Improving the wide angle accuracy of the screen propagator for elastic wave propagation

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Summary

The wide angle modification for scalar screen propagators has been extended to the elastic case. Compared with the original elastic screen method, the new approach gives much more accurate phase for models with large velocity contrast and for wide propagating angles. The proposed method offers us an improved one-way propagator for elastic wave modeling and imaging.

Introduction

Fast propagators in complex heterogeneous media, especially for 3-D models, are crucial to the application of seismic methods in complex structures including the development of interpretation, modeling, imaging and inversion methods. An elastic screen method has been proposed to calculate the propagation of elastic waves (Wu, 1994, 1996; Xie and Wu, 1995, 1996). Based on one-way wave equations, these methods are very fast compared with the conventional full wave finite-difference method. On the other hand, they are still wave equation based methods and are more accurate than many ray theory based asymptotic methods. These advantages make it a very good propagator for elastic wave modeling and imaging. The derivation of a screen method usually requires that the velocity perturbation is small or the incident angle is small. However, the real models used in modeling and imaging practices may contain very large velocity contrasts. Under these situations, the conventional screen method can give the correct phase only within a small scattering angle, and can not generate satisfactory results for modeling and migration. Several attempts have been made to improve the wide angle capacity of the screen method for scalar waves. The PSPI method (Gazdag and Sguazzero, 1984) uses multi-reference velocities and then generates the solution using interpolations. The Fourier finite-difference method (Ristow and Ruhl, 1994), wide angle improvement method (Xie and Wu, 1998) and the modified pseudo screen method (Jin, Wu and Peng 1998) all use a finite-difference method to modify the screen solution. Using the generalized Bremmer coupling series and the Hamilton path integral, deHoop et al. (1998) gave a general formulation for the screen method. Based on their formulation, an accurate solution can be obtained by summing higher order terms in series. Huang et al. (1998) suggested an extended local Born/Retyov method to improve the angular accuracy. The above improvements are all for the acoustic or scalar wave screen propagators. In this paper, we will extend the wide angle modification method of Xie and Wu (1998) to elastic wave propagation.

Brief description of the method

Based on the one-way wave equation and scattering theory, the screen solution of a forward propagating elastic wave passing through an inhomogeneous thin slab between z_0 and z_1 can be represented as the superposition of plane P- and S-waves (Wu, 1994, Xie and Wu 1995, 1996)

$$\mathbf{u}_{scrn}(\mathbf{x}_T, z_1) = \frac{1}{4\pi^2}\int d\mathbf{K}_T \left[\mathbf{u}^P_{scrn}(\mathbf{K}_T, z_1) + \mathbf{u}^S_{scrn}(\mathbf{K}_T, z_1)\right] e^{i\mathbf{K}_T\cdot\mathbf{x}_T} \quad (1)$$

where $\mathbf{K}_T$ is the transverse wavenumber of plane waves, $\mathbf{x} = \mathbf{x}_T + z\mathbf{e_z}$ is the position vector, superscripts P and S denote P- and S-waves, and

$$\mathbf{u}^P_{scrn}(\mathbf{K}_T, z_1) = e^{i\gamma_\alpha \Delta z}\left[\mathbf{u}^P_0(\mathbf{K}_T, z_0) + \mathbf{U}^{PP}_f(\mathbf{K}_T, z_0) + \mathbf{U}^{SP}_f(\mathbf{K}_T, z_0)\right] \quad (2)$$

$$\mathbf{u}^S_{scrn}(\mathbf{K}_T, z_1) = e^{i\gamma_\beta \Delta z}\left[\mathbf{u}^S_0(\mathbf{K}_T, z_0) + \mathbf{U}^{SS}_f(\mathbf{K}_T, z_0) + \mathbf{U}^{PS}_f(\mathbf{K}_T, z_0)\right] \quad (3)$$

where $\mathbf{u}^P_{scrn}$ and $\mathbf{u}^S_{scrn}$ are screen solutions for P- and S-waves, $k_\alpha = \omega/\alpha_0$ and $k_\beta = \omega/\beta_0$ are background P and S wavenumbers, α_0 and β_0 are P- and S-wave reference speeds, $\gamma_\alpha = (k_\alpha^2 - \mathbf{K}_T{}^2)^{1/2}$ and $\gamma_\beta = (k_\beta^2 - \mathbf{K}_T{}^2)^{1/2}$ are vertical components of these wavenumbers. $e^{i\gamma_\alpha\Delta z}$ and $e^{i\gamma_\beta\Delta z}$ are phase shift operators. $\mathbf{U}_f$ denotes forward scattered waves, and superscripts PP, PS, SP and SS indicate the scattering between different wave types. Detailed expressions of these scattered fields can be found in Xie and Wu (1995).

For the elastic screen method, the calculation of interaction between the incident wave and an inhomogeneous slab is separated into several steps. First, the scattering terms are calculated. $\mathbf{U}^{SP}_f$ and $\mathbf{U}^{PS}_f$ give the coupling between different wave types. $\mathbf{U}^{PP}_f$

and $\mathbf{U}_f^{SS}$ are forward scattering for the same wave types, i.e., P to P and S to S, which contribute only to the phase of the forward propagating P- and S-waves. Two phase shift operators, $e^{i\gamma_\alpha \Delta z}$ and $e^{i\gamma_\beta \Delta z}$, give P- and S-wave phase shift through the background velocities, respectively. Similar to the scalar wave case, the elastic screen method gives correct results only within small scattering angles, there are errors for large velocity contrast and large scattering angles. The above analysis shows, for elastic phase screen method, the coupling and propagation are calculated separately. Having obtained the coupling terms, the P- and S-waves can propagate through the model independently, i.e., their propagations are decoupled. Instead of modifying coupled waves, this permits us to modify P- and S-wave propagations separately.

Based on this idea, the method introduced by Xie and Wu (1998) for scalar wave can be adopted to improve the phase of P- and S-waves. The equation for modifications are

$$\frac{\partial \mathbf{u}^P(\mathbf{K}_T, z)}{\partial z} = -ik_\alpha \left[\frac{a\left[(\frac{1}{n_\alpha}*) - 1\right] \frac{\mathbf{K}_T{}^2}{k_\alpha^2}}{1 - b\left[1 + (\frac{1}{n_\alpha^2}*)\right] \frac{\mathbf{K}_T{}^2}{k_\alpha^2}} \right] \mathbf{u}^P(\mathbf{K}_T, z) \tag{4}$$

and

$$\frac{\partial \mathbf{u}^S(\mathbf{K}_T, z))}{\partial z} = -ik_\beta \left[\frac{a\left[(\frac{1}{n_\beta}*) - 1\right] \frac{\mathbf{K}_T{}^2}{k_\beta^2}}{1 - b\left[1 + (\frac{1}{n_\beta^2}*)\right] \frac{\mathbf{K}_T{}^2}{k_\beta^2}} \right] \mathbf{u}^S(\mathbf{K}_T, z) \tag{5}$$

where $n_\alpha = \alpha_0/\alpha$ and $n_\beta = \beta_0/\beta$ are Fourier transforms of the refraction index for P- and S-waves, and "$*$" denotes the convolution in wavenumber domain. $a = 0.5$ and $b = 0.25$ are Padé expansion coefficients, their values can be further optimized to give the best results. The elastic screen solution $\mathbf{u}^P_{scrn}$ and $\mathbf{u}^S_{scrn}$ from (2) and (3) are used as input to equations (4) and (5). Solve these equations to implement the wide angle correction, and use the output to replace the $\mathbf{u}^P_{scrn}$ and $\mathbf{u}^S_{scrn}$ in equation (1). Finally, equation (1) gives the modified solution for elastic wave propagation. Equations (4) and (5) can be solved with a finite-difference method in space domain, which usually gives better stability than the wavenumber domain method. Similar to the screen method, the entire problem can be solved using duel domain approach shuttling between space and wavenumber domain.

Figure 1 compared the dispersion curves of different approaches. Shown in this figure are vertical wavenumber k_z versus horizontal wavenumber $\mathbf{K}_T$ for a plane P- or S-wave penetrating a slab with constant velocities. To show the accuracy with large velocity perturbations, we chose reference velocities as half of the true velocities. The velocity contrasts $\alpha/\alpha_0 = \beta/\beta_0 = 2.0$ (i.e., 100% velocity perturbations). The inner circle is the accurate dispersion curve. The outer circle is the dispersion curve in the background velocity, i.e. the phase shift solution. The dispersion curve from the phase screen method gives correct phase in the vertical direction but has large errors at wide angles. The wide angle version using (4) and (5) gives much better results. As can be seen in the figure, if we optimize the parameters a and b, the result can be further improved.

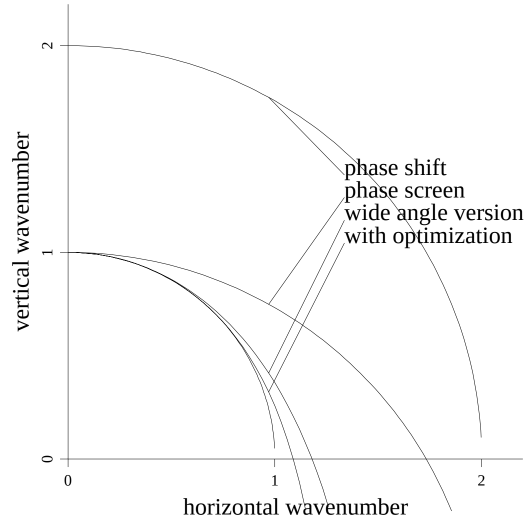

Figure 1: Comparison between dispersions from different approaches. The P-wave velocity $\alpha = 2\alpha_0$ (velocity perturbation is 100%). The vertical and horizontal coordinates are vertical and horizontal wavenumbers, Different curves indicate different approaches.

Numerical examples

To test the phase accuracy of different approaches, numerical examples are calculated using both elastic screen method and the wide angle method of this paper. Shown in Figures 2 and 3 are snap shots of the wavefields. The model is a homogeneous layer with $\alpha = 4.0km/sec.$ and $\beta = 2.4km/sec.$ We chose the reference velocities $\alpha_0 = 2.0km/sec.$ and $\beta_0 = 1.2km/sec.$, which are equivalent to 100% velocity perturbations for both P and S velocities. A complex source that radiates both P- and S- waves with a radiation pattern is used. Figure 2 shows snap shots from the elastic screen method. As expected, it gives a correct phase only in the small angle for both P- and S-waves. Figure 3 shows snap shots using modification equations (4) and (5). The wide angle phase accuracy has been considerably improved.

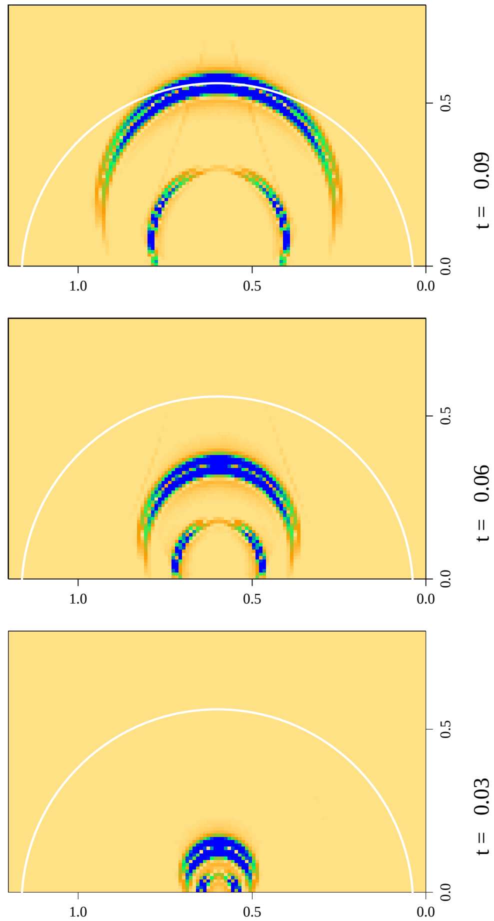

Figure 2: Snap shots from the elastic phase screen method. Velocity perturbations are 100% for both P- and S- waves. Complex source radiate both P- and S-waves.

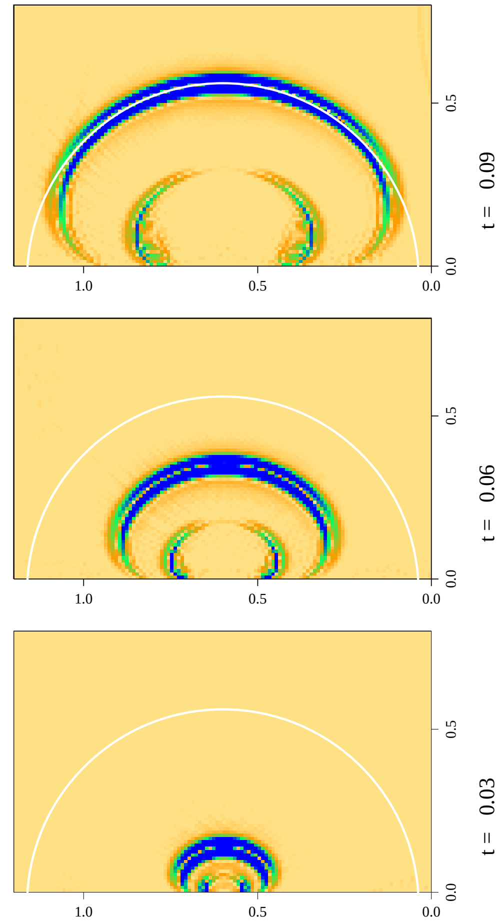

Figure 3: Similar to that in Figure 2, except the modification equations (4) and (5) are used for P- and S-waves.

The next one is a two layer model. The velocities at the source side are $\alpha = 2.0km/sec.$ and $\beta = 1.2km/sec$, which have been chosen as background velocities. On the other side of the interface, $\alpha = 3.0km/sec.$ and $\beta = 1.8km/sec.$ The velocity perturbations are 50% for both P and S velocities. A P-wave source is used to generate waves in the model. Figure 4 shows snap shots from phase screen method. Figure 5 shows snap shots using modification equations. Both P- and S-wave fronts have been improved.

Conclusions

The elastic screen method (Xie and Wu, 1995, 1996) is based on the one-way wave equation and small angle, small perturbation approximations. It provides an efficient way for calculating the propagation of elastic waves in a laterally heterogeneous model. However, for models with large velocity perturbations, the elastic screen method gives large phase error at wide scattering angles. In this study, the wide angle modification method for scalar waves (Xie and Wu, 1998) has been adopted to elastic waves. The resultant method can improve the phase accuracy for elastic wave propagation at wide angle and large velocity perturbations. This greatly increased the modeling and imaging capabilities of the elastic screen method in models with large velocity contrasts.

Acknowledgments

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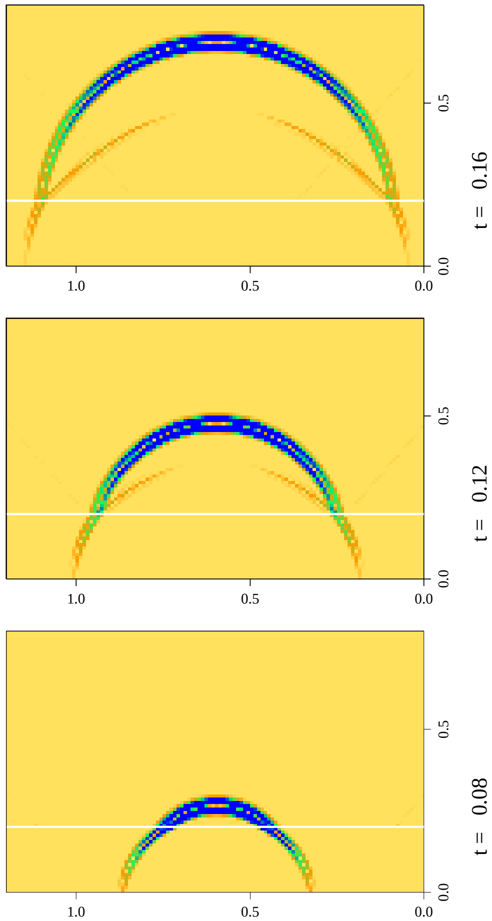

Figure 4: Snap shots from the elastic screen method. Across the interface, velocity perturbations are 50% for both P- and S- waves. A P-wave source is used.

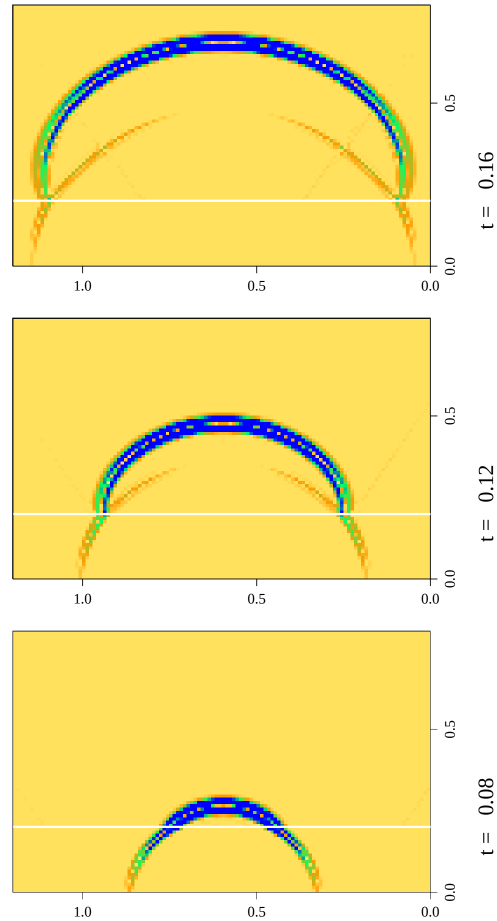

Figure 5: Similar to that in Figure 4, except the modification equations (4) and (5) are used for P- and S-waves.

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