Improve the wide angle accuracy of screen method under large contrast

Xiao-Bi Xie and Ru-Shan Wu, Institute of Tectonics, University of California, Santa Cruz*

Summary

Phase screen method is a high efficient one way wave equation method which can be used either in forward modeling or seismic migration. However, the phase screen method is accurate only for small perturbations or for small incident angles which prevent it from applying to models with large velocity contrasts. In this study, a general formulation of the screen method is derived for wide angle under large velocity contrast. The derivation is on the plane wave decomposition which avoid the small perturbation assumption. Using different approximations the general formulation leads to different approaches. Our results can be used to design different dual domain algorithms. It can also lead to hybrid methods which combine the screen method and spatial domain finite-difference method together to solve the one way wave equation.

Introduction

The phase screen method is based on the one way wave equation method. It can be used as a high efficiency propagator for forward propagation (Wu, 1994; Xie and Wu, 1995, 1996) or seismic migration (Wu and Xie 1994; Huang and Wu, 1996; Wu and Jin, 1997). The derivation of phase screen method usually requires that the velocity perturbation is small or the incident angle is small. However, the real models used in exploration practices may contain very large velocity contrasts. For example, in the Gulf of Mexico, the salt dunes have velocities which are twice or even three times larger than the surrounding media. Under these situations, the phase screen method can give correct phase only within a small angle, which can not generate satisfactory results for migrations. To improve the accuracy of phase screen method under wide angle, Wu, Huang and Xie (1995), Wu and de Hoop (1996) proposed a wide angle version for the screen method which sometimes called pseudo-screen method. Compared with the phase screen method, which takes only zeroth order term in the expansion, pseudo-screen method can give better phase for wide angle waves. However, as mentioned above, these methods are derived from a small perturbation basis, the wide angle error caused by the large velocity contrast still exist. In this paper, we will give a generalized formula for the screen method. which can be applied to wide angle under large velocity contrast. From this general formulation, different approachs can be obtained. For simplicity, we will discuss 2D case only, but extend the results to 3D is straightforward.

Brief description of the method

Under large velocity contrast, the perturbation theory is no longer valid. We will start from the 2D scalar wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_0^2 n^2\right)p(x,z)=0 \tag{1}$$

where $p(x,z)$ is the wavefield, $n(x,z)=c_0/c(x,z)$ is the refraction index, c_0 is a reference velocity and $c(x,z)$ is the velocity of the media, $k_0=\omega/c_0$ is a reference wavenumber. Factoring equation (1) will generate two one way wave equations. One is for forward propagation and the other is for backward propagation. The equation governing the forward propagated waves is

$$\frac{\partial p(x,z)}{\partial z}=i\sqrt{n^2k_0^2+\frac{\partial^2}{\partial x^2}}\,p(x,z) \tag{2}$$

For a media with small velocity perturbations or under small angle approximations, the second term in the square root is small, the square root can be expanded using Taylor series. However, under the large perturbation and wide angle the Taylor expansion of the square root function is no longer valid. An alternative is to use Padé approximation (Collins 1990). From equation (2) and using first order Padé approximation, we have

$$\frac{\partial p(x,z)}{\partial z}=ink_0\left[1+\frac{a\frac{1}{n^2k_0^2}\frac{\partial^2}{\partial x^2}}{1+b\left[\frac{1}{n^2k_0^2}\frac{\partial^2}{\partial x^2}\right]}\right]p(x,z) \tag{3}$$

where a and b are expansion coefficients. For first order expansion $a=0.5$ and $b=0.25$. Considering the wave field can be expressed as a superposition of plane waves

$$p(x,z)=\int dk_{0x}\psi(z,k_{0x})e^{ik_{0x}x} \tag{4}$$

where $\psi(z,k_{0x})e^{ik_{0x}x}$ is a plane wave component, $\psi(z,k_{0x})$ is its amplitude and k_{0x} is the transverse wavenumber. Applying the Fourier transform to x in equation (3), we have the equation for $\psi(z,k_{0x})$

$$\frac{\partial\psi(z,k_{0x})}{\partial z}=i\left[k_{0z}-k_0(\frac{\delta c}{c}*)\right.$$
$$\left.+k_0\frac{a(n*)\eta}{1+b\eta}-k_0\frac{a\zeta}{1+b\zeta}\right]\psi(z,k_{0x}) \tag{5}$$

where $\eta = -(1/n^2*)(k_{0x}^2/k_0^2)$ and $\zeta = -k_{0x}^2/k_0^2$. Note equation (5) is already in the wavenumber domain. $(n*)$, $(1/n^2*)$ and $(\delta c/c*)$ are Fourier transforms of n, $1/n^2$ and $\delta c/c$, and "$*$" denotes the convolution in wavenumber domain. On the right hand side of equation (5), there are four terms. The first term gives the phase change caused by the background velocity, which equivalent to the phase shift solution where no velocity perturbation is considered. The second term, combined with the first term, gives the phase screen solution, which correctly gives the phase shift for zero angle incidence but cause errors for wide angle incidence, especially for large perturbations. The following are two first order Padé expansions. The first one results from the square root function itself, and the second comes from the expansion of k_{0z}. These terms modify the phase screen solution

To make the calculation easier, the last two terms can be combined and give

$$\frac{\partial\psi(z,k_{0x})}{\partial z} \approx i\left[k_{0z} - k_0(\frac{\delta c}{c}*) - k_0\frac{a\left[(\frac{1}{n}*)-1\right]\frac{k_{0x}^2}{k_0^2}}{1-b\left[1+(\frac{1}{n^2}*)\right]\frac{k_{0x}^2}{k_0^2}}\right]\psi(z,k_{0x}) \quad (6)$$

where we have omitted the fourth order terms of k_{0x}/k_0. Equation (6) is easier to calculate but still has good accuracy.

To obtain formulas for screen method, considering the wave propagate through a thin slab perpendicular to the z direction and has velocity $c(x,z)$ between z_0 and z_1. The slab is buried in a homogeneous media with a reference velocity c_0. If the slab is thin enough, the change of $n(x,z)$ along z direction can be omitted within $\Delta z = z_1 - z_0$, and equation (5) or (6) can be intergrated with z. For a simple discussion, keeping only the leading term in the rational expansion in equation (6), we can derive a formula similar to the pseudo screen method. Under this approximation, the wavefield is

$$\psi(z_1,k_{0x}) = \exp\left\{i\left[\Delta z k_{0z} - \Delta z k_0\left(\frac{\delta c}{c}*\right) - \Delta z k_0 A\frac{k_{0x}^2}{k_0^2}\right]\right\}\psi(z_0,k_{0x}) \quad (7)$$

where $A = (1/2)[(1/n*) - 1]$. On the right hand side, there are three terms. The first two terms gives the phase screen solution. The last term modifies the phase screen solution for wide angle incidence and large perturbations. Formulas keeping more terms are also possible.

In Figures 1 and 2 we compared the accuracies of different approximations. Shown in this figure are calculated phases versus accurate phase for a plane wave penetrating a slab with constant velocity c. To show

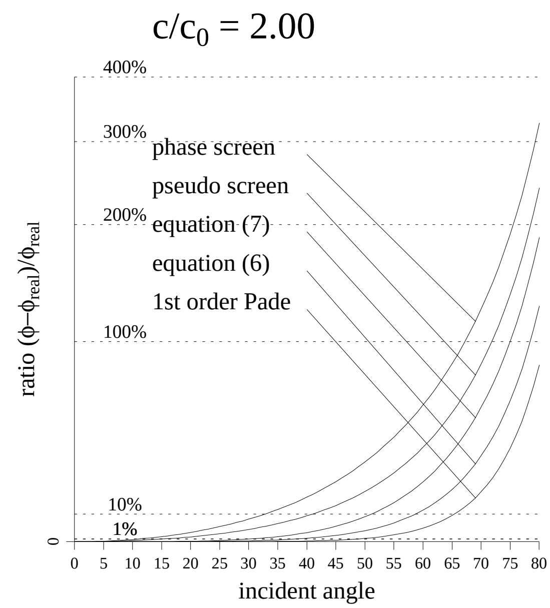

Figure 1: Comparison of phase errors for different approximations. The velocity contrast $c/c_0 = 2.0$.

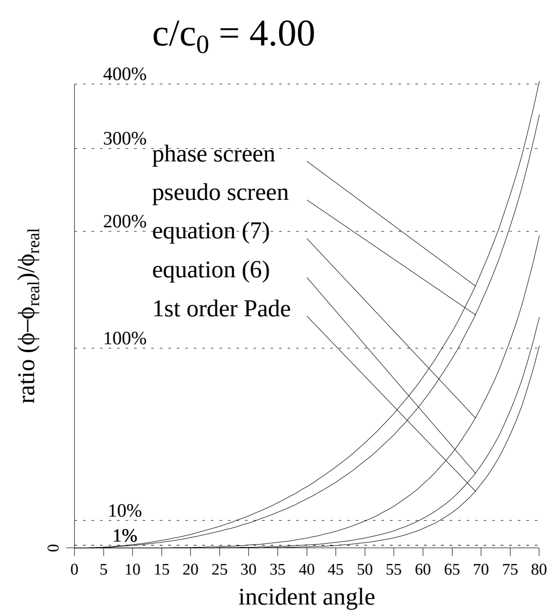

Figure 2: Comparison of phase errors for different approximations. The velocity contrast $c/c_0 = 4.0$.

accuracies under large velocity perturbations, a velocity contrast $c/c_0 = 2.0$ (i.e., 100% velocity perturbation) is chosen in Figure 1. The vertical coordinate is the relative phase error, and the horizontal coordinate is the incident angle. Different curves are for phase screen method, pseudo screen method, and various approximations of this study. Given a threshold error of 1%, the phase screen method gives correct phase only up to 10 degrees, the pseudo screen method gives correct phase to about 15 degrees. The equation (7), although keeping only one term, gives correct phase up to 30 degrees. Equation (6) gives accurate phase to 40 degrees. The Padé approximation, equation(5), gives even higher accuracy but is less efficient. In Figure 2, the velocity contrast is $c/c_0 = 4.0$. The phase screen method gives correct phase only within 5 to 7 degrees, while the equations (6) and (7) can give correct phase up to 30 to 40 degrees.

Numerical implementations

Equations (5) or (6) can be seen as a general form of the screen method. From this equation, several approaches can be derived. It can be implemented in either the wavenumber domain or in space domain. Note that k_{0z} is a function of k_{0x}. The operation between k_{0z} and $\psi_(z_0, k_{0x})$ is wavenumber domain multiplication. The operations of $(\delta c/c*)$, $(1/n*)$, etc., to $\psi_(z_0, k_{0x})$ are wavenumber domain convolutions. Generally speaking, it will be more efficient if we replace the wavenumber domain convolution with a space domain multiplication. The dual-domain technique, which permits both operations be taken in their favorite domains and shuttle the wavefield between the domains with fast Fourier transforms, is the most efficient way to calculate the propagation. A dual-domain formula for equation (7) can be written as

$$\psi(x, z_1) = \left(\mathcal{F}^{-1} - i\Delta z k_0 A \mathcal{F}^{-1}\frac{k_{0x}^2}{k_0^2}\right) \times e^{i\Delta z k_{0z}} \mathcal{F} e^{-i\Delta z k_0 \left(\frac{\delta c}{c}\right)} \psi(x, z_0) \quad (8)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote Fourier and inverse Fourier transforms. For media without large perturbations, modification can be omitted, and the equation degenerate to the usual phase screen solution. If the higher order perturbations can be neglected, the equation degenerate into the pseudo screen method (Wu and Huang 1995; Wu and de Hoop 1996). For models where large perturbations are very localized, a spatial domain version may be more efficient.

$$\psi(x, z_1) = \left[1 + i\Delta z \frac{A}{k_0}\frac{\partial^2}{\partial x^2}\right] \times \mathcal{F}^{-1} e^{i\Delta z k_{0z}} \mathcal{F} e^{-i\Delta z k_0 \left(\frac{\delta c}{c}\right)} \psi(x, z_0) \quad (9)$$

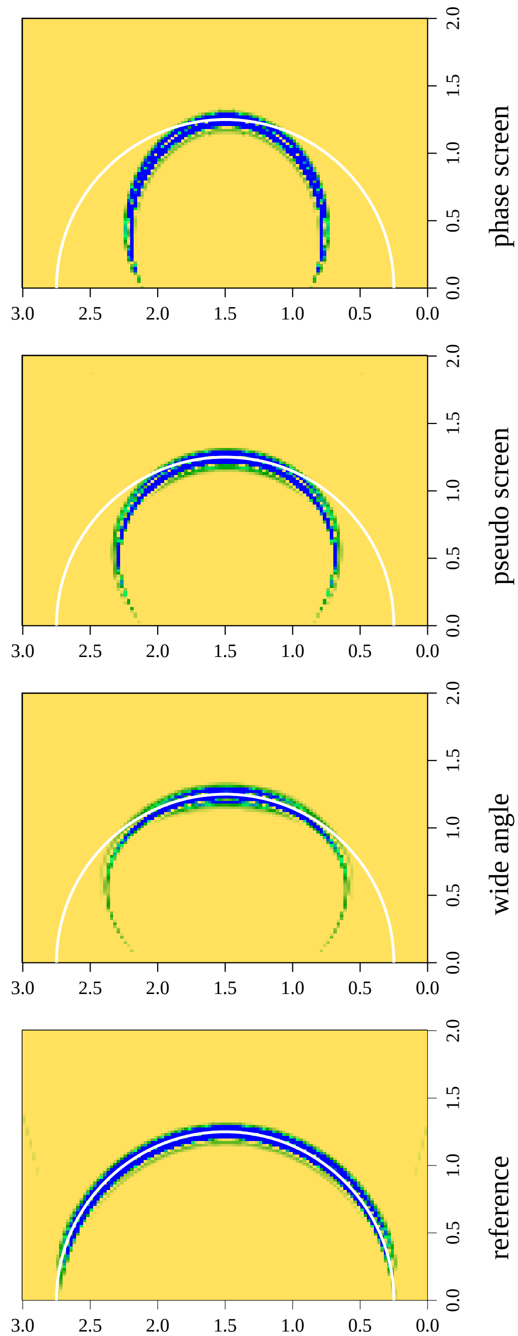

Figure 3: Comparison of wavefield generated using different methods. The velocity contrast $c/c_0 = 2.0$. From top to bottom, the snap shots are from phase screen method, pseudo screen method (using Taylor expansion) and equation (8) in this paper. As a reference, the bottom one gives the correct phase.

In space domain, the modification terms can be applied only to regions where the perturbations are large and which is more flexible than in the wavenumber domain.

Note $\psi_{PS} = \mathcal{F}^{-1} e^{i\Delta z k_{0z}} \mathcal{F} e^{-i\Delta z k_0 \left(\frac{\delta c}{c}\right)} \psi(x, z_0)$ is the phase screen solution. The above equations separated the phase screen solution from the residuals. The rest part of the solution can be seen as residual field for wide angle and large perturbations. These equations can also be solved with a hybrid method by first calculate phase screen solution and then using finite-difference method to solve the residual fields. Similar treatment can also be applied to equations (5) and (6). Since the residual field is relatively weak and easier to deal with, and the phase screen method is relatively efficient, the hybrid method is a good candidate for high efficiency high accuracy wave propagation.

Numerical examples

To test the phase accuracies of different approaches, Numerical examples are calculated using both phase screen method, pseudo screen method and the wide angle method of this paper. Shown in Figure 3 are snap shots of the wavefields. A Greene's source (Greene, 1984) is used in the calculation. This is a wide angle source for one way wave equation method and can cover the angle up to about 45 degrees. The model is a homogeneous layer with $c = 2c_0$ and $c_0 = 1.5$ km/s, which equivalent to a homogeneous layer with a velocity of 3 km/sec. As a reference, the bottom panel gives a correct wave field which is calculated directly using the real velocity. The arc indicate the theoretical wave front. The first snap shot (the top panel) is from phase screen method. The phase screen method gives correct phase in the zero angle direction but has very large errors for wide angles. The second snap shot is from pseudo screen method with a first order Taylor approximation. The wide angle phase response is better than the phase screen method but still gives large errors. The third snap shot is obtained using equation (8) or (9). Although we kept only the leading term in these equations, the phase accuracy has been considerablely improved.

Conclusions

A general form of the screen method is given in this study. The derivation is based on the plane wave decomposition and avoid the small perturbation assumption. The results can be applied to wide angle wave propagation and migration under large velocity perturbations. From this general formula, different approachs including the phase screen method and pseudo screen method can be derived using different approximations. From this equation, we can also derive formulas leading to different dual domain implementations. It can also generate hybrid methods which combined the screen method and spatial domain finite-difference method together to solve the one way wave equation.

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