Free surface boundary condition and the source term for one-way elastic wave method

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Summary

One-way wave equation method is very efficient. It can be used as forward and backward propagators in seismic modeling and migration. However, unlike the full wave method, the one-way method is usually derived for a source free situation. The source is added to the calculation as a boundary condition. The free surface effect at both the source side and receiver side are often not considered. Based on the plane wave decomposition, this study gives a free surface and source condition for elastic one-way wave equation method. This boundary condition can be applied at either the source side or the receiver side. With this boundary condition, many of the free surface effects, e.g., reflected P and S-waves, secondary surface waves, etc., can be generated. This boundary condition can be easily applied to reflection, VSP and cross hole modelings and seismic migration. It is especially useful for one-way methods based on the wavenumber domain Fourier transforms.

Introduction

One-way wave propagation method of acoustic and elastic waves have drawn broad interest recently in seismic forward modeling and migration (e.g., Claerbout, 1985; Stoffa, et al., 1990; Wu, 1994, 1996; Wu and Xie, 1994; Xie and Wu 1995,1996; Huang and Wu, 1996). Although, one-way wave equation method in inhomogeneous media has been extensively studied in the literature, few detailed studies have been made on the implementation of source term and free boundary conditions. In modeling and imaging practice, source and receivers are usually very close to the free surface. Many problems will involve the interaction between waves and free surface boundaries. Compared with an infinite space, the existence of a free surface boundary will add complexity to the received waves. For example, a buried explosion source generates not only direct down going P-wave. It also generates down going PP and PS surface reflected waves. A receiver located on the ground surface receives both incident wave and reflections from the surface. A VSP or cross hole observation will receive the reflections from the free surface. In many cases, the surface wave will also be generated. Wapenaar (1990), Charara and Tarantola (1996) introduced source and boundary conditions for acoustic media. In this study, we will introduce source and boundary conditions for elastic media. When combine the free surface boundary condition into the one-way wave equation algorithms, many phenomena related to free surface can be properly simulated.

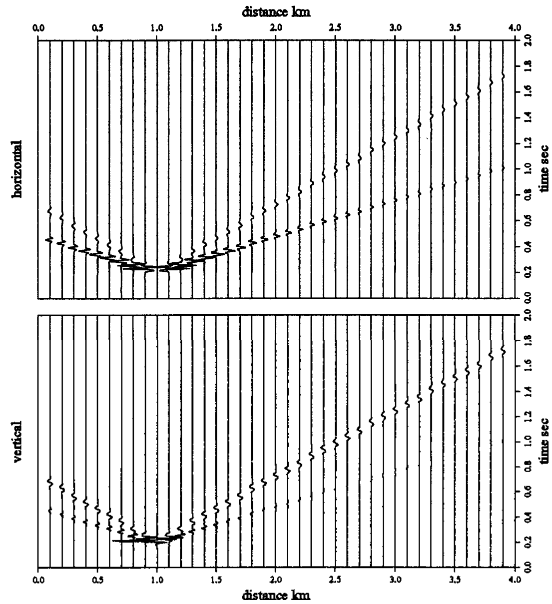

Figure 1: Response of the free surface to a buried explosion source. The first high frequency arrival is composed of both direct P-wave, reflected PP and PS waves. The later low frequency arrival is the Rayleigh wave generated by the interaction between the incident wave and free boundary.

Brief description of the method

Consider a half space with a free surface at z = 0 and choose the downward direction as the positive z. For simplicity, we discuss a 2D problem in x -z plane, and take P-wave incidence as an example. To extend these conditions to three dimensions and S-wave incidence is straightforward. At depth z, wave field u can be expressed as

$$\mathbf{u}(x, z) = \frac{1}{2\pi} \int dK_T \left[\mathbf{u}^P(K_T, z) + \mathbf{u}^{PP}(K_T, z) \right.$$

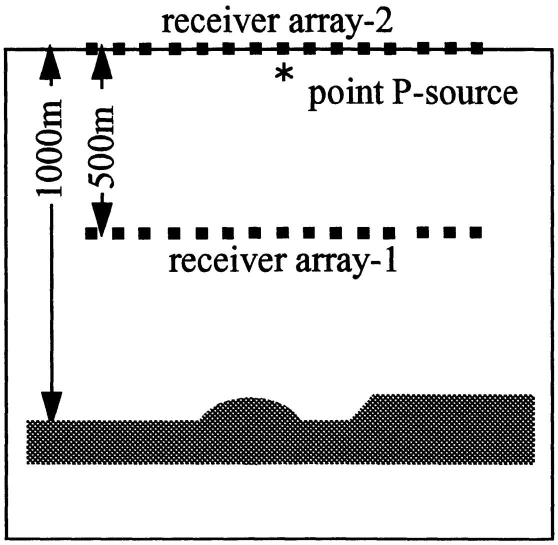

Figure 2: Configuration of the model. An explosion source is located at a depth of 50 m. There is a low velocity layer at the depth of 1000 m. The velocity perturbation within the low velocity layer is 20 % for both P and S-waves. The free surface boundary condition is used on the upper boundary. The receiver array at the 500 m depth is used to obtain down going waves, and the receiver array at the upper boundary is used to record the reflections from the model.

$$+\mathbf{u}^{PS}(K_T, z)\big]\, e^{iK_T\cdot x} \qquad (1)$$

where the integrand is composed of plane P and S-waves, K_T is their horizontal wavenumber, x = $x\mathbf{e}_x$ + $z\mathbf{e}_z$ is the position vector, $\mathbf{u}^P$ is the incident P-wave, $\mathbf{u}^{PP}$ and $\mathbf{u}^{PS}$ are down going reflected P and S-waves from the free surface. They can be expressed as

$$\mathbf{u}^P(K_T, z) = S(\omega, K_T)\mathbf{k}^P exp[ik_z^P|z - z_0|] \qquad (2)$$

$$\mathbf{u}^{PP}(K_T, z) = S(\omega, K_T)\mathbf{k}^P R^{PP} exp[ik_z^P(z + z_0)] \qquad (3)$$

$$\mathbf{u}^{PS}(K_T, z) = -S(\omega, K_T)(\mathbf{k}^S \times \mathbf{e}_y)R^{PS} \times exp[i(k_z^P z_0 + k_z^S z)] \qquad (4)$$

where $S(\omega, K_T)$ is the amplitude of incident P-wave, k_z^P and k_z^S are vertical wavenumbers of P and S-waves, $\mathbf{k}^P$ and $\mathbf{k}^S$ are wavenumber vectors for P- and S-waves, R^{PP} and R^{PS} are free surface reflectivities for *PP* and *PS* waves, $\mathbf{e}_y$ is a unit vector in y-direction. z_0 is a certain depth at which the incident wave is known. The incident wave can be either the wave from a source or reflections from the structure. The above system satisfies both one-way wave equation and the boundary condition. This means, for an incident wave, either from a source or from a one-way calculation, we can obtain the free surface effect by simply add reflections to meet the boundary condition. Since this method uses the plane wave superposition to express the wave field, it is especially useful for one-way methods based on the Fourier transform in wavenumber domain, e.g., the split step Fourier method (Stoffa, et al., 1990), PSPI method (Gazdag and Sguazzero, 1984) and complex screen method (Wu, 1994, 1996; Xie and Wu, 1995, 1996; Huang and Wu, 1996), etc.

The above boundary condition can be easily combined into a one-way wave equation method. For receiver side problems, simply replace $S(\omega, K_T)$ in equations (2) to (4) with the amplitude of incident plane waves, then substitute $\mathbf{u}^P$, $\mathbf{u}^{PP}$ and $\mathbf{u}^{PS}$ into equation (1). The $\mathbf{u}(x, 0)$ gives the wavefield at ground surface.

For source side problem, $S(\omega, K_T)$ can be replaced with a source term. In this way, all down going waves, including direct wave from the source and reflections from the free surface, can be obtained. Once obtained all down going waves at depth z, further extrapolation of the wavefield can start from this level. For convenience, we can choose the starting level at z = 0. There will be no problem for down going reflections $\mathbf{u}^{PP}$ and $\mathbf{u}^{PS}$ to start from z = 0. For direct wave $\mathbf{u}^P$, which is radiated from the source at z_0, we can back propagate it from the source level z_0 to the free surface z = 0 by multiply it with a propagator $exp(-ik_z^P z_0)$. Then do the downward extrapolation together with the reflected waves.

Numerical examples

To test the boundary condition and source term introduced in the last section, we first calculate the response of a free surface to a point explosion source. For a 2D case, a P-wave source term is

$$S(\omega, K_T) = \frac{-L(\omega)}{2\rho\alpha^2 k_z^P} exp[-iK_T x_s] \qquad (5)$$

where *L(w)* is the spectrum of the source time function, ρ and α are density and P-wave velocity, and x_s is the horizontal location of the source. Substitute equation (5) into equations (1) to (4) and set z = 0, we can obtain the free surface response. Figure 1 gives the response to a source buried in a depth of 50 m. The first arrival is composed of both direct P-wave, reflected PP-wave and reflected PS-wave. On the ground surface, both these waves have the same horizontal slowness. The second arrival with relatively low frequency is Rayleigh wave generated by the interaction between the incident P-wave and the free surface. From this example we can see, without the free surface condition, only direct P-wave can be received. The free surface boundary condition gives more complicated and realistic waveforms. To reach the Rayleigh pole in the complex wavenumber plane, the vertical wavenumber already extended to the imaginary domain. This example shows the broad range of applicability of the boundary condition.

The boundary condition can be applied to a one-way wave equation method at the source side to give secondary waves generated by the interaction between the source and the free surface, or at the receiver side to give the distortion to incident waveforms. In the next example the source and free surface condition is used as input to a one-way elastic complex screen method. With this method, we can calculate both downward waves and primary reflections from the structure (Xie and Wu 1995, 1996). The configuration of the source, receivers and model are shown in Figure 2. An explosion source is located at a depth of 50 m. There are two sets of receivers. The first receiver array is located at a depth of 500 m, and used to obtain the forward (downward) waves. The second receiver array is located on the ground surface and used to record the reflections from the model. The background parameters for the model are $\alpha = 3.6km/s$, $\beta = 2.08km/s$ and $\rho = 2.2\text{g/cm}^3$. There is a low velocity layer located at a depth of 1000 m. The low velocity layer has an irregular upper interface and a 20 % velocity perturbation for both P and S-waves.

Shown in Figure 3 are synthetic seismograms of down going waves at depth 500 m. The upper penal is for horizontal components and the lower penal is for vertical components. In addition to the P-wave directly from the source, there are also surface reflected down going PP and PS waves (in seismology literatures they usually called pP and pS waves). The PP wave can be seen immediately following the first arrival due to a shallow source. The second clear arrival is PS. Shown in Figure 4 are reflected waves recorded at the ground surface. From each down going P or S-wave, there will be reflected P and S-waves. Both the shape of the irregular interface and many different wave types add the complexity to the waveforms. Without the free boundary condition, the surface reflected arrivals will be missed in the seismograms.

In the above example, the boundary condition is applied to the source side. The same boundary condition can be applied to the receiver side. If heterogeneities exist close to the surface or there are topographic fluctuations, they will serve as secondary sources once illuminated by the incoming wave. These secondary sources, just like our first example, will excite surface waves on the ground surface and distort the received waveforms.

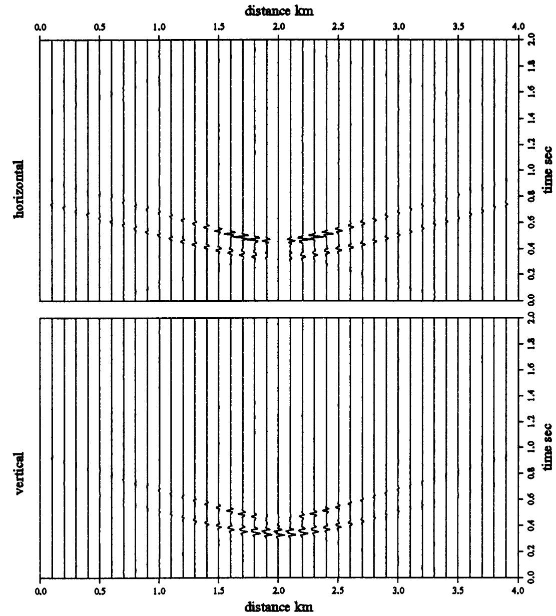

Figure 3: Down going waves from the source and the free boundary reflections. The first arrival is P-wave directly from the source, which is closely followed by the free surface reflected wave PP. The second clear arrival is free surface reflected wave PS.

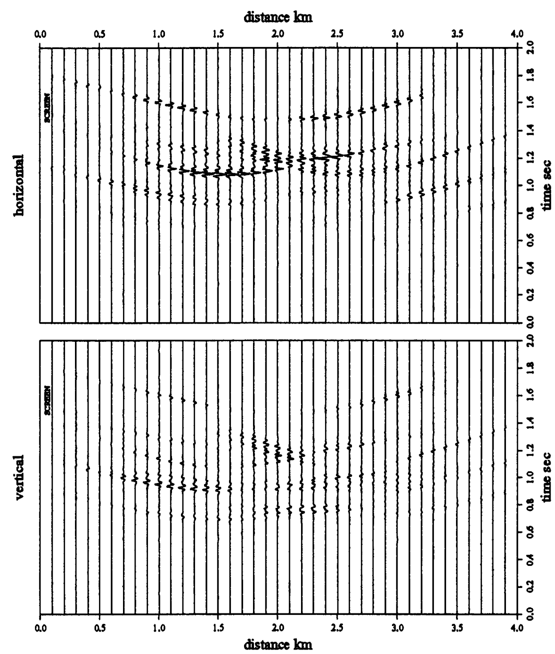

Figure 4: Reflections recorded at the upper boundary. There are three sets of down going waves, and each of them will generate P and S reflections on the interface. They made the reflected signal rather complicated.

Conclusions

Based on the plane wave decomposition, a source and free boundary condition is designed for elastic one-way wave equation method. This boundary condition can be applied to a one-way wave equation method at either the source side or at the receiver side. Many of the free surface effects, (e.g., free boundary reflections, secondary surface waves, etc.) can be generated by this method. This boundary condition can be applied to seismic imaging, reflection modeling, VSP and cross hole simulations, etc. It is especially useful for one-way methods based on the wavenumber domain Fourier transforms.

Acknowledgements

This research was supported by the Office of Naval Research through contract N00014-95-1-0093-01, and by the United States Department of Energy under ACT1 project. The facility support from the W.M. Keck Foundation is also acknowledged. Contribution Number 328 of the Institute of Tectonics, University of California, Santa Cruz.

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