Seismic wave scattering in 3D random media: A finite-difference simulation and slowness domain analysis
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Summary
We develop a slowness analysis method for investigating the seismic wave scattering in complex velocity models. The wavefield is decomposed into mixed space-time and slowness-frequency domains. The scattering process can be better investigated in these mixed domains. We conduct finite-difference simulations for velocity models with different random fluctuations and apply the slowness analysis to the synthetic wavefield. The results reveal that in random velocity models with different correlation lengths along different directions, the scattered waves show different radiation patterns. It is possible using this property to determine the dominant direction of subsurface heterogeneities such as the crack alignment in the reservoirs.

Introduction
Small scale heterogeneities, e.g., velocity fluctuations, fractures, aggregate of small cracks, are widely existed in the subsurface environment. Their detailed properties (intensity, distribution and the alignment) are closely related to many important issues such as the reservoir characterization. Though it is usually difficult to directly image these small scale structures, investigate waves scattered from these heterogeneities can provide useful statistical information about their properties (Burns, et al., 2006; Zheng, et al. 2011; Hall and Wang, 2012; Sun, et al. 2012).

The scattering process is controlled by many characteristic scales, e.g., wavelength, frequency and correlation length of heterogeneities. The scattered waves often propagate off the incident direction forming different scattering angles and radiation patterns. These patterns are related to the properties of elastic parameters and their distributions in the model. For example, small scale heterogeneities tend to generate wide-angle forward and backward scatterings while large scale heterogeneities generate more forward scatterings (Wu, 1989). The perturbation of elastic parameters λ, μ, ρ or a crack each has its own scattering pattern and responses differently for P or S wave incidence. The scattered field may change with elapsed times forming variable couds. Investigating all these scattering properties carried in the wavefield can provide useful constraints to the heterogeneities, including material parameters and their distributions.

Numerical methods such as the finite-difference, finite-element or spectral element methods are highly flexible in handling complex velocity models. Thus, they play an important role for investigating seismic wave scattering (e.g., Frankel and Clayton, 1986; Coates and Schoenberg, 1995; Vlastos, et al. 2003; Xie, et al. 2005; Willis, et al. 2006; Hall and Wang, 2012). However, due to the complex processes associated with the scattering, it is often difficult to reveal the wave-material interactions by simply studying the spatial-time wavefield. The scattered waves developed at certain spatial-time can propagate towards various directions and contain different frequency components. In order to characterize the detailed scattering process, it is preferred to study the wavefield from all these aspects simultaneously. Motivated by this idea, we develop a method to investigate the synthetic wavefield in the localized mixed domains (e.g., time, space, direction and frequency). As examples, we apply this method to the elastic wave scattering in 3D random velocity models. The results reveal that the scattered waves carrying useful information which can be used for characterizing the small scale heterogeneities in the model.

Converting the Wavefield to Mixed Domains
Several methods can be used to transfer spatial-time domain data into slowness (or equivalently wavenumber) domain, for example, the FK analysis (Zhang, et al. 2006) or slant stacking (Xie and Lay, 1994; Xie, et al., 2005). Here we employ a local slant stacking method to conduct slowness analysis. The localized operation guarantees the result can be projected to space and slowness domains. Following Xie et al. (2005) and Yan and Xie (2010, 2012), we simultaneously separate the elastic wave into P and S wave modes and decompose them into a superposition of local plane waves (or beams) propagating in different directions. First, the wavefield $u(x,t)$ is converted from the time domain to frequency domain. Then the mixed mode wavefield $u(x,\omega)$ is decomposed into P and S components localized in both space and slowness

$$u^p(x,\omega) = \int W(x-x') \hat{e} \cdot \left[u(x',\omega) \cdot \hat{e}\right] e^{-i\omega(x-x')} dx'$$

$$u^s(x,\omega) = \int W(x-x') \hat{e} \left[u(x',\omega) \times \hat{e}\right] e^{-i\omega(x-x')} dx'$$

where $u^p(x,\omega)$ and $u^s(x,\omega)$ are P and S waves in local slowness domain, $W(x-x')$ is a space window which samples a piece of wavefield centered at $x$, $\hat{e} = \hat{p}$ is the slowness vector, $\hat{e}$ is a unit vector towards the propagation direction, $\hat{p}$ is the absolute value of slowness with $p_p = 1/\tau_p$ and $p_s = 1/\tau_s$ are for P and S waves, and $\tau_p$ and $\tau_s$ are the average P and S wave velocities in the sampling domain.
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window. Transforming \( u^p(p, x, \omega) \) and \( u^s(p, x, \omega) \) back to the time domain, we obtain \( u^p(p, x, t) \) and \( u^s(p, x, t) \). By further applying the bandpass filter with central frequency \( \omega_0 \) to these time series, we obtain bandpass filtered signals \( u^p(p, x, \omega_0, t) \) and \( u^s(p, x, \omega_0, t) \). These signals denote wave packet components arriving at the space location \( x \) at time \( t \), with propagation direction along \( p \) and central frequency \( \omega_0 \).

Table 1. 3D velocity models used in FD simulation

<table>
<thead>
<tr>
<th>No.</th>
<th>Model descriptions (Parameters are listed in Table 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Random model with isotropic power spectrum. Model parameters for background, perturbations and correlation lengths refer to B1, R2 and C1 in Table 2.</td>
</tr>
<tr>
<td>2</td>
<td>Random model with anisotropic power spectrum. Model parameters refer to B1, R2 and C2 in Table 2.</td>
</tr>
<tr>
<td>3</td>
<td>Three-layered model with an intermediate high-speed layer located between depths 0.6 and 0.8 km. Parameters for top and bottom layers are in B1 and R1, for middle layer are in B2 and R1.</td>
</tr>
<tr>
<td>4</td>
<td>Similar to No.3, except the parameters for the intermediate layer are in B2, R3 and C2.</td>
</tr>
<tr>
<td>5</td>
<td>Similar to No.3, except the parameters for the intermediate layer are in B2, R3 and C3.</td>
</tr>
<tr>
<td>6</td>
<td>Similar to No.3 but there are crack-like inclusions in the middle layer. The thin low velocity inclusions are extended in ( x )- and ( z )-directions (refer to Figure 6d).</td>
</tr>
</tbody>
</table>

Table 2. List of model parameters.

<table>
<thead>
<tr>
<th>Background parameters</th>
<th>rms perturbations</th>
<th>Correlation lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_s ) = 3.0 km/s</td>
<td>( \delta V_s/V_s = 0 )</td>
<td>( a_s = 50 m )</td>
</tr>
<tr>
<td>( V_p = 1.765 ) km/s</td>
<td>( \delta V_p/V_p = 0 )</td>
<td>( a_p = 50 m )</td>
</tr>
<tr>
<td>( \rho = 2.20 ) g/cm(^3)</td>
<td>( \delta \rho/\rho = 0 )</td>
<td>( a = 50 m )</td>
</tr>
<tr>
<td>( V_s = 3.5 ) km/s</td>
<td>( \delta V_s/V_s = 3% )</td>
<td>( a_s = 200 m )</td>
</tr>
<tr>
<td>( V_p = 2.06 ) km/s</td>
<td>( \delta V_p/V_p = 3% )</td>
<td>( a_p = 25 m )</td>
</tr>
<tr>
<td>( \rho = 2.25 ) g/cm(^3)</td>
<td>( \delta \rho/\rho = 1% )</td>
<td>( a = 250 m )</td>
</tr>
<tr>
<td>( V_s = 2.8 ) km/s</td>
<td>( \delta V_s/V_s = 5% )</td>
<td>( a_s = 25 m )</td>
</tr>
<tr>
<td>( V_p = 2.5 ) km/s</td>
<td>( \delta V_p/V_p = 5% )</td>
<td>( a_p = 200 m )</td>
</tr>
<tr>
<td>( \rho = 2.20 ) g/cm(^3)</td>
<td>( \delta \rho/\rho = 1% )</td>
<td>( a = 250 m )</td>
</tr>
</tbody>
</table>

Scattering of a plane wave in random models

We first investigate a down-going plane \( P \)-wave propagating through different random models. The first model is described in Table 1 as No.1 with parameters listed in Table 2. It has isotropic random fluctuations for \( v_p \), \( v_s \) and \( \rho \). Shown in Figure 1a is the FD generated wavefield snapshot overlapped on the velocity model. The Ricker wavelet has a central frequency of 60 Hz. We can see the distorted down-going wave front followed by the scattered waves. To investigate the detailed properties of the scattered wavefield, we conduct the slowness analysis mentioned above.

Figure 1. Wavefield snap shots for a down-going plane \( P \) wave through different random models. The wavefields are overlapped on the velocity model, with (a) model No.1, and (b) and (c) different views for model No.2 (refer to text). Shown in Figure 2 is the slowness analysis result. On the top of the figure are 3-component synthetic seismograms from the center of the sampling window. The waveforms have been band pass filtered between 20 and 60 Hz. The large-amplitude signal in the \( z \)-component composed of the incident wave and early scatterings, and the subsequent vibrations in all three components are scattered waves. The rms wave amplitude is projected on the slowness sphere, which is further separated into upper and lower hemispheres with their map views shown as circles. The center of each circle is for \( \theta = 0^\circ \) (down-going) and \( \theta = 180^\circ \) (up-going); the outer circle and dashed inner circle are for \( \theta = 90^\circ \) and \( \theta = 45^\circ \). Around the circle is the
azimuth angle, with the horizontal and vertical directions are for x- and y-slowness. The maximum value is labeled below each circle. Each column is obtained from a 0.05 s time window and aligned with the seismograms. The 4 rows from top to bottom are for up-going P, down-going P, up-going S and down-going S waves, respectively. For this specific geometry, down-going waves are forward scattered waves and up-going waves are backscattered waves.

For the P-wave, the early forward scattering (P down) is along the z-direction. However, with the time elapse, the scattering becomes more and more wide angled. The reason is that the small-angle forward scattering tends to have the same direction as the primary wave, thus following the primary wave front closely, while the wide angle scatterings are left behind and more abundant in the later coda. This can also be observed from the snapshot in Figure 1a. For the backscattered waves (P up), the early scattered waves are more narrow angled but the later coda are more isotropic. Similar result can be observed from the scattered S-wave. At early stage the forward scattered waves (S down) are mostly small angled and gradually change to more wide angled. The backscattered waves (S up) are more isotropic. Due to the symmetric reason, the P-to-S scatterings at both $\theta = 0^\circ$ and $\theta = 180^\circ$ are very weak. In this model, $k_x = k_y \approx 4$. In the early stage, the forward scattering is stronger than the back scattering.

Shown in Figure 3 is the slowness analysis similar to that shown in Figure 2, except the velocity model No.2 in table 1 is used. This velocity model has different correlation lengths along x- and y-directions (C2 in table 2). In other words, the model looks more like composed of small scale heterogeneities along the y-direction ($k_y \approx 2$) and composed of large scale heterogeneities along the x-direction ($k_x \approx 17$). As a result, the interactions between the plane P-wave and the heterogeneities generate abundant wide angle scatterings along the y-direction and narrow angle scatterings along the x-direction. This can be seen from the waveform snapshots shown in Figure 1b (along y-direction) and 1c (along x-direction).

The slowness analysis can quantify the characteristics of the scattered waves. In Figure 3, the P-to-P forward scattering has wide scattering angles at azimuth close to y-direction but very narrow scattering angles in x-direction. Similarly, for P-to-S forward scattering (S down), the wide angle energy are along y-direction. Particularly, the wave energy focuses on the direction $\theta = \cos^{-1}(v_s/v_p) \approx 54^\circ$. This results from that the high speed P-wave continually generates low speed S scattering which has a scattering angle of about 54 degrees. The backscattered waves do not follow this mechanism thus without having a dominant direction. At later times, for both P and S, as well as for forward and backward scatterings, the direction focusing is relatively smeared, perhaps due to multiple scatterings. The above results show that the scattered waves carry important information of the heterogeneities including their different correlation lengths along different directions.
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Figure 4. Wavefield snapshots and slowness analyses for a spherical wave incident on different random layers, with (a) a 3-layer constant velocity model, (b) and (c) models with random layers, (d) model with crack-like inclusions.

Scattering of a Spherical Wave in Random Layers

Figure 4 illustrates the wavefield snapshots of a spherical incident P-wave overlapped on different velocity models, along with the slowness analyses for reflected P and S waves and their coda waves. The horizontal distance is along y-direction and the depth is along z-direction. The wavefield is sampled near $x = 0$ km, $y = 0.5$ km, $z = 0.2$ km, and waveforms are band pass filtered between 20 and 60 Hz before conduct the slowness analysis.

Shown in 4a is the result for model No.3 in table 1. This is a constant layered model. All incident and reflected waves have been labeled and the result is for reference. The three slowness hemispheres on the right hand side give the energy distributions for primary down-going P and selected reflected P and S waves (indicated by brown lines), all having spot-like distributions. Shown in 4b is the result for model No.4 in table 1. In this model, the random layer has a short correlation length along the y-direction and long correlation length along x- and z-directions. The primary waves have been subtracted before conduct the slowness analysis and the results are for pure scattered waves. Shown in the three slowness hemispheres are energy distributions for selected up-going coda waves following reflected P and S waves. Because the heterogeneities have abundant short-wavelength components along the y-direction, both scattered P and S waves have wide-angle scatterings azimuthally along y-direction. Some waves are scattered backward towards the source direction.

Shown in 4c is the result for model No.5 in Table 1. The random middle layer has a short correlation length along the x-direction. Thus, both scattered P and S waves have wide-angle scatterings azimuthally along x-direction. Figure 4d is for model No.6 in table 1. We randomly insert crack-like thin inclusions in the middle layer. These inclusions are aligned along x-direction and occupy about 10% of the entire volume of the layer. Within these inclusions, the P and S wave velocities are 5% lower than the surrounded media. Thus the model parameters change faster along y-direction. In the scattered P and S waves, we observe wide-angle scatterings azimuthally along y-direction but not as strong as those shown in 4b. In all these results, the back scattered S waves are relatively weak.

Conclusion

We propose a slowness analysis method for investigating wave propagation and scattering in complex velocity models. This method can be applied for analyzing seismic scattering problems in random models including velocity heterogeneities or cracks. With this method, we investigate elastic wave scattering in 3D random velocity models with different correlation lengths along different directions. The results demonstrate that scattered waves carry important information of the random structure. By analyzing the slowness and frequency spectra of scattered waves, it is possible to obtain statistical properties of subsurface structures such as the dominant direction and density of cracks in the reservoirs.
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References


