Extracting angle domain information from migrated wavefield

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Summary

The recent development of wave equation based migration methods provided accurate propagators for seismic wave extrapolation. These propagators brought the possibility that many analysis and inversion can be made at the depth using migrated wavefield. In this study, we propose an approach for extracting angle domain information from migrated wavefield. The method is based on localized plane wave analysis and can be used for almost any migration method. Then, using the concept of the local image matrix, useful information (angle domain image gathers, reflector dips, etc.) can be further extracted from this angle related information. Numerical examples are conducted to demonstrate the applications of this method.

Introduction

The recent developments of wave equation based migration methods (e.g., Stoffa et al., 1990; Ristow and Ruhl, 1994; Jin, et al., 1998; Xie and Wu, 1998; Huang and Fehler, 2000; Xie et al., 2000; Biondi, 2002) provided accurate propagators for seismic wave extrapolation. In addition to generate high quality subsurface images, these propagators also allow many analyses and inversions, which previously could only be carried on the surface, to be made at the depth using migrated wavefield. For example, the AVO analysis using offset information is traditionally applied to surface data. Now, the surface data can be extrapolated to the target area and then analysis can be made at the depth. The advantage of this technique is that ambiguities caused by the complicated propagation can be effectively eliminated. These analyses can be made by investigating migrated wavefield in different domains. The angle domain common image gather is often very useful for velocity updating and AVA analysis. Several robust approaches (Prucha, et. al., 1999; Mosher, 2001; Liu, et. al., 2001; Rickett and Sava, 2001) have been developed for this purpose. The target of this study has two fold. First, we propose a new approach to extract angle domain information from migrated wavefield. This method is based on a local plane wave analysis, which does not rely on a specific propagator such as those in common-offset domain or offset plane wave domain. It can be applied to almost any wave propagation method. Second, we propose further analysis on angle related information based on the local image matrix concept. Useful information about the reflectors can be obtained by properly sorting the energy in the image matrix. Numerical tests using simple models and the 2D SEG/EAGE Salt model are conducted to demonstrate this method.

Angle domain analysis for migrated wavefields

A basic scattering/reflection experiment can be obtained by illuminating the target using an incident wave in direction θ^I and observe the scattered energy leaving the reflector in direction θ^R. Figure 1 is a cartoon showing this process. Collecting all possible illumination/reflection directions forms an image matrix $I(\theta^I, \theta^R)$, which is the partial image as a function of incident and scattering angles.

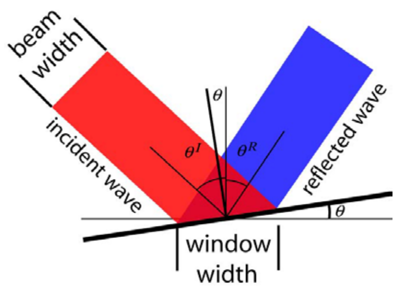


Figure 1. A basic reflection/scattering observation.

Within the seismic migration process, the down going wavefield from the source and the upper going wavefield from receivers are both downward extrapolated. The angle related analysis can be made right at the target region. To decompose the wavefield into localized plane wave superposition and obtain the local image matrix, we use the windowed Fourier transform

$$I(\mathbf{K}_T^I, \mathbf{K}_T^R; \mathbf{x}_T, z, \omega) = C \cdot F\left[w(\mathbf{x}_T, \mathbf{x}'_T)u^I(\mathbf{x}'_T, z, \omega)\right] F\left[w(\mathbf{x}_T, \mathbf{x}'_T)u^R(\mathbf{x}'_T, z, \omega)\right]$$

where u^I and u^R are incident and scattered waves, w is a 2D moving window in the transverse space direction $\mathbf{x}'_T$ and centered at $\mathbf{x}_T$. A finite-sized window is required for localized wave number analysis under finite frequencies. The size of the window provides the trade off between the spatial and angle resolutions. $F[\cdot]$ is a 2D Fourier transform. C is the normalization factor. The observation is centered at the horizontal position $\mathbf{x}_T$ and depth z, and generated by an incident plane wave with local wavenumber $\mathbf{K}_T^I$ and a reflected plane wave with local wavenumber $\mathbf{K}_T^R$. These transverse wavenumbers can be transformed to angles via $\theta(K_T) = \sin^{-1}(K_T/k)$, with $k = \omega/\alpha$ as the local wavenumber and α as the local velocity. The result can be summed up over frequency ω. Similarly, the local image matrix can be calculated using a beam forming (slant stack) method directly in the space domain.

$$
\begin{aligned}
I(\theta^I, \theta^R; \mathbf{x}_T, z, \omega) \\
&= C \cdot \sum_{x_T'} \left[w(\mathbf{x}_T, \mathbf{x}_T') u^I(\mathbf{x}_T', z, \omega) \exp\left[ik(\mathbf{x}_T' - \mathbf{x}_T) \sin\theta^I \right] \right] \\
&\times \sum_{x_T'} \left[w(\mathbf{x}_T, \mathbf{x}_T') u^R(\mathbf{x}_T', z, \omega) \exp\left[ik(\mathbf{x}_T' - \mathbf{x}_T) \sin\theta^R \right] \right]
\end{aligned}
$$

For some wave equation based migration methods, e.g., the common offset migration, offset plane wave migration (Mosher, et al., 1996; Mosher and Foster, 1998; Jin and Wu, 1999; Prucha, et. al., 1999; Jin et al. 2000; Rickett and Sava, 2001) and wavelet transform based method (Wu et. al., 2000; Wu and Chen, 2001), their propagators intrinsically carry the angle information. The local scattering matrix can be easily constructed using this information.

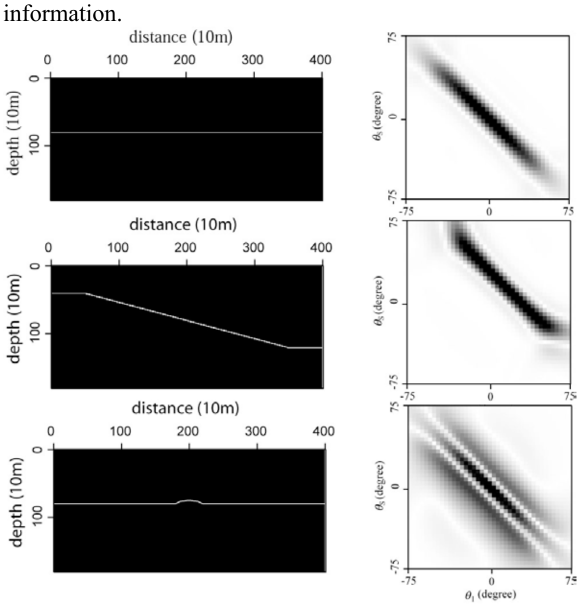


Figure 2. The local image matrixes for typical structures. From upper to bottom are for flat, dipping and curved reflectors. On the left are velocity models and on the right are image matrixes. For details see the text.

Figure 2 shows features of the local image matrixes for some typical reflectors. The left panels are velocity models of different reflectors, and on the right are image matrixes with their horizontal and vertical axes as incident and scattering angles. The local image matrixes are obtained at the center of the model using a 32 grid space window. For a locally horizontal reflector (upper panel), the energy is concentrated at the diagonal of the matrix, which meets the reflection principle, i.e., incident angle equals the reflection angle. For a dipping reflector (middle panel), the energy still forms a strip parallel to the diagonal but shifts for a distance. If the reflector is more complicated than a flat interface, for example a curved interface, random layers, uneven rough interface, etc., their image matrixes will appear to be more complicated (lower panel).

Depending on analysis purposes, the energy in the image matrix can be sorted in different ways. Figure 3 is a sketch showing possible analyses. The horizontal and vertical axes denote incidence and scattering angles. Investigating energy distribution along the diagonal direction (from upper-left to lower-right) gives angle related information that can be used in velocity updating, AVA analysis etc. For a dipping reflector, its energy will be shifted form the main diagonal. The offset of the energy measured along the upper-right to lower-left direction carries the dipping information of the structure. Other useful image gathers may also be constructed by properly summing up the energy. For example, integrating the energy along the horizontal direction gives the common scattering angle gather, and summing up the energy along the vertical direction gives the common illumination angle gather, etc.

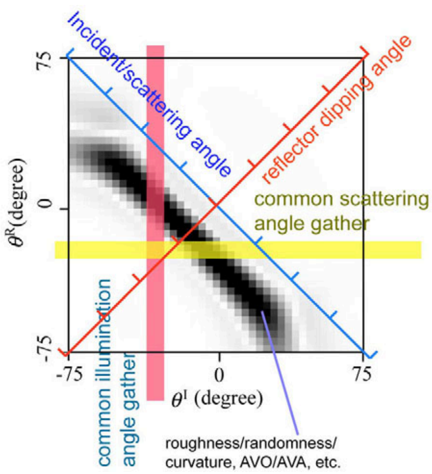
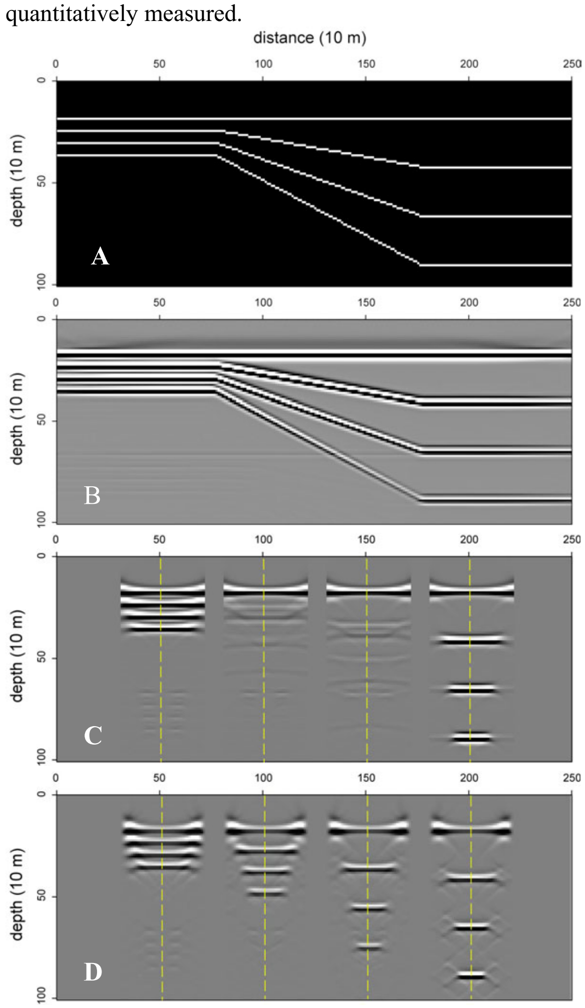


Figure 3. Cartoon showing the local image matrix and various types of analyses can be made.

A simple numerical example

To show the applications of the angle domain analysis, we use a simple 2D model. The velocity model is shown in Figure 4A. It is composed of a 3.5 km/s constant background and four 4.0 km/s reflectors. Sixty shots on the surface are used to illuminate the model and a wide-angle generalized screen propagator (Xie and Wu, 1998; Xie et. al. 2000) is used for the wavefield extrapolation. The migrated image is shown in Figure 4B. The angle information is calculated at horizontal locations 50, 100, 150 and 200, respectively. Figure 4C gives the angle domain image gathers using the diagonal energy only. As expected, image gathers reflect horizontal structures very well but do not reflect the dipping structures. In figure 4D the energy in the image matrix is summed along the upper-right to lower-left direction. With the off diagonal energy add in, the angle image gathers for the dipping part of the structure are properly produced. Normal incidences are automatically aligned on the center line. Due to the illumination condition, the angle coverage decreases with the depth. These angle image gathers can be used as the basis for velocity updating and AVA analysis. As has been mentioned above, the shift of energy distribution from the diagonal gives the measurement of the dip angle. By summing up the energy along the upper-left to lower-right direction, the energy distribution along upper-right to lower-left direction (refer to the cartoon in Figure 3) gives the information about reflector dips. Shown in Figure 4E are measurements of dipping angles. At distances 50 and 200, where the reflectors are flat, all energy peaks are centered at diagonals. For profiles at distances 100 and 150, the first reflector is flat but followed by three reflectors with increasing dipping angles. The shift of energy peaks from the center lines reveals their dip angles which can be quantitatively measured.



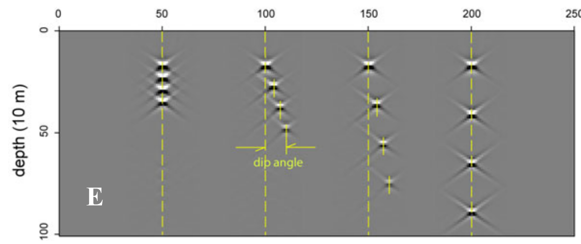


Figure 4. A simple numerical example. For details see the text.

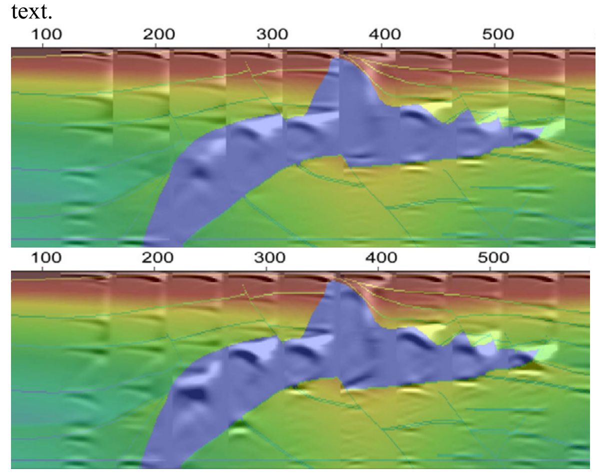


Figure 5. Angle domain image gathers for the SEG/EAGE Salt model.

SEG/EAGE Salt model

As the next example, we apply the analysis method to the SEG/EAGE Salt model. First, we calculate the angle domain image gathers. Shown in the upper panel of Figure 5 is the angle image gather using only the diagonal energy. Although it properly gives image gathers for near horizontal reflectors, it fails at steep dip structures such as the left flank of the lower salt boundary. Shown in the lower panel are angle image gathers including off diagonal energy. The image gathers for the lower salt boundary are clearly shown in the figure. The results also show that for the lower boundary the angle coverage is very narrow. Figure 6 shows the dip angle estimates for the sub salt structures. Two enlarged portions show the details of the result. In the left portion, the lower salt boundary dips toward the left with a pretty steep angle. In the right portion, sub salt structures show different dip angles. The lower salt boundary dips toward the left. The base line is horizontal. Other reflectors are toward the right with mild dip angles. All these features are clearly shown and angles can be quantitatively measured using their shifts.

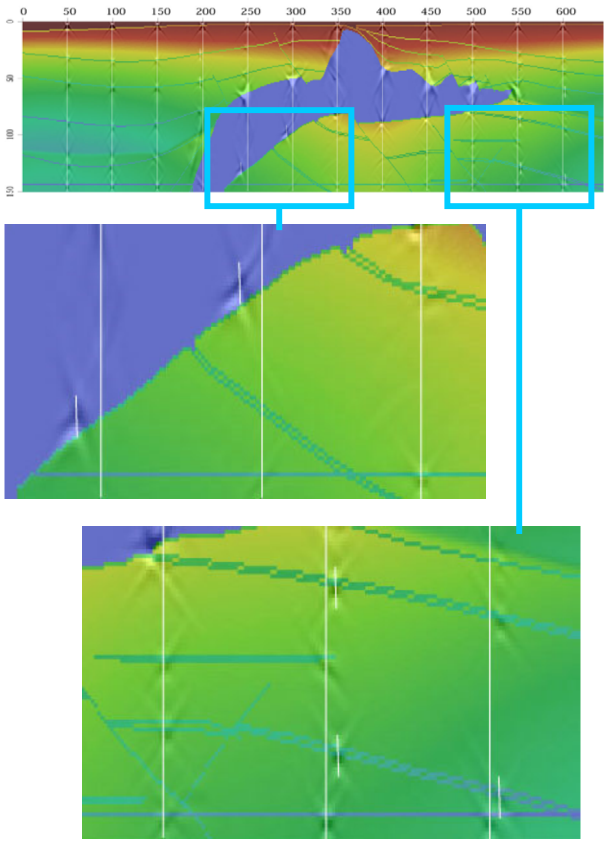


Figure 6. Determining dip angles of subsurface structures.

Conclusion

In this research, we proposed an approach for extracting angle domain information from migrated wavefield. The local image matrix can be generated using downward extrapolated source and receiver waves. The image matrix carries useful information about the subsurface reflectors. Properly sorting the energy within the image matrix allow different observations be made for velocity updating, AVA analysis and determining the structure geometry.

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