A Poynting vector based illumination and resolution analysis method for full wave RTM
Xiao-Bi Xie*, Institute of Geophysics and Planetary Physics, University of California at Santa Cruz; Zhe Yan, Institute of Geophysics and Geomatics, China university of Geosciences

Summary
A full-wave broadband method is proposed for seismic illumination and resolution analysis. The Poynting vector method is adopted to decompose the wavefield into its angle components. By assuming that the finite-difference calculated wavefield preserves the source spectrum, the broadband signal can be converted to the frequency domain, and used to calculate the point spreading function, which can further be used for the illumination and resolution analysis. Combining these techniques, we can avoid intensive calculations, massive input/output and huge storage. The resulted method is highly efficient and particularly suitable to team with the RTM for resolution analysis. Numerical examples are presented to validate this method and demonstrate how to correct the migrated depth image using resolution functions.

Introduction
Illumination and resolution analysis provides useful information for evaluating the migration system, including effects from both acquisition geometry and overburden structures. By compensating these effects, the depth image can be improved. Rickett (2003) applied wave-equation-based illumination analysis to normalize the depth image. Based on the ray tracing method, Gelius et al. (2002), Sjoeberg et al. (2003), Lecomte (2008) derived the resolution function, and used it to correct the image in the wavenumber domain. Based on the one-way propagator, Xie et al. (2005) proposed to correct the image using the point spreading function (PSF). Wu et al. (2004, 2006), Cao and Wu, 2009, Mao and Wu (2011) used wavelet transform to calculate the resolution function and conducted image correction. Cao (2013) and Yan et al. (2014) tested image correction for the RTM image.

In this paper, we propose an efficient illumination and resolution analysis method under the full-wave frame, in which the angle-domain information is extracted using the Poynting Vector. In the rest part of this paper, we briefly review the relationship among seismic image, and illumination and resolution functions. Then, present how to decompose the wavefield into angle domain, and form the PSF for image correction. Numerical examples are used to demonstrate the proposed method in illumination and resolution analysis.

Theory and Method
Using a survey system composed of a source at \( \mathbf{x}_s \) and a receiver at \( \mathbf{x}_r \) to investigate a small target region \( I(x) \) in the vicinity of location \( \mathbf{x} \), the seismic image can be expressed as (see e.g., Xie et al. 2005; Yan et al. 2014)

\[
I(\mathbf{x}, \mathbf{x}', \omega) = \int_{\mathfrak{V}_s} M(\mathbf{x}') R(\mathbf{x}, \mathbf{x}', \omega) d\mathbf{k}', \tag{1}
\]

where the resolution function \( R(\mathbf{x}', \mathbf{x}') \) maps the velocity perturbation \( M(\mathbf{x}') \) to image \( I(\mathbf{x}') \). Because of the limitations in the system, e.g., the incomplete acquisition aperture or complex overburden structure, the mapping is usually distorted and smears the image. Thus \( R \) is also known as the point spreading function, which is the response of the imaging system to a point scatter (Chen and Schuster, 1999; Gelius, et al., 2002; Gibson and Tzimeas, 2002; Xie et al., 2005; Cao 2013) Equation (1) is similar to a convolution, except that \( R \) is localized, i.e., it is defined near location \( \mathbf{x} \) and varies in the space due to the variable illumination. Introducing the local Fourier transform, the space convolution in (1) can be converted into the wavenumber domain multiplication (Gelius, et al., 2002; Xie et al., 2005, 2006; Lecomte, 2008, Cao, 2013)

\[
I(\mathbf{x}, \mathbf{k}_s, \omega) = R(\mathbf{x}, \mathbf{k}_s, \omega) \cdot M(\mathbf{x}, \mathbf{k}_s), \tag{2}
\]

where

\[
M(\mathbf{x}, \mathbf{k}_s) = \int_{\mathfrak{V}_s} M(\mathbf{x}') e^{i \mathbf{k}_s \cdot \mathbf{x}'} d\mathbf{k}', \tag{3}
\]

\[
R(\mathbf{x}, \mathbf{k}_s, \omega) = 2k_s^2 \delta(\omega) s(\omega) \sum_{\mathbf{k}_s} A(\mathbf{x}, \mathbf{k}_s, \mathbf{x}, \mathbf{x}_s, \omega), \tag{4}
\]

and

\[
A(\mathbf{x}, \mathbf{k}_s; \mathbf{x}_s, \mathbf{x}_r; \omega) \sim k_x \cdot k_s \cdot k_y.
\]

\[
G'(\mathbf{x}, \mathbf{k}_s, \omega) G'(\mathbf{x}_s, \mathbf{k}_s, \omega) \times \tag{5}
\]

\[
G(\mathbf{x}, \mathbf{k}_s; \mathbf{x}_s, \omega) \times G(\mathbf{x}_s, \mathbf{k}_s; \mathbf{x}_s, \omega)
\]

where \( \mathbf{k}_s = \mathbf{k}_i + \mathbf{k}_o \) is the target dipping wavenumber, \( \mathbf{k}_i \) and \( \mathbf{k}_o \) are incident and scattered wavenumbers near the image point, and \( R(\mathbf{x}, \mathbf{k}_s, \omega) \) is the wavenumber domain PSF. \( G(\mathbf{x}, \mathbf{k}_s, \omega) \) are wavenumber domain Green’s functions from both source and receiver to the image point. Equation (5) converts the response from acquisition coordinate \( \{ \mathbf{k}_i, \mathbf{k}_o \} \) to the target dipping coordinate \( \mathbf{k}_s \) (refer to Figure 1).

Given the acquisition system and the overburden structure, methods were proposed to calculate \( R(\mathbf{x}, \mathbf{k}_s) \) or \( R(\mathbf{x}, \mathbf{x}', \omega) \), followed by using it in seismic illumination and resolution analysis (e.g., Xie et al., 2005, 2006; Lecomte, 2008). It is relatively straightforward to calculate
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the \( R(x,k_x) \) using the frequency domain equations shown above. For a time domain method such as the RTM, theoretically, one can use Fourier transform to convert the entire time-space wavefield into frequency domain, followed by calculating the PSF using frequency domain equations. However, such a procedure often involves intensive calculations, massive input/output and huge storage, and is computationally inefficient. Alternatively, we propose using the Poynting vector to decompose the wavefield into its angle component directly in time domain. Then convert the broadband signal into its frequency component by assuming the frequency contents are unchanged during the propagation. Finally, this frequency-angle domain information is used in equations (4)–(5) to create the PSF for image correction. The resulted method is highly efficient.

Figure 1. Cartoon showing the coordinate system.

Figure 2. Wave propagation directions calculated using the local slowness analysis method (energy peaks), and the Poynting vector method (red arrows).

The FD calculated source wave can be decomposed into a superposition of local beams with incident angle \( \theta \)

\[
u(x,\theta;\mathbf{x},t) = \int s(\omega)G(x,\theta;\mathbf{x},\omega)e^{i\omega t}d\omega.
\]

where \( s(\omega) \) is the source spectrum. We calculate the average mean square amplitude of \( u(x,\theta;\mathbf{x},t) \) within a short time interval \( T \)

\[
\frac{1}{T} \int u^2(x,\theta;\mathbf{x},t)dt = \int s(\omega)s^*(\omega)G(x,\theta;\mathbf{x},\omega)G^*(x,\theta;\mathbf{x},\omega)d\omega.
\]

Assuming that, during propagation, the wavefield keeps its spectrum unchanged, we can convert the broadband signal \( u \) to the frequency domain

\[
G(x,\theta;\mathbf{x},\omega) = \frac{1}{T} \int u^2(x,\theta;\mathbf{x},t)dt = \int s(\omega)s^*(\omega)G(x,\theta;\mathbf{x},\omega)d\omega
\]

Similarly, we have

\[
G(x,\theta;\mathbf{x},\omega) = \frac{1}{T} \int u^2(x,\theta;\mathbf{x},t)dt = \int s(\omega)s^*(\omega)G(x,\theta;\mathbf{x},\omega)d\omega
\]

where \( \theta_s \) is the local scattered angle.

To substitute equations (8) and (9) into (5), the coordinate transforms involve first rotating the incident-scattering angles \( \theta_i, \theta_s \) to dipping-reflection angles \( \theta_i, \theta_r \) with \( \theta_i = (\theta_i + \theta_s)/2 \) and \( \theta_r = (\theta_s - \theta_i)/2 \) (refer to Figure 1), followed by converting them to the dipping wavenumber domain \( k_d, \theta_d \) with \( k_d = 2k_0 \cos \theta_d \) and \( \theta_r = \theta_d \) [refer to Alkhalifah (2015) and Xie (2015)]. The two transforms can be combined into

\[
\theta_i = \theta_i + \cos^{-1}\left(\frac{k_d}{2k_0}\right) \quad \text{and} \quad \theta_s = \theta_s - \cos^{-1}\left(\frac{k_d}{2k_0}\right)
\]

With these transforms and a proper Jacobian, (8) and (9) can be substituted into (4) and (5) to form the monotonic wavenumber domain PSF

\[
R(x,k_x,\omega) = \frac{2k_0^2s(\omega)s^*(\omega)}{T^2 \int |s(\omega)s^*(\omega)|d\omega} \times \sum_x \sum_k \int u^2\left(x,\theta_i + \cos^{-1}\left(\frac{k_d}{2k_0}\right);x,t\right) dt
\]

\[
\times \int u^2\left(x,\theta_s - \cos^{-1}\left(\frac{k_d}{2k_0}\right);x,t\right) dt
\]

\[
\times \frac{1}{k_0}\left[1 - \left(\frac{k_d}{2k_0}\right)^2\right]^{1/2}
\]

Numerically integrate it over frequency, we have

\[
R(x,k_x,\omega) = \int R(x,k_x,\omega)d\omega.
\]

By deconvolving \( R \) from equation (2), the corrected image \( I' \) can be obtain as

\[
I'(x,k_x) = I(x,k_x)/R(x,k_x),
\]

which should depict the subsurface velocity perturbation \( M(x,k_x) \) better.
Numerical Examples

Poynting vector. Poynting vector gives the wave energy flux direction. Therefore, by calculating the Poynting vector, we can obtain the wave propagation direction at a specific time and space location (Yoon et al. 2004, 2011). The resulted method is highly efficient. As a comparison, Fig. 2 compares the wave propagation directions in the BP salt model, calculated using both local slowness analysis method (energy peaks) and the Poynting vector method (red arrows). Although the Poynting vector method has the weakness in a complicated wavefield (Jin et al., 2014), this situation usually accounts for a very small portion in the entire space-time domain. In the illumination calculation, the Green’s functions are used for both the source and receiver wavefields, which largely relaxes the requirement of dealing with complex wavefields.

Figure 3. Broadband wavenumber domain PSF at selected locations in the 2D SEG/EAGE salt model.

The PSF. The monotonic and broadband PSFs can be calculated using equations (11) and (12). Illustrated in Fig. 3 are 15 Hz broadband wavenumber domain PSFs calculated at selected locations in the SEG/EAGE salt model. The acquisition system is composed of 325 shots with an interval of 48.8 m. Each shot has 176 left-side receivers, with an interval of 24.4 m. Note, the maximum detectable wavenumber is proportional to $2k_p$. Therefore, in high-velocity region, the resolution is low and vice versa. At shallow depth the illumination covers a wide angle range. However, under the salt body, the angle coverage is uneven.

Image correction for 2D SEG/EAGE salt model. The PSF carries the full information regarding the image distortion, including those from the acquisition geometry and overburden structures. According to equation (13), by dividing PSF from the image in wavenumber domain, the uneven illumination can be compensated, and the overall quality of the image can be improved. As the first example, we apply the illumination and resolution analysis to the 2D SEG/EAGE salt model. The result is demonstrated in Fig. 4, where 4a is the conventional RTM image. The area marked by the white square is chosen to demonstrate the correction procedure, which is shown from 4b to 4f. Using the sampling window to scan the entire image and repeatedly using the correction process mentioned above, we obtain the corrected image for the entire model which is shown in Figure 4g. Comparing to 4a, subsalt structures in 4g are more balanced, and several steep-dip structures are more emphasized.

Image correction for Sigsbee 2A model. The acquisition system is composed of 500 shots with an interval of 45.7 m. Minimum and maximum offsets are 0 and 7932 m and the receiver interval is 22.9 m. The source is a 20 Hz Ricker wavelet. The conventional RTM image is shown in Fig. 5a, and the vertical illumination is shown in Fig. 5b. We see that near horizontal structures below the overhang part of the salt body are poorly illuminated, which is responsible for the missing image in the RTM result (circled by ellipses). The image corrected by the PSF is shown in Figure 6a. Comparing 6a with 5a, the image quality is significantly improved. In general, the amplitudes are more balanced, particularly in the subsalt region. By zoom to the areas labeled by white squares, illustrated in Figs. 6b and 6d are enlarged details in the corrected image. Compared with the conventional images shown in 6c and 6e, the corrected images have consistent layered structures which can be traced closer to the salt flank, improved focusing of point scatters, and generally sharper images.
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The correction also eliminated low-wavenumber artifacts seen at the shallow part in 5a.

Computational Issues and the Efficiency

The illumination and resolution analysis is a useful tool as a supplement to seismic imaging but it usually demands intensive calculations and huge storage, which prevent it from being used in practice. This problem is particularly severe for a time-domain migration such as the RTM. We introduce several techniques to mitigate the efficiency and storage problem encountered in the broadband analysis, i.e., i) measuring the wave propagation direction using the Poynting vector, ii) select a number of most energetic phases to calculate and store, instead of measure the entire wave train, iii) assuming the wavefield preserves the source spectrum instead of actually calculating Fourier transforms for waveforms. For a typical 2D case, if the illumination analysis is conducted along with the RTM, compared to the time spend by the RTM itself, the angle decomposition using the Poynting vector at a 1x1 interval takes approximately 10-20% of extra time. Creating the PSF and conduct the image correction at a 5x5 interval will take about 20% additional time.

Figure 5. (a) The conventional RTM image of the Sigsbee 2A model, and (b) the vertical illumination.

Discussion and Conclusion

A full-wave-equation-based broadband method is proposed for illumination and resolution analysis. The finite-difference propagator is used to calculate the Green’s functions. The Poynting vector method is adopted to decompose the wavefield into angle components. To conduct the resolution analysis, the frequency information is required to convert the angle-domain information into the wavenumber-domain information. To avoid saving the entire space-time wavefield and formally carry out Fourier transforms at all space locations, we assume that the wavefield keeps the spectrum of the source wavelet and directly convert the broadband signal into its spectrum.

Figure 6. (a) Corrected depth image. To illustrate the details, (b) and (d) are enlarged corrected image indicated by white squares in (a), and (c) and (e) are correspondent conventional images.

By combining the above mentioned techniques together, the proposed method is highly efficient and flexible. It is particularly suitable for illumination analysis in RTM imaging. If the source wavefield generated in the RTM is used as Green’s functions, the illumination and resolutions analysis only takes an extra time that is a fraction of the time used for the migration imaging. Numerical examples are used to demonstrate how to calculate the PSF and correct depth images.
EDITED REFERENCES
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