dip, and rake angles) are less affected (Figure 5e–5h).

**Influence of $\epsilon$ and $\delta$ on SMT inversion**

Similar to the previous section “Influence of $V_{P0}$ on SMT inversion,” we also evaluate the impact of errors in $\epsilon$ and $\delta$. Two models (marked as MA1 and MA2) with the same $V_{P0}$ but different $\epsilon$ and $\delta$ from the true model M0 are selected. Details of these three models M0/MA1/MA2 are shown in Figure 2b and 2c. Because $\epsilon$ and $\delta$ govern the horizontal P-wave velocity and the variation in the P-wave velocity away from the vertical direction, respectively, errors in these parameters (MA1 and MA2) may change the arrival times and amplitudes at some receivers. Although differences in arrival times between the synthetic and observed data can be partially compensated by shifting the time, misfits in synthetic amplitudes may still affect the SMT inversion. The inversion results using models MA1 and MA2 for mechanisms S1 and S2 are listed in Figure 6. From the results, we make the following three observations for the tests with a NoiseLevel from 0 to 4:

1) The maximum misfits of the DC/CLVD/ISO percentages are approximately 10% for S1 (Figure 6b) and 50% for S2 (Figure 6d), when the noise level increases to four. This phenomenon agrees with the results when using incorrect $V_{P0}$ (Figure 5a–5d), which means that $\epsilon$ and $\delta$ can also affect the percentages of SMT inversion, especially for the dip-slip mechanism (S2).

2) The inverted DC components are still accurate for S1 (the omega angles are less than 2°, and the beach balls are nearly the same, Figure 6e and 6f), whereas for S2, the omega angles reach 21° (Figure 6g and 6h), and the beach balls are different from the correct ones (Figure 6i and 6j). Comparing the results with the noise in the data (Figure 4a–4d), it is evident that the influence of $V_{P0}$ errors on the inverted fault-plane solutions of S1 is at the same level, whereas its influence on the DC components of S2 is relatively large.

**Figure 4.** Inversion results from synthetic waveforms with different levels of noise. (a) Percentage misfits of the inverted DC (the solid red line), CLVD (the dashed blue line), and ISO (the dotted green line) components for strike-slip source S1 (100% DC) with NoiseLevel from 0 (no noise with S/N = +∞) to 4 (the noise is four times the signal with S/N = 0.25). (b) Beach balls and omega angles of the inverted results for S1 (beach balls and omega angles only reflect the DC components). (c) Percentage misfits of the inverted results for dip-slip source S2 (100% DC). (d) Fault-plane misfits and omega angles of the inverted results for S2. (e and f) Inversion results for normal fault S3. (g and h) Inversion results for reverse fault S4.
angles reach approximately 20° and the beach balls are slightly different from the true ones (Figure 6g and 6h). These results also agree with the test results in Figure 5e–5h, which means that $V_p$, $\varepsilon$, and $\delta$ can affect SMT inversion when the source mechanism is dip slip (S2).

3) The percentages of the DC, CLVD, and ISO components are more easily influenced by incorrect $\varepsilon$ and $\delta$, with a 10% misfit for S1 and a 50% misfit for S2 (Figure 6a–6d), whereas the fault-plane solutions (represented by the beach balls) are affected at a lower level (Figure 6e–6h).

INVERSION OF FIELD DATA

Surface microseismic data set and model

To test the SMT inversion using the proposed method, we apply it to a microseismic field data set in southern China. This data set is obtained from a star-like surface array (Figure 7a), which has already been used in the numerical tests and sensitivity analysis (Figure 1). The acquisition system is composed of 1771 geophones distributed along six crosslines every 20 m. The aperture of the receiver array is approximately 6935 m in the $x$-direction (west to east) and 5738 m in the $y$-direction (south to north). The waveform data provided in this data set are vertical-component ground velocities, and the frequency band of the geophones is 1–500 Hz.

The microseismic events are induced by hydraulic fracturing injection to a shale gas reservoir from a vertical well (Figure 7b and 7c). The surface projection of the fracturing perforation is located at 2796 m in the east and 2327 m in the north. We select two microseismic events with a high S/N from this data set. These two events are completely separated in the time domain. The time length of the field data is 501 ms, with a sampling interval of 1 ms. The synthetic seismograms are resampled to the same rate before the inversion. The waveform data of these two events, after static correction and proper filtering, are displayed in Figure 8.

The subsurface structure revealed by the reflection seismic profile indicates that the studied area is dominated by horizontal layers. Therefore, we construct a horizontally layered 3D velocity model by smoothing P-wave well-logging data (already used in the numerical test shown in Figure 2a as model M0). The target layer is between 3716 and 4120 m in depth, which corresponds to the Longmaxi Shale. The velocity anisotropy parameters $\varepsilon$ and $\delta$ for each layer (Figure 2b and 2c as model M0) are set according to the well-logging data and previous studies in this area (Deng et al., 2015; Liu et al., 2017; Li et al., 2019).

Imaging source locations

Unlike for the synthetic data set where the source location is known, for the field data set, we have to obtain the source location.
Because model M0 (Figure 2) used for SMT inversion accounts for anisotropy, a method based on the backward-propagating wavefield (Gajewski and Tessmer, 2005; Artman et al., 2010) is applied to obtain the source location. Wave modeling is still conducted using the SGFD method. Because only first-arrival P-waves can be clearly identified in the field data set (Figure 8), we use this phase for backpropagation and stacking. The length of the sampling window is 60 ms, which contains roughly two cycles for signals with a 30 Hz dominant frequency. The stacked images for events no. 1 and no. 2 are displayed in Figures 9 and 10, respectively. According to the maximum energy distributions, the source locations of these two events are determined to be (2780, 2250, 3880 m) and (2790, 2315, 3800 m) in the order of east, north, and depth, respectively. The depths of these two microseismic sources are located in the target shale layer (3716–4120 m depth), and their surface projections are located around the hydraulic fracturing perforation (2796 m in the east and 2327 m in the north).
Figure 8. Observed velocity records of microseismic events (a) no. 1 and (b) no. 2. There are 1771 traces in total (the locations are shown in Figure 7a), and the recording length is 501 ms. The amplitudes corresponding to the color scale are approximately $-10^{-4}$ to $10^{4}$ m/s.

Figure 9. Source imaging results for locating event no. 1. The maximum energy is located at (2780, 2250, and 3880 m) in the order of east, north, and depth, respectively. (a–c) A depth slice and two vertical slices, respectively.
SMT inversion for two microseismic events

Only vertical-component data are available in this surface microseismic monitoring case. For microseismic events no. 1 and no. 2, based on their source locations, we adopt the proposed microseismic SMT inversion method and compute their source moment tensors (Figures 11 and 12). For event no. 1, the inverted \( \mathbf{m}_1 \) is \((-0.64, 0.45, -0.03, -0.07, -0.14, -0.18) \times 10^{11} \text{ N}\cdot\text{m}\), the fault strike/dip/rake angles are \((52.6^\circ, 77.8^\circ, -4.1^\circ)\) and \((143.4^\circ, 86.0^\circ, -167.7^\circ)\), and the percentages of the DC, CLVD, and ISO components are 69.0%, -20.6%, and -10.4%, respectively (Figure 11c and 11d). The negative signs of the ISO and CLVD components indicate some volume loss in the source region, potentially due to the closure of a preexisting fracture or some other mechanisms linked to compaction accompanied by shear. Figure 11a and 11b shows maps of the selected geophones and a comparison of their recorded (black) and synthetic (red) waveforms. Similarly, for event no. 2, the inverted \( \mathbf{m}_2 \) is \((-0.61, 0.28, -0.04, -0.04, -0.01, 0.06) \times 10^{11} \text{ N}\cdot\text{m}\), the fault strike/dip/rake angles are \((44.1^\circ, 85.7^\circ, 2.7^\circ)\) and \((313.9^\circ, 87.3^\circ, 175.7^\circ)\), and the DC, CLVD, and ISO components occupy 54.4%, -25.5%, and -20.1%, respectively, of the total SMT. The observed (black) and synthetic (red) waveform fitting results of the selected geophones and their locations are also shown in Figure 12a and 12b.

Comparing the inverted SMT results of the two events (Figures 11 and 12), we find that their beach balls are very similar, the moment magnitudes of these two events are 1.2, and their DC percentages are approximately 60%. Research via acoustic emission monitoring has shown that the main fracture is formed by the interactions between cracks in the rock damage zone, and that the macroscopic shear rupturing composes the opening or closing of secondary cracks at the ends and surroundings of the fracture surface (Lei et al., 2000). Because the distance between events no. 1 and no. 2 is small, it is likely that they have similar focal mechanisms and are induced by similar seismic origins.

Evaluation of the SMT solutions

The SMT inversion results of the microseismic field data are evaluated via three criteria:

1) Waveform fitting between the observed and synthetic data:

From the fitting results of these two events (Figures 11b and 12b), it is shown that the synthetic waveforms are very close to the actual recorded data, which indicates the reliability of the SMT solution.
2) Comparison with the results from numerical tests:
In this paper, the acquisition system and velocity model in the numerical tests are the same as those used in field data processing. Comparing the inverted $m_1$ and $m_2$ with the two source mechanisms (strike-slip S1 and dip-slip S2, Figure 4a–4d) in numerical tests, we find that the beach balls of these two microseismic events (no. 1 and no. 2, Figures 11c and 12c) are similar to the strike-slip mechanism (Figure 4b). The numerical tests indicate that the strike-slip SMT inversion is relatively stable (a percentage misfit of less than 15%, an omega angle of less than 4°, marked as S1 in Figures 4–6).

3) Inversion tests using partial field data:
We conduct several SMT inversions using randomly selected partial data sets, namely, the jackknife tests (statistical resampling tests, e.g., Pesicek et al., 2014; Hicks and Rietbrock, 2015), and we compare the computed results with the inverted $m_1$ and $m_2$. To create these partial data sets, we randomly select 70%, 50%, or 20% of the traces from the original 1771 traces. For each percentage, 100 partial data sets are generated and inverted for the SMT. Figures 13 and 14 show the deviations of the inverted parameters relative to the results using the whole data set ($m_1$ and $m_2$). Columns (a–c) in Figures 13 and 14 refer to the results using 70%, 50%, and 20% of the total traces, respectively. Shown in each subfigure are histograms of frequencies per 100 random tests for the given errors, with the horizontal coordinates indicating the errors (absolute errors for angles and DC, CLVD, and ISO components and percentage errors for the scalar moment tensor). The rows from top to bottom are for the strike, dip, and rake angles; the DC, CLVD, and ISO components; and the scalar moment tensor, respectively. The numbers in the upper right of each subfigure denote the mean value and the standard deviation.
Seismic field data show that the proposed method is robust and stable. and synthetic data. Numerical tests and applications of surface microseismic source mechanism inversion method to the 3D anisotropic elastic wave equation. SMT is inverted based on the moment-tensor model to represent the microseismic source and adopts the SGFD method to calculate the synthetic seismograms. In this paper, a microseismic source mechanism inversion method is introduced, which may affect the synthetic waveforms and lead to errors in the SMT inversion result on data noise and errors in anisotropy parameters. According to the results, SMT inversion can tolerate moderate noise in the data (four times the signal). After shifting the time between the observed and synthetic data, we find that the SMT inversion results, especially the percentages of the DC, CLVD, and ISO components, can still be significantly influenced by incorrect vertical velocity and velocity anisotropy parameters. This means that the inverted non-DC (CVLD and ISO) components in the seismic focal mechanism inversion may be a result of an imperfect medium model. Moreover, the numerical tests indicate that, using a surface acquisition system, inversions of a strike-slip source are more stable than those of a dip-slip one and the worst results are from the 45° dip-slip (normal or reverse) fault. This is because, for a strike-slip source, there are four P-wave radiation lobes falling into the array aperture, whereas for a dip-slip source, there are only two lobes from the P-wave radiation pattern that fall into the array aperture, and for a 45° dip-slip fault, basically only one radiation lobe can enter the array aperture (see the beach balls in Figure 4). This enables the surface array to better constrain the nodal planes of a strike-slip source than that of a dip-slip source or a 45° dip-slip fault. These conclusions are also consistent with results from other studies on SMT inversion (e.g., Staněk et al., 2014). Moreover, we find that velocity anisotropy and vertical velocity errors can affect the SMT inversion of a dip-slip mechanism (percentage misfit approximately 50% and omega angle approximately 20°). This indicates that failure to account for anisotropy may significantly affect the reliability of an SMT solution, especially the percentages of the DC, CLVD, and ISO components.

Other than anisotropy, the inversion of the microseismic source mechanism involves many other issues, e.g., the coverage of acquisition systems and uncertainty in the source location and source time function. Notice that the results shown here partially benefit from the wide source-receiver coverage. Differences in inverted parameters should be expected if different acquisition geometries are used. Source imaging has always been an important issue in microseismic studies, and the source time function may also affect SMT inversion results. In addition, compared with a 1D model, a 3D structure is probably closer to the true one. A 3D model has lateral variations, which may affect the synthetic waveforms and lead to errors in the SMT solution. In conclusion, we believe that more studies are needed.
in high-resolution 3D medium modeling and high-fidelity source mechanism inversions. By including velocity anisotropy into SMT inversion, the proposed approach has the potential to improve the study of the non-DC mechanisms of hydraulic fracturing-induced microearthquakes.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (grant nos. 41974156 and 41804050). The surface microseismic data were provided by the China National Petroleum Corporation. The authors thank the anonymous reviewers for their helpful suggestions.

DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.

APPENDIX A

SGFD METHOD

In anisotropic media, the 3D elastic wave equation is

\[
\rho \frac{\partial v_i}{\partial t} = \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial v_l}{\partial x_k} \right) + \rho f_i,
\]

where \( v_i \) is the particle velocity, \( c_{ijkl} \) is the stiffness tensor, and \( \rho \) is the stress tensor. In the SGFD method (Virieux, 1984, 1986; Graves, 1996), the differential operator is approximated by the following equation:

\[
\frac{df}{dt} \approx \frac{1}{\Delta t} \sum_{m=1}^{L} \left[ f \left( a_0 + \frac{2m-1}{2} \Delta a \right) - f \left( a_0 - \frac{2m-1}{2} \Delta a \right) \right].
\]

The following stability conditions (Levander, 1988; Graves, 1996) should be met to ensure accuracy of the SGFD scheme:

\[
\Delta t < 0.495 \cdot \frac{\Delta x}{v_{\text{max}}} - \frac{\Delta x}{\lambda_{\text{min}}} \leq \frac{\Delta x}{5}.
\]

REFERENCES


Hicks, S. P., and A. Rietbrock, 2015, Seismic slip on an upper-plate normal fault during a large subduction megathrust rupture: Nature Geoscience, 8, 955–960, doi: 10.1038/ngeo2585.


Biographies and photographs of the authors are not available.