Target oriented full-wave equation based illumination analysis
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Summary
We propose a full-wave equation based illumination analysis method with target oriented capability. The full-wave finite-difference propagator is used to extrapolate the source and receiver side wavefields to the subsurface target area. A time-domain local-slowness analysis method is used to decompose the wavefields into local angle domain. The local illumination matrix can be constructed and different types of illumination measurements can be derived. This illumination analysis method does not have angle limitations. Thus this approach can handle structures with their dipping angles beyond 90 degrees and is particularly useful to provide illumination analysis for reverse-time migration.

Introduction
The seismic illumination analysis quantifies the capability that an acquisition system can image the subsurface structures. The illumination measurements can be used to optimize the acquisition survey design, evaluate the image quality, and make corrections to seismic image, resulting in more accurate subsurface physical parameter retrieval. Although the early attempts in illumination estimates adopted oversimplified model e.g., homogeneous velocity model, horizontal target and symmetric ray paths, recently it has developed quickly. The illumination calculation can be considered a special type of modeling. First, seismic waves from sources and receivers need to be propagated through overburden structures to reach to the targets. Second, the propagation directions for both waves need be determined at the target location since targets with different dipping angles can only be illuminated by waves at certain directions. Finally, the illumination measurement can be calculated from incident-scattering waves from different directions.

The ray-based methods can handle smoothly varying heterogeneous velocity models and has no angle limitations. Particularly, because the ray method naturally carries the wave propagation directions, it has been widely used for illumination calculations (Schneider and Winbow, 1999; Muerdter and Ratcliff, 2001; Gelius, et al., 2002). However, the high-frequency asymptotic approximation causes intrinsic difficulties for this method being used in complex overburden structures.

Recently, the one-way propagator is widely used in seismic imaging. At the same time, methods specifically for extracting angle information from the wavefield are developed (Xie et al., 2002; Wu and Chen, 2002, 2003; Sava and Fomel, 2003). Combining the two techniques together, a one-way wave equation based method is developed for illumination analysis (Xie et al. 2003, 2005, 2006; Wu and Chen 2002, 2006; Wu et al. 2003; Jin et al. 2003, 2006). This is an efficient approach and is consistent with most one-way wave equation based migration methods. However this approach cannot properly handle the wide-angle signals such as turning waves.

Figure 1: Sketch showing the calculation of illumination measurements, with (a) -45 degree ADR map in a constant velocity model and from a source-receiver pair, (b) source-side and (c) receiver-side slowness analyses, (d) local illumination matrix and (e) illumination as a function of reflection angle for a dipping reflector.

In the last few years, reverse-time migration regains the attention in seismic community partially because it does not have the angle limitation and can properly image the steeply dipping structures with turning waves. It is also a challenge to conduct the illumination analysis that is consistent with the reverse-time migration. To overcome the angle limitation of the one-way propagator, Xie and Yang (2008) proposed a method which uses the full-wave finite-difference method as the propagator and uses a time domain local slowness analysis method to determine the angle information and calculate the illumination. Cao and Wu (2008) proposed another full-wave method which uses frequency domain split-step local plan-wave decomposition for angle analysis. In this paper, we will further develop the method of Xie and Yang (2008) and present the target...
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oriented illumination analysis using the full-wave propagator. The new method does not have angle limitations usually encountered by a one-way approach. In addition, it accurately solves the energy transmission in a complex velocity model. This method is particularly useful in providing illumination analysis for reverse-time migration.

Formulations for Different Illumination Measurements

The illumination analysis for a target can be started from investigating an incident source beam towards the target at angle \( \theta_s \) and a scattered beam leaving the target towards the receiver at angle \( \theta_g \). This pair of incidence-scattering beams forms a basic illumination measurement. Note, due to multi-pathing, scattering and reflection, etc., the waves from a source can come to a target from multiple directions, and so is the case for waves leaving the target back to the receiver. We then collect all possible incidence and scattering beams from one pair of source-receiver and they give complete description of illumination for the target. In frequency domain, such collection of beams can be expressed with an illumination matrix (Xie, et al. 2006)

\[
A\left(\omega, \mathbf{r}_s, \theta_s; \mathbf{r}_g, \theta_g\right) = \frac{2\omega^2}{v^2} I\left(\omega, \theta_s, \mathbf{r}_s, \mathbf{r}_g\right) I\left(\omega, \theta_g, \mathbf{r}_g, \mathbf{r}_s\right)
\]

where \( \omega \) is the frequency, \( v \) is local velocity, \( I \) is the energy flux in \( \theta \) direction, \( \mathbf{r} \) is the target location, \( \mathbf{r}_s \) and \( \mathbf{r}_g \) are source and receiver locations respectively.

Equivalently, we can calculate a similar illumination matrix using broad band time domain wavefield. Since the time-domain wavefield does not directly provide the propagation direction, we first decompose the Green’s function into angle domain beams using a method similar to that used by Xie et al. (2005)

\[
G\left(\theta_s, \mathbf{r}_s, \mathbf{r}_g, t\right) = C \int W\left(\mathbf{r} - \mathbf{r}'\right) \delta\left(\mathbf{r}' - \mathbf{p}_s - \mathbf{r}\right) d\mathbf{r}'
\]

where \( G\left(\theta, \mathbf{r}_s, \mathbf{r}_g, t\right) \) is the Green’s function generated by the FD method, \( G\left(\theta, \mathbf{r}_s, \mathbf{r}_g, t\right) \) is its angle-domain decomposition. \( \mathbf{p}_s \) is the slowness vector, \( W \) is a space window centered at \( \mathbf{r} \), \( C \) is a normalization factor. A similar equation can be found for scattering beam \( G\left(\theta_g, \mathbf{r}_g, \mathbf{r}_s, t\right) \).

Considering the frequency and time domain equivalents

\[
\omega^2 I\left(\omega, \theta_s, \mathbf{r}_s, \mathbf{r}_g\right) \approx \int \left[ \frac{\partial G\left(\theta_s, \mathbf{r}_s, \mathbf{r}_g, t\right)}{\partial t} \right]^2 dt,
\]

\[
I\left(\omega, \theta_g, \mathbf{r}_g, \mathbf{r}_s\right) \approx \int \left[ G^2\left(\theta_g, \mathbf{r}_g, \mathbf{r}_s, t\right) \right] dt
\]

the local illumination matrix can be expressed as

\[
A\left(\mathbf{r}, \theta_s, \theta_g; \mathbf{r}_s, \mathbf{r}_g\right) = \left(2/v^2\right) \int \left[ \frac{\partial G\left(\theta_s, \mathbf{r}_s, \mathbf{r}_g, t\right)}{\partial t} \right]^2 dt \cdot \int G^2\left(\theta_g, \mathbf{r}_g, \mathbf{r}_s, t\right) dt
\]

For an acquisition system composed of multiple sources and receivers, its illumination matrix can be obtained by summing up the contributions from individual source-receiver pairs

\[
A\left(\mathbf{r}, \theta_s, \theta_g\right) = \sum_{\mathbf{r}_s} \sum_{\mathbf{r}_g} A\left(\mathbf{r}, \theta_s, \theta_g; \mathbf{r}_s, \mathbf{r}_g\right)
\]

In the above equations, the angles \( \theta_s \) and \( \theta_g \) are related to the acquisition coordinate. With coordinate transforms \( \theta_d = \left(\theta_s + \theta_g\right)/2 \) and \( \theta_r = \left(\theta_s - \theta_g\right)/2 \), we can obtain the illumination matrix \( A\left(\mathbf{r}, \theta_d, \theta_r\right) \) defined in target coordinate system, where \( \theta_d \) and \( \theta_r \) are target dipping angle and reflection angle, respectively. Comparing to the acquisition coordinate, the target coordinate is more suitable for investigating the target oriented illumination.

Numerical Examples for Illumination Calculations

We use numerical examples to show the calculations of different illuminations.

Local illumination matrix. Figure 1 illustrates the basic concept of the illumination analysis. Shown in Figure 1a is a -45 degree ADR map, calculated in a constant velocity model, overlapped with the source and receiver waves. The slowness analyses shown in 1b and 1c give the wave propagation directions. At the target location, we construct
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a local illumination matrix (shown in 1d) in which the horizontal and vertical coordinates are incidence and scattering angles while the two diagonals are dipping angle and reflection angle, respectively.

Shown in Figure 2 are local illumination matrices located at selected locations in a constant velocity model. The sources cover the surface with a left side receiver geometry from 0 to 5.4 km. For this specific model the illuminations are limited within ±90° for both θ_i and θ_t (the inner square) and the reflection angle coverage decreases with the increase of the depth. Shown in Figure 3 are similar illumination matrices in a model composed of a depth dependent background and a salt dome with steep and overhang flanks. The acquisition system is the same as that used in Figure 2. For most of these local illumination matrices, the dipping angle coverage is beyond ±90°. The local illumination matrices become complex closing to the salt body, partially affected by the reflection waves generated on the salt body. Note that the area can be covered by full-wave illumination method (the entire outer square) is much larger than that can be covered by a one-way propagator based illumination method (the inner square). This makes the full-wave method can investigate structures with dipping angles beyond ±90° (e.g., overhung structures).

The acquisition dip response. The acquisition dip response (ADR) is the total illumination for a target with a given dipping angle. The ADR can be obtained from the illumination matrix by substituting the dipping angle θ_d with the target dipping angle and integrate the matrix over reflection angle θ_r (Xie and Yang, 2008)

\[ D(r, θ_d) = \int A(r, θ_d, θ_r) dθ_r. \] (6)

The background in Figure 1a shows a −45° ADR generated by a pair of source-receiver in a constant velocity model. To help understand the calculation, a small −45° target is added in the figure. Shown in Figures 4a-d are ADRs for -135, -90, -45, and 0 degrees in a v(z) model and generated by a pair of source-receiver. The model is 15 km in horizontal by 3.6 km in depth and the velocities at the top and bottom are 1.5 and 3.5 km/s, respectively. A 20-Hz Ricker wavelet is used in the FD calculation. As can be seen from the figure, turning waves in a v(z) model illuminate structures with their dipping angle exceeding ±90°, which is the limitation of a one-way wave equation based method. To help clearly see the illumination at large depths, the automatic gain control (AGC) is used in calculations for a single source-receiver pair.

The FD calculated Green’s functions naturally contain direct waves, reflections, turning waves, refractions and multiples, etc. The full-wave illumination analysis includes contributions from these waves, while a one-way propagator usually cannot handle up-going waves. Shown in Figure 5 are ADRs for a two-layer velocity model. The velocities in the top and bottom layers are 2.5 and 4.0 km/s, and the interface is located at depth 3.12 km. We use this model to demonstrate how to analyze contributions from different waves which can be separated by propagation directions and time windows. Shown in Figure 5a is the −45° ADR which is generated by direct “down-down” waves. Figure 5b is the −120° ADR generated by “up-up” reflections. Figure 5c is the −120° ADR obtained from the
direct source wave and reflected receiver wave, i.e. the “down-up” waves. The later two ADRs cannot be properly handled by a one-way propagator based illumination analysis.

Illumination as a function of target reflection angle: For a locally planar target with a normal vector pointing to direction \( \theta_n \), the target-oriented illumination as a function of reflection angle can be obtained from the illumination matrix (Xie, et al., 2006)

\[
D(r, \theta_r, \theta_d) = A(r, \theta_d, \theta_r)^{\frac{\theta_d}{\theta_n}}. \tag{7}
\]

Figure 1d illustrates the calculation of \( D(r, \theta_r, \theta_d) \). The dashed line passes through the energy peak is \( \theta_d = -45^\circ \) and the coordinate along the line is reflection angle \( \theta_r \). The illumination as a function of reflection angle can be directly measured in the illumination matrix along this line. As an example, the illumination distribution for a -45º target is shown in Figure 1e as a fan-shaped distribution. Note the maximum illumination is slightly biased from the normal direction, resulting from the finite offset between the source and receiver (see 1a and 1d).

Figure 6 compares the target oriented illumination with the reverse-time image for the salt dome model. Figure 6a shows the reverse-time image (Xie and Wu, 2006) where only one shot is used and the receivers cover trace numbers 0 to 320. Shown in 6b is the illumination as a function of reflection angles along the bottom reflector and salt flank. The illumination estimate is consistent with the migration image. The horizontal interface and vertical salt flank are properly illuminated and imaged. However, the overhung part of the salt flank (indicated by the arrow) is missing from the image. The target oriented illumination properly indicates that there is an illumination hole at this part of the salt flank.

The volumetric ADR: Figure 7 shows the volumetric ADR for the salt dome model shown in Figure 3. We use 46 surface shots to generate the illumination. Figure 7a is the -135º ADR map. The overhung structure of left salt flank can be illuminated by the adopted acquisition geometry. The weak illumination from the reflection waves can be found at the left side of the dome structure, and there is a shadow area on the right side of the dome. Similarly, Figure 7b indicates that the -90 degree left salt flank can be well illuminated. The shadow area is much smaller than that in Figure 7a. Figures 7c-d show that structures with small dipping angles can be evenly illuminated.

Conclusions

We propose a full-wave equation based illumination analysis method which has the target oriented capability. The full-wave finite difference propagator is used to extrapolate the source and receiver waves to the subsurface target. A time domain local slowness analysis is used to decompose the wave fields into local angle beams and form the local illumination matrix. ADRs beyond 90 degrees can be extracted from the illumination matrix. The current approach has no angle limitations and is particularly useful for providing illumination information for reverse-time migration.

Acknowledgments

This research is supported by the WTOPI Consortium at the University of California, Santa Cruz. The facility support from the W.M. Keck Foundation is also acknowledged.
EDITED REFERENCES
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