

Finite-frequency sensitivity analysis and migration velocity updating

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Summary

The finite-frequency sensitivity theory has been used in velocity tomography for transmitted waves. In this paper, we expand its application to seismic migration and propose a wave-equation based migration velocity updating approach. This method naturally incorporates the wave phenomena and is best teamed with the wave-equation based migration method for velocity analysis. Meantime, it still keeps the simplicity of a ray-based velocity analysis method although the rays have been replaced by the more accurate sensitivity kernels. We also discuss practical issues for calculating sensitivity kernels and constructing inversion system. Numerical examples are calculated to validate both the theory and algorithm.

Introduction

The prestack depth migration is one of the most important techniques for retrieving subsurface information from reflection seismic data. However, satisfactory depth image depends on accurate migration velocity model. In practice, depth migration often starts from using a less accurate initial model. The resulted image is usually distorted but contains information for further correcting the model. The improved model is then used for the next round of migration. In an iterative way, the velocity model is refined and the depth image is improved. The entire process is called the migration velocity analysis. Generally, it can be seen as a special type of velocity tomography, except the information is extracted from the depth image rather than from seismic data directly.

Traditionally, both velocity tomography and migration velocity updating have been dominated by the ray-based techniques which assuming an infinitely high frequency. When a broadband signal propagates through a complex region, the ray theory often poorly approximates the actual wave propagation. Given this situation, a robust velocity updating method based on the wave equation theory is highly demanded. The sensitivity of finite-frequency signal to velocity perturbation has been extensively investigated and successfully used in velocity tomography (e.g., Woodward, 1992; Vasco et al., 1995; Dahlen et al., 2000; Huang et al., 2000; Zhao et al., 2000; Spetzler and Snieder, 2004; Jocker, et al., 2006; and Liu et al., 2008). Recently, researchers start to test this algorithm for migration velocity analysis (e.g., Sava and Biondi, 2004; de Hoop,

2006; Xie and Yang, 2007, 2008a, b; Fliedner et al., 2008; and Sava and Vlad, 2008).

In this paper, we briefly introduce the finite-frequency sensitivity theory. Based on this, we explore how this method can be used for migration velocity analysis. The new method is wave-equation based and incorporates the wave motion phenomena including scattering, interfering and multipathing, etc. With this new approach, we build an inversion system which converts the observed residual moveout (RMO) into velocity corrections. Several practical issues related to this method are also discussed. We investigate different numerical algorithms for calculating sensitivity kernels and different ways to store huge kernel files. Synthetic data set is used to validate the finite-frequency sensitivity theory and numerical algorithms.

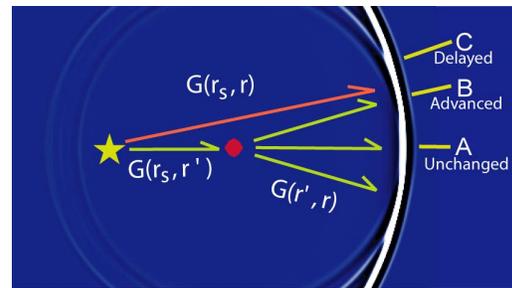


Figure 1. How a small scatter affects the travel time of a finite-frequency signal.

The Finite-Frequency Sensitivity Kernels

The seismic ray method based on high-frequency approximation has long been used for velocity tomography and migration velocity updating. According to the ray theory, if velocity perturbations are right on the ray path, they will influence the travel time measurement along the ray. Otherwise, the wave front should not be affected. However, for a real-world finite-frequency signal, the situation is quite different. The wavefield snapshot in Figure 1 illustrates how a scatter (a piece of positive velocity perturbation) can affect the wave front movement. When the source wave hits a scatter, it generates scattered waves. In the wavefield, what we actually observed is the superposition of the primary plus scattered waves. Due to the interference, the combined wave front may show phase advance or delay relative to the otherwise unperturbed wave front. In Figure 1, we see the large amplitude primary

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wave and the weak scattered wave jointly generate a distorted wave front. The velocity perturbation affects the travel speed of the wave front in a way much complex than that predicted by the ray theory. For example, according to the ray theory, the wave front at point A should be affected the most but it is almost unaffected. The ray theory predicts that the wave front at points B and C will not be affected. However, the wave front at B showing phase advance and at C showing phase delay.

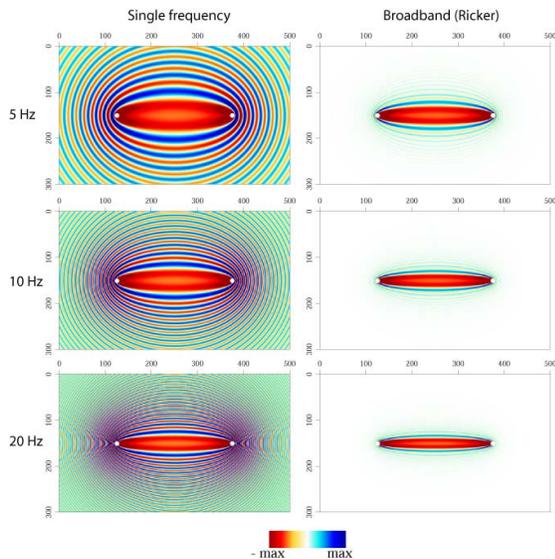


Figure 2. Monotonic and broadband sensitivity kernels under different frequencies. The source to receiver distance is 5 km.

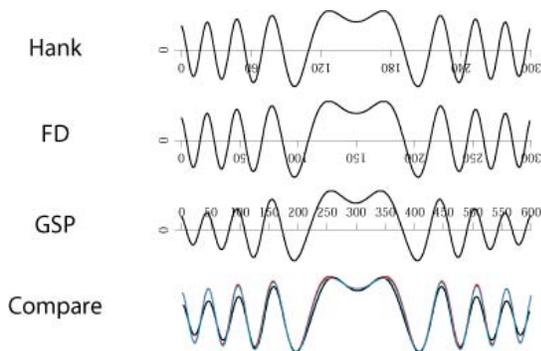


Figure 3. Comparison between 2D sensitivity kernels calculated using different methods.

The finite-frequency sensitivity theory presents a more accurate picture on how a wave can sense the velocity model variations. Several authors derived equations for calculating sensitivity kernels for transmitted wave (e.g., Woodward, 1992; Spetzler and Snieder, 2004; Xie and Yang, 2008b). The single frequency travel time sensitivity kernel K^F can be expressed as

$$K^F(\mathbf{r}, \mathbf{r}_S, \mathbf{r}_G) = \text{imag} \left[2k_0^2 \frac{G(\mathbf{r}; \mathbf{r}_S)G(\mathbf{r}; \mathbf{r}_G)}{G(\mathbf{r}_G; \mathbf{r}_S)} \right], \quad (1)$$

where $k_0 = \omega/v_0(\mathbf{r})$ is the background wavenumber, G is the Green's function, \mathbf{r}_S and \mathbf{r}_G are source and receiver locations. Shown in Figure 2 are examples of sensitivity maps from a point source to a receiver in a homogeneous 2D model. The left column is for monotonic waves and the right column is for broadband waves. Different rows are for different frequencies. The sensitive area is far broader than a thin ray. We call these sensitivity maps “travel time sensitivity kernels” which tell how finite-frequency signals radiated from a source can “sense” velocity variations along its way to the receiver. The sensitivity shows both positive and negative signs indicating the same perturbation may cause either positive or negative travel time delays depends on its location. For single frequency kernels, its sensitivity shows oscillations in a broad region. The central non-zero region is the first Fresnel zone. For a broadband kernel, its oscillating parts are mostly canceled out due to interference and its non-zero sensitivity is mainly located in the center. With the increase of frequencies, the broadband sensitivity kernel becomes narrower and more like a ray. The sensitivity kernel links the observed travel time residual to the velocity perturbation and sets up a basis for velocity tomography.

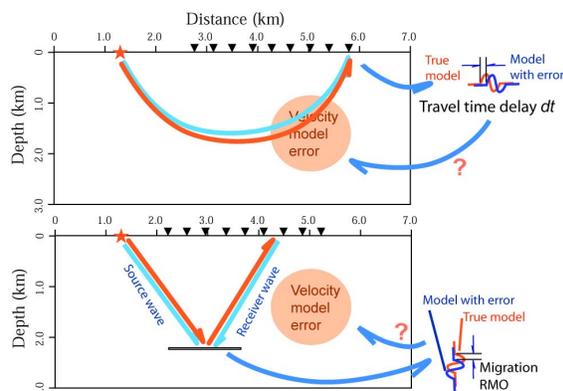


Figure 4. Top panel: velocity tomography using transmitted waves, and bottom panel: migration velocity updating using the RMO.

Depends on the velocity model and our purpose, different algorithms can be adopted to calculate sensitivity kernels. In a homogeneous velocity model, the analytical solutions can be used. For heterogeneous models, we can use the full-wave FD method or one-way propagator method. The FD method is accurate but rather expensive while the one-way method is more efficient and can be used for practical data processing. For a 2D homogeneous model, Figure 3 compares single frequency kernels along a cross section in the middle of the source and receiver and perpendicular to the geometric ray. From top to bottom are kernels calculated using the Hankel function, the full-wave FD and

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the one-way propagator. Comparison at the bottom shows the general consistency between different methods.

Sensitivity Kernel for Migration Velocity Updating

Finite-frequency sensitivity kernels have been used for solving many tomography problems with great success. The major obstacle that prevents this method from being used in migration velocity analysis is that these finite-frequency sensitivity kernels are mostly for transmitted waves and the information is extracted from the data domain (e.g., traveltimes delays). However, in seismic migration, the information must be extracted from the depth image (e.g., the RMO) instead of from the data. The cartoon in Figure 4 illustrates the difference. Here, the key issue is how to convert the observed RMO into velocity perturbation and back project it into model space to update the velocity field.

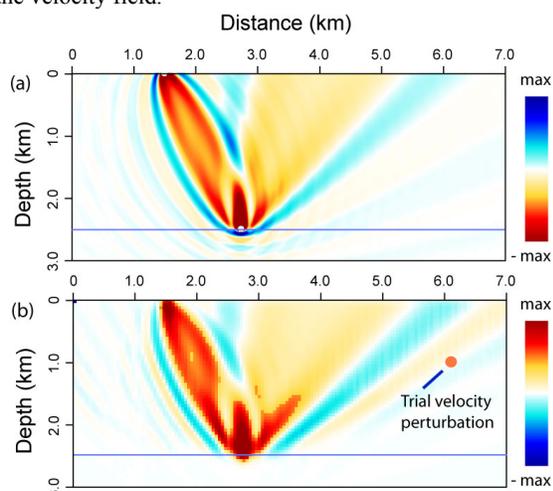


Figure 5. The sensitivity kernels for a shot gather image, with (a) theoretical sensitivity kernel, and (b) the sensitivity map directly measured from migration imaging (see Xie and Yang 2008b).

Based on the Born and Rytov approximations, Xie and Yang (2007, 2008b) derived a broadband sensitivity kernel relating the RMO in prestack depth migration to velocity variations in migration velocity model. The kernel is formulated for the shot-record prestack depth migration and shot-index common image gather (CIG). Thus no expensive angle-domain analysis is required. An efficient method based on the one-way propagator and multiple-forwardscattering and single-backscattering approximation (Xie and Wu, 2001, Wu, et al., 2006) is used to calculate the sensitivity kernel. To check the validity of the theory and numerical method, we also designed a method to directly measure the sensitivity map from the migration process. Shown in Figure 5a is the theoretically calculated sensitivity kernel for a shot gather image, and 5b is the sensitivity map directly measured from migration imaging. The general consistency between these kernels validates the theory and the numerical algorithm.

The Inversion System for Velocity Updating

Based on the finite-frequency sensitivity theory (Xie and Yang, 2008b), the observed RMO can be linked to the migration velocity model error through an integral equation

$$\delta R(\mathbf{r}_{S1}, \mathbf{r}_{S2}, \mathbf{r}_I) = -\sum_k \int_{V(\mathbf{r}_k)} m(\mathbf{r}') K^B(\mathbf{r}', \mathbf{r}_{S1}, \mathbf{r}_{S2}, \mathbf{r}_I) dV', \quad (2)$$

where $m(\mathbf{r}) = \delta v(\mathbf{r})/v_0(\mathbf{r})$ is the unknown velocity error to be inverted, $v_0(\mathbf{r})$ is the initial velocity, $\delta R(\mathbf{r}_{S1}, \mathbf{r}_{S2}, \mathbf{r}_I)$ is the observed relative RMO, \mathbf{r}_I is the image location, \mathbf{r}_{S1} and \mathbf{r}_{S2} are locations of two sources in the same CIG, $K^B(\mathbf{r}, \mathbf{r}_{S1}, \mathbf{r}_{S2}, \mathbf{r}_I)$ is the broadband differential kernel which combines the sensitivities from a pair of shots in a CIG, $V(\mathbf{r}_k)$ is the small cell after partition the velocity model. In equation (2), $(\mathbf{r}_{S1}, \mathbf{r}_{S2}, \mathbf{r}_I)$ is the index for RMO data and (\mathbf{r}_I) is the index for unknown velocity perturbations. Once we have the relative RMO data δR and the differential kernel K^B , we can invert the velocity perturbation $m(\mathbf{r}_I)$ and use it to update the velocity model.

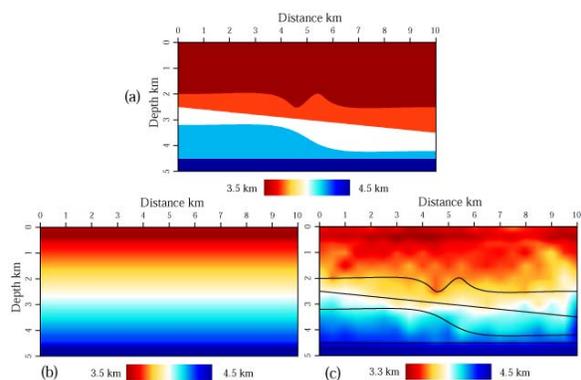


Figure 6. Velocity models in updating process, with (a) true velocity v_0 , (b) initial model and (c) model after two iterations.

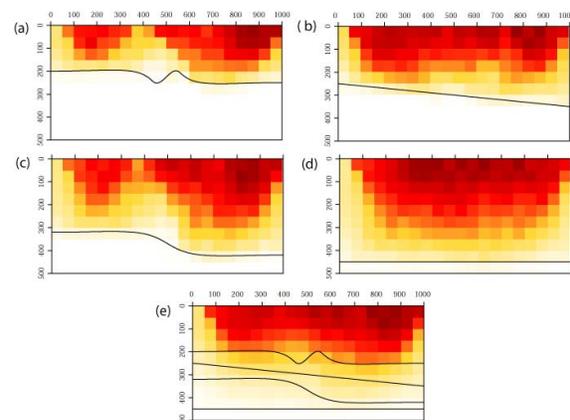


Figure 7. Coverage of sensitivity kernels in the model space. Panels (a) to (d) are coverage for individual reflectors and panel (e) is the coverage for all kernels.

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Testing Velocity Updating Using a Synthetic Data Set

We use the following numerical example to demonstrate the migration velocity updating process. The synthetic data set is generated using a fourth-order scalar-wave FD method and a 5-layer velocity model shown in Figure 6a. A total of 31 evenly distributed surface sources are used in the calculation and the source time function is a 17.5 Hz Ricker wavelet. On each reflector, we select 31 image points to calculate the shot index CIG and the RMO is measured using cross correlations between traces. The broadband sensitivity kernels are calculated using the above mentioned one-way and one-return method. Each kernel is calculated using 60 frequencies and the same 17.5 Hz Ricker wavelet is used for the source function. The least squares method by Lawson and Hanson (1974) is used to solve equation (2). To discretize the integral equation, we partition both the model and kernels into 0.5×0.5 km cells. Within cells we use the bilinear function to interpolate the velocity.

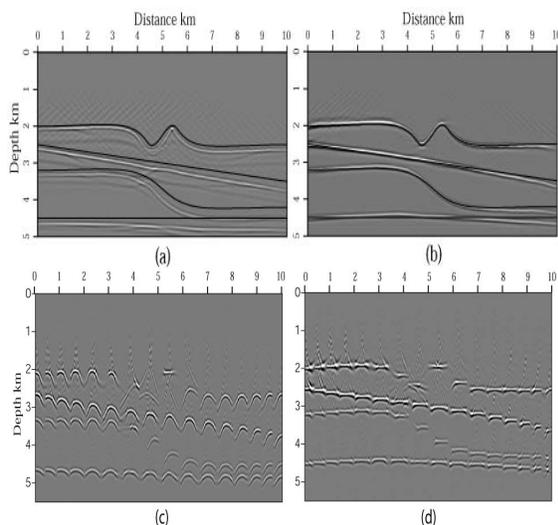


Figure 8. Depth image improved in the velocity updating process. (a) Image calculated using the initial velocity model; (b) image calculated using the updated model; (c) CIGs in the initial velocity model; and (d) CIGs in the updated velocity model.

Similar to the “ray coverage” used in the ray based tomography, we can calculate the “kernel coverage” and use it to estimate the quality of inversion. Illustrated in Figure 7 is the kernel coverage in the 5-layer model. We calculate this figure by summing up all positive values for all kernels which are actually used in the inversion. Panels (a) to (d) are kernel coverage for the 4 individual reflectors, and (e) is the total coverage from all kernels. Due to the configuration of the acquisition system and reflector geometry, the entire model is not evenly covered by sensitivity kernels. However, even with limited numbers of shots and image points, the entire model above the deepest reflector is properly illuminated and there is no blind region in the model. This is the advantage of the wave

equation based method comparing to the high-frequency ray-based method.

The initial model in Figure 6b is a 1-D model with a linear vertical gradient. The prestack depth migration in the initial model generates a depth image which is shown in Figure 8a. The dark curves overlapped on the image are locations of reflectors (interfaces) in the true velocity model. We pick reflector locations from the initial image and use them to calculate finite-frequency sensitivity kernels in the initial model. After two iterations, we obtain an updated velocity model which is shown in Figure 6c. The depth image calculated using the updated velocity is shown in Figure 8b. In general, we see the updated velocity model approaches the true velocity model and the image of the reflectors approaches interfaces in the true velocity model.

For a further comparison, Figures 8c and 8d illustrate the shot-index CIGs calculated from the initial and updated velocity models. We see most of the gathers are flattened during the velocity updating process. Certain errors can be seen at the lower-right corner in the final image (Figure 8b). These errors may be resulted from that the dipping structures deflect the reflection waves outside the acquisition aperture.

Conclusions

We briefly discuss the finite-frequency sensitivity theory which is traditionally only for transmitted signals in seismic velocity tomography. We expand this method from transmitted wave to migration velocity updating. The new method is formulated for the shot-record prestack depth migration and shot-index CIG. Thus no expensive angle-domain analysis is required. Based on this sensitivity kernel, we build an inversion system which converts the observed RMO to the correction in migration velocity model. Different methods are tested for calculating sensitivity kernels. The method based on the one-way propagator and multiple-forwardscattering and single-backscattering approximation is efficient and can be used for practical data processing. Using a synthetic data set, we demonstrate how to update the velocity model using this approach. The result shows, after a few iterations, the quality of the depth image is improved and the CIGs are flattened. This is a wave-equation based method which naturally incorporates the wave phenomena and is best teamed with the wave-equation based migration method for velocity analysis. However, it still maintains the simplicity of a ray-based velocity analysis method although rays have been replaced by the more accurate sensitivity kernels.