AVA analysis based on RTM angle-domain common image gather
Rui Yan* and Xiao-Bi Xie, IGPP, University of California, Santa Cruz

Summary
We propose an alternative approach of AVA analysis based on RTM angle-domain common image gathers (ADCIGs). For each shot, the incident angle is measured from the most energetic phase of single-frequency wavefields. Using local slant stacks, the source wavefields are projected to the incident angle direction. The dip of reflector is obtained from the stacked depth image. By assuming mirror reflection, the reflection angle can be calculated from the dip angle and incident angle. Shot-domain common image gather are formed by deconvolution imaging condition of decomposed source wavefields and original/decomposed receiver wavefields and then are sorted into ADCIG by reflection angle. AVA is extracted from ADCIGs and analyzed in cross-plot domain. A layered reservoir model is used to demonstrate the accuracy of AVA calculation. Another reservoir model illustrates that the proposed AVA analysis method can clearly separate reservoir reflections from background reflections.

Introduction
Dynamic information about the wavefields such as AVA (amplitude versus angle) analysis is crucial in geological interpretation of reservoirs. A traditional procedure for AVA analysis is to examine the alignment of images created with multi-offset data. This method works well for generally flat media. However, complex overburden stratigraphic structures generate complex seismic wavefields and may obscure the signals from the reservoir. If the surface seismic data is migrated into target areas, the propagation effects will be removed (Mosher, 1996).

Reverse time migration (RTM) is a powerful tool to collapses the surface seismic traces to subsurface images. First of all, RTM can produce high-fidelity subsurface images, providing better confidence to locate the correct position of AVA anomalies. In terms of amplitude preservation, RTM is a dip-unlimited accurate propagator, thus it has great advantage over ray-based method and one-way propagator. Nevertheless, a dynamically correct imaging condition is still needed. Cross-correlation of source and receiver wavefields is a stable imaging condition, but it is not good to extract the reflectivity. To recover accurate AVA amplitudes, the cross-correlation normalized by source illumination (Claerbout, 1971) is preferred. Many authors (Valenciano and Biondi, 2003; Guitton, 2007; Schleicher et al., 2008; Chattopadhyay and McMechan, 2008) have investigated this imaging condition.

Angle information is not explicit in RTM. To retrieve the AVA behavior of the reflections, the most crucial step is to build ADCIGs (angle-domain common image gathers) from RTM. In recent years, plenty of methods have been proposed and they mainly fall into three categories. In the first category, ADCIGs are obtained by first decomposing both source and receiver wavefields into localized beams with different directions, followed by an imaging condition applying to these localized beams around the image point (Xie and Wu, 2002). This method has been applied to RTM (e.g., Zhang et al., 2010; Xu et al., 2011; Yan and Xie, 2011).

In the second category, ADCIGs are extracted during the imaging stage. The extended images with space and/or time lags are calculated, and then are transformed into angle-domain by a slant stack (Sava and Fomel, 2003, 2006). The ADCIGs are extracted for RTM by some researchers (e.g., Zhang et al., 2007; Vyas et al., 2010).

Under the third category, the Poynting vector or the polarization vector of the P-wave is calculated and used to determine the wave propagation direction. Some authors (e.g., Zhang and McMechan, 2011a, b; Yoon et al., 2011) chose to calculate the source wave direction only and focused on the contributions of one or a few most energetic phases. Then, with known reflector dipping angle and assuming mirror reflection (i.e., the reflector is locally planar), the ADCIG is calculated. Others (e.g., Dickens and Wimbow, 2011) computed the energy flux directions for both the source and receiver wavefields and extracted angle dependent reflectivity at the imaging point.

In this paper, we propose a RTM-based AVA analysis. First, we describe the angle decomposition techniques. Second, we measure the incident angle from single-frequency source wavefields. Third, we extract the dip information from the migration image. By taking advantage of dip information, we produce true-amplitude ADCIGs. Fourth, AVA behavior is analyzed in cross-plot domain. Fifth, two numerical experiments are used to demonstrate the AVA results extracted from subsurface image.

Angle Decomposition
Here we target to investigate the AVA feature of P-P reflection, so we use acoustic RTM to extrapolate the seismic data. First, the time-domain wavefields are transformed to frequency domain and then are decomposed into local plane waves using local slant stacks:

$$u(\theta, x, \omega) = \int W(x - x') u(x', \omega) e^{i\omega\phi(x')} dx', \quad (1)$$

where $u(x', \omega)$ stands for either source or receiver wavefield and $u(\theta, x, \omega)$ is its decomposed local plane
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wave whose propagation direction is $\theta$, $W(x' - x)$ is a local spatial window function centered at location $x$. $\mathbf{p} = \hat{e} / v$ is the slowness vector, where $v$ is the average velocity within the sampling window, $\hat{e}$ is the unit vector specifying the wave propagation direction.

Angle-domain deconvolution imaging condition

To apply the cross-correlation imaging condition to the decomposed plane waves, we derive the partial image for an image location $x$:

$$I(\theta_s, \theta_g, x) = \int\left[ u_s(\theta_s, x, \omega) \cdot u_g^*(\theta_g, x, \omega) \right] d\omega,$$  (2)

where $I(\theta_s, \theta_g, x)$ is the angle-domain partial image; $u_s(\theta_s, x, \omega)$ is the local incident (source-side) plane wave and $u_g^*(\theta_g, x, \omega)$ is the local scattered (receiver-side) plane wave; $\theta_s$ is the local incident angle and $\theta_g$ is the local scattering angle.

To obtain the true-amplitude image, we normalize the partial image by the strength of source illumination, i.e., the deconvolution imaging condition. The imaging condition can be expressed as:

$$T(\theta_s, \theta_g, x) = \frac{I(\theta_s, \theta_g, x)}{S(\theta_s, x)},$$  (3)

where the source illumination is

$$S(\theta_s, x) = \int\left[ u_s(\theta_s, x, \omega) \cdot u_s^*(\theta_s, x, \omega) \right] d\omega.$$  (4)

We can convert the representation $T(\theta_s, \theta_g, x)$ to $\tilde{T}(\theta_s, \theta_g, x)$ by a coordinate transform (e.g., Xie and Wu, 2002)

$$\theta_d = \frac{\theta_s + \theta_g}{2},$$  (5)

and

$$\theta_r = \frac{\theta_s - \theta_g}{2},$$  (6)

where $\theta_d$ is the target dip angle and $\theta_r$ is the reflection angle (See Figure 1). Since the AVA response of the reflector is reflection-angle-dependent, it is related to angle-domain partial image in the following way

$$R(\theta_r, x) = \tilde{T}(\theta_r, \theta_d, x).$$  (7)

Incident angle

The single-frequency wavefields (usually at dominant frequency) is used to measure the propagation direction of source wavefields. For each image point, we use equation 1 to calculate every possible angle component of single-frequency source wavefields. The direction with the maximum coming energy will be picked to calculate the incident angle. Though the final image is contributed by many arrivals from various directions, it is safe to choose the most energetic phases for the purpose of AVA analysis.

Figure 1. The schematic sketch of local angle-domain analysis.

Structure dip

The dip information can be extracted from the migration image. We select an image area around each image point and transform the local image into wavenumber domain through a Fourier transform. In the wavenumber domain, we compute the energy along all directions, and the direction with the maximum energy will be considered as the normal direction of the reflector. This approach works well in areas with clear dip. The areas with ambiguous dips are usually not suited for AVO analysis thus will not cause problem.

Given both the incident angle and the dip angle, the reflection and scattering angles can be obtained. Thus, we can directly project both source and receiver wavefields to the incident and scattering directions to form a partial image as a function of the reflection angle. Note, in this process, the wavefield decomposition around all directions is not required. This can tremendously reduce the computation cost. Optionaly, we can choose not to decompose the receiver wavefield. Instead, the image gather can be directly formed by the decomposed source wavefield and the original receiver wavefield.

AVA analysis

For relatively small reflection angles, the reflectivity of compressive wave can be approximated by an equation (Foster and Keys, 1999; Foster et al, 2010):

$$R(\theta) = A + B \cdot \sin^2 \theta,$$  (8)

where $\theta$ is the reflection angle. $A$ is the intercept and $B$ is the slope of the reflection coefficient.

For small perturbations in velocity and density at a reflection interface, the intercept and slope can be approximated by
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\[ A = \frac{\Delta V_p}{V_p} + \frac{\Delta \rho}{\rho}, \]  
\[ B = \frac{\Delta V_p}{V_p} - 4 \frac{\Delta V_s^2}{V_{p}^2} \left( \frac{\Delta \rho}{\rho} + \frac{\Delta V_s}{V_s} \right), \]

where \( V_p \), \( V_s \) and \( \rho \) are the averages of P-wave velocity, S-wave velocity and density above and below the reflecting interface, respectively; \( \Delta V_p \), \( \Delta V_s \) and \( \Delta \rho \) are the differences in P-wave velocity, S-wave velocity and density between these layers, respectively.

Let \( \gamma = \frac{V_s}{V_p} \), the relation between \( B \) and \( A \) is

\[ B = \left( 1 - 8 \gamma^2 \right) A - 4 \gamma \Delta \gamma + \left( 4 \gamma^2 - 1 \right) \frac{\Delta \rho}{2 \rho}. \]

In a cross-plot of \( A \) versus \( B \), if there is little contrast in \( \gamma \), the reflection tends to fall on the fluid line

\[ B = \left( 1 - 8 \gamma^2 \right) A. \]

### Table 1. Reservoir model

<table>
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<th>Porosity</th>
<th>P-velocity</th>
<th>S-velocity</th>
<th>Density</th>
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<td>Shale</td>
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<td>1668</td>
</tr>
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<td></td>
<td>23%</td>
<td>3170</td>
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<td></td>
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<td>23%</td>
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<td></td>
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<td></td>
<td>25%</td>
<td>3399</td>
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</tr>
</tbody>
</table>

### Numerical examples

We use a simple layered reservoir model to investigate the AVA behavior of reservoir reflections. The background medium is shale with constant velocity. A horizontal reservoir layer (hydrocarbon-saturated porous sand) is located in the background. Different pore fluids (gas, oil or brine) are filled into the reservoir layer. Shown in Table 1 are elastic parameters of shale and nine types of reservoirs. Nine pairs of intercepts and slopes are calculated by equation 9 and 10 and are cross-plotted in Figure 2.

In Figure 2, the reflections from the top of sand fall on a trend approximately parallel to the fluid line but with a negative intercept in the cross-plot domain. The reflections from the base of sand fall on a parallel trend on the opposite side of the fluid line. Each trend is defined by a \( V_p/V_s \) contrast (See equation 11). Hydrocarbon reservoirs typically have a lower \( V_p/V_s \) than the surrounding formation, and increasing pore fluid compressibility even reduces \( V_p/V_s \). Thus gas produces the greatest departure from the fluid line, followed by oil and brine. Porosity is another rock property that has an impact on seismic responses. Increasing the porosity decreases acoustic impedance, but do not significantly affect \( V_p/V_s \). As a result, porosity changes move the AVA response along the trend parallel to fluid line.

![Figure 2: Effects of changes in reservoir properties on AVA response in A versus B cross-plot domain.](image-url)

In a cross-plot of \( A \) versus \( B \), if there is little contrast in \( \gamma \), the reflection tends to fall on the fluid line

\[ B = \left( 1 - 8 \gamma^2 \right) A. \]

In Figure 2, the reflections from the top of sand fall on a trend approximately parallel to the fluid line but with a negative intercept in the cross-plot domain. The reflections from the base of sand fall on a parallel trend on the opposite side of the fluid line. Each trend is defined by a \( V_p/V_s \) contrast (See equation 11). Hydrocarbon reservoirs typically have a lower \( V_p/V_s \) than the surrounding formation, and increasing pore fluid compressibility even reduces \( V_p/V_s \). Thus gas produces the greatest departure from the fluid line, followed by oil and brine. Porosity is another rock property that has an impact on seismic responses. Increasing the porosity decreases acoustic impedance, but do not significantly affect \( V_p/V_s \). As a result, porosity changes move the AVA response along the trend parallel to fluid line.

![Figure 3: The comparison of theoretical angle-dependent reflectivity and the corresponding AVA extracted from source-illumination-normalized ADCIGs.](image-url)
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The bottom of the reservoir is reversed by sign. The upper panel is the shale/gas reflection. The middle panel is the shale/oil reflection. The lower panel is the shale/brine reflection.

For the simple model, the elastic seismic data are generated using elastic finite difference code, and acoustic RTM is employed to migrate the pressure of the seismic data. We select a vertical line in the model, calculate the AVA responses, and compare them with the theoretical reflection coefficients (Aki and Richards, 1980) in Figure 3. The extracted AVA response agrees very well with the theoretical reflectivity. In this simple case, the proposed method can obtain reliable AVA responses.

In the second example, we design an elastic model (Figure 4a) to conduct AVA analysis. A hydrocarbon reservoir is located in the middle of the model. There are multiple thin layers in both the background and the reservoir. The P-wave velocity is shown in Figure 4a. For the background shale, $V_p/V_s$ is about 1.9. The reservoir is oil-saturated sands and its $V_p/V_s$ is about 1.6. We carry on a seismic experiment which contains 71 shots ranging from 0.5 km to 7.5 km and 301 double-side receivers with a maximum offset of 6 km. The source time function is a Ricker wavelet with dominant frequency 30 Hz.

Shown in Figure 4b is the stacked migration image produced by cross-correlation imaging condition and 4c is the dip angle of the reflectors estimated from 4b. ADCIGs are obtained from shot-domain CIGs sorted by reflection angle. Figure 5 depicts the AVA responses computed from subsurface ADCIGs in cross-plot domain. The reflections from shale, which have little contrast in $V_p/V_s$, fall near the fluid line. The reflections from the top of reservoir appear at the lower-left side of the fluid line and the bottom reflections appear at the upper-right side of the fluid line. Both can be clearly distinguished from the background reflections. Even with medium dips, the AVAs from the top of reservoir still stand out from the background.

Conclusions
We provide a procedure to generate true-amplitude ADCIGs and analyze their AVA behaviors. Assuming the most energetic phase, we determine the incident angle from the single-frequency source wave. The dip angle is obtained from the image. The reflection angle is calculated from the incident and dip angles. Source-illumination-normalized common image gathers are formed and mapped into ADCIGs using known reflection angles. The proposed method is very efficient and can retrieve reliable AVAs even with variable dip angles.

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EDITED REFERENCES

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