G048

Resolution Analysis of Seismic Imaging

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SUMMARY

We define and formulate the resolution of an imaging system based on the inverse theory and local angle domain decomposition of Green's functions. The resolution defined in this way includes both the effects of acquisition system and imaging (migration) process. The theory and method are based on wave theory and no asymptotic approximation is made in the calculation. It represents and quantifies the actual resolution we observed in the migrated images. The resolution taking into account only the influence of the acquisition system (frequency band and spatial aperture) and neglecting the factors such as the errors in backpropagation (migration) can be considered as the resolution limit of the system, the best resolution an acquisition system can get. Theoretical analysis and numerical examples are given to show the importance of propagator accuracy in the evaluation of resolution in complex media.

Introduction

Spatial resolution has been studied by Beylkin et al. (1986) based on generalized Radon transform and the mapping of domain integration (frequency band and spatial aperture) into a spectral coverage in spatial frequency domain (wavenumber domain). The mapping is done by simple ray-tracing which does not take into account of the frequency dependence and other wave phenomena. Since then, the topic has been investigated by many authors from different points of view (Chen and Schuster, 1999, 2001; Gelius et al., 2002; Gibson and Tzimeas, 2002). While these analyses are useful tools for the resolution problem, most analyses are formulated to calculate the influences of acquisition system to resolution, and did not look at other factors in the imaging/migration process, such as the accuracy of the propagators, errors in velocity model, etc. Some authors called such a resolution as the resolution limit, the best resolution of an acquisition system can get. In this research, we formulate the *resolution of imaging system*, which includes the acquisition system and imaging (migration) process. The analysis is based on the inverse theory and the local angle domain decomposition of Green's functions. This resolution of imaging system represents and quantifies the actual resolution we observed in the migrated images and provides a theoretical basis for the estimation of different factors influencing the resolution and image quality in complex media. The other important feature of our formulation is that the resolution problem is treated totally based on wave theory and no high-frequency asymptotic approximation is made.

Formulation of spatial resolution based on inverse theory

Spatial resolution can be studied under the general frame of inversion theory. Resolution operator or the discrete form, resolution matrix has been defined to quantify the resolution of parameter inversion by a particular inversion scheme (Aki and Richards, 1980; Tarantola, 1987). The resolution matrix has dependence on both the acquisition system and the inversion scheme. Assume the acquisition process can be modeled by

$$Fm = d \qquad (1)$$

where F is the forward modeling operator and d is the data. If we adopt a specific inversion operator B to apply to the data we can get a set of model parameters m_I, which is different from the real m,

$$m_I = Bd \qquad (2)$$

Substituting (2) into (1) we get the relation between the inverted model and the real model

$$m_I = BFm \quad , \qquad (3)$$

and the ***resolution matrix (or operator)*** is defined as

$$R = BF \qquad (4)$$

For an exact inversion of a well-posed problem, we should have

$$R = I \qquad (5)$$

where I is the identity matrix. For most the cases, the resolution matrix is not an identity matrix and the spreading of the matrix elements along the diagonal give some quantitative measure of the parameter resolution of the inversion. For the sake of simplicity, here we will use the noiseless formulation. For the stochastic approach (Tarantola, 1987) a similar derivation can be obtained.

The ***imaging problem*** can be formulated as a specific inverse problem. We assume the media can be decomposed into a smooth variation of velocity and a sharp jumps of impedance (discontinuities). The velocity distribution of the background media can be derived with different approaches and is assumed known in the "imaging problem". The unknowns in the imaging problem are the parameter strengths and their locations (distribution). For the problem of ***spatial resolution,*** we assume the scattering coefficients everywhere are ***unity*** and therefore are known. Then the resolution matrix is totally defined by the spatial resolution.

In the following, we use the migration operator (simply backpropagation integral) as the inversion operator (here the imaging operator) to show the effects of different factors to the final resolution. We can write the space-domain formulations for modeling (acquisition process) and imaging (migration) as:

$$u_s(\omega, x_g; x_s) = -k^2 \int_V d\underline{x}' G_M(\omega, \underline{x}'; x_s) s(\underline{x}) G_M(\omega, x_g; \underline{x}') \quad \begin{cases} x_s \in A_s \\ x_g \in A_g(x_s) \end{cases} \tag{6}$$

$$I(\underline{x}) = \int d\omega \int_{A_s} dx_s W_s(\omega, x_s) G_I^*(\omega, \underline{x}; x_s) 2\int_{A_g} dx_g W_g(\omega, x_g) \frac{\partial G_I^*(\omega, \underline{x}; x_g)}{\partial z} u_s(\omega, x_g, x_s) \tag{7}$$

where u_s is the scattered wave field observed on the surface at x_g excited by a source on the surface at x_s. G_M is the Green's function of the modeling (acquisition) process, which may include all the factors (geometric spreading, intrinsic and scattering attenuation, boundary scattering, etc.) for the real heterogeneous media; G_I is the Green's function of inversion process, which could be quite different from G_M (see Figure 1). The integrations on the receiver aperture and source aperture are for the prestack migration process. *W*'s are the weighting functions for integrations. The integration on the receiver aperture A_g is the Rayleigh integral which simulates the backpropagation process. The integration on frequency is from the imaging condition which states that the downward extrapolated source field and the scattered field will meet at zero time at the scattering points. In (7) we used the cross-correlation imaging condition. Other imaging conditions for correcting the imaging amplitude can be also applied, but the general conclusion of resolution analysis will not be influenced.

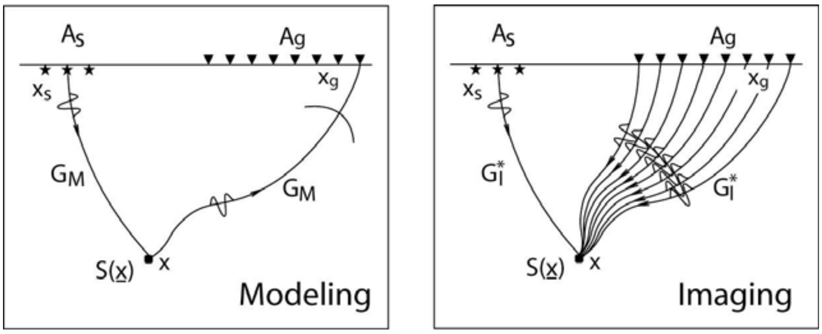

Figure 1. Schematic diagrams showing the modeling (data acquisition) and imaging (inversion) processes.

Write modeling and imaging process (6) and (7) into operator form, resulting in

$$\mathbf{U}(\omega, x_g, x_s) = \mathbf{F}(\omega, x_g, x_s \mid \underline{x}_0)\mathbf{S}(\underline{x}_0) \tag{8}$$

$$\mathbf{I}(\underline{x}) = \mathbf{B}(\underline{x} \mid \omega, x_s, x_g)\mathbf{U}(\omega, x_s, x_g) \tag{9}$$

where **F** is the acquisition (modeling) operator and **B** is the imaging operator which invert the data **U** into the image **I**. Therefore the resolution operator is obtained as

$$\mathbf{R}(\underline{x}, \underline{x}_0) = \mathbf{B}(\underline{x} \mid \omega, x_s, x_g)\mathbf{F}(\omega, x_g, x_s \mid \underline{x}_0) \tag{10}$$

The kernels for the resolution operator can be obtained as

$$\begin{aligned} R(\underline{x}, \underline{x}_0) = &-\int d\omega k^2 \int_{A_s} dx_s W_s(\omega, \underline{x}, x_s) G_I^*(\omega, \underline{x}; x_s) G_M(\omega, \underline{x}_0; x_s) \\ &\times 2\int_{A_g} dx_g W_g(\omega, \underline{x}, x_g) \frac{\partial G_I^*(\omega, \underline{x}; x_g)}{\partial z} G_M(\omega, x_g; \underline{x}_0) \end{aligned} \tag{11}$$

For zero-offset (zero receiver aperture) acquisition, or monostatic measurement in radar terminology, the above defined resolution matrix is degenerated to

$$\begin{aligned} R(\underline{x}, \underline{x}_0) = &-2\int d\omega k^2 \int_{A_s} dx_s W_s(\omega, \underline{x}, x_s) G_I^*(\omega, \underline{x}; x_s) \times \\ &\times G_M(\omega, \underline{x}_0; x_s) \frac{\partial G_I^*(\omega, \underline{x}; x_s)}{\partial z} G_M(\omega, x_s; \underline{x}_0) \end{aligned} \tag{12}$$

Note that the amplitude function in (12) is different from the exploding-reflector modeling. In the latter case, (12) is further simplified to

$$R(\underline{x},\underline{x}_0) = -2\int d\omega k^2 \int_{A_g} dx_g W_g(\omega,\underline{x},x_g) \frac{\partial G_I^*(\omega,\underline{x};x_g)}{\partial z} G_M(\omega,x_g;\underline{x}_0) \tag{13}$$

The above derived resolution matrix (operator) is in fact the ***spatial resolution matrix (resolution operator)*** for the whole acquisition and migration process (the ***imaging system***). The matrix element $R(\underline{x},\underline{x}_0)$ is called the ***resolving kernel*** of the resolution operator, which is the ***point spreading function*** (PSF)of the imaging system. In this way the point spreading function (impulse response) is defined under the guidance of general inversion theory.

If we set the scatterer's distribution $s(x') = \delta(x' - x_0)$ in equation (6) and substitute it into equation (7), we see the equivalence of the imaging process to the calculation of resolution matrix. This gives us the numerical procedure of calculating the resolution matrix or PSF for any imaging system.

If we assume that an exact Green's function G_M is used as the inverse propagator G_I, then the effect of imperfect propagators can be eliminated, and the resolution is totally determined by the data aperture. The resolution derived this way is a theoretical limit of the acquisition system similar to the resolution studied by previous investigations (Beylkin et al., 1986; Gelius et al., 2002; Gibson and Tzimeas, 2002).

Angular-spectral representation of point spreading function (PSF)

Angular-spectral representation of resolution of PSF is more intuitive. We can see directly the information coverage in the local angle domain. We perform local 3D Fourier transform on $R(\underline{x},\underline{x}_0)$ with respect to x with coordinate center at $\underline{x}_0$:

$$R(\underline{K},\underline{x}_0) = \int_v d\underline{x} e^{-i\underline{K}\cdot(\underline{x}-\underline{x}_0)} W(\underline{x},\underline{x}_0) R(\underline{x},\underline{x}_0) \tag{14}$$

where $\underline{K}$ is the 3D wavenumber vector and $W(\underline{x},\underline{x}_0)$ is a 3D window function to localize $R(\underline{x},\underline{x}_0)$. For simplicity, we set the weighting functions W_s and W_g in migration as unity. Substituting (11) into (14) we obtain

$$R(\underline{K},\underline{x}_0) = -\int_{A_f} d\omega k^2 \int d\underline{K}_s \int_{A_s} dx_s G_I^*(\omega,\underline{x}_0,\underline{K}_s;x_s) G_M(\omega,\underline{x}_0;x_s) \\ \times 2\int_{A_g} dx_g \frac{\partial G_I^*(\omega,\underline{x}_0,\underline{K}-\underline{K}_s;x_g)}{\partial z} G_M(\omega,x_g;\underline{x}_0) \tag{15}$$

where $G(\omega,\underline{x}_0,\underline{K};x')$ is the beamlet decomposition (or local plane-wave decomposition) of the Green's function (Wu and Chen, 2002) around $\underline{x}_0$, and x' is the source or receiver position.

The influence of propagator accuracy to the resolution

To show the difference between the real resolution for an imaging system and the theoretical resolution limit, we give an example of imaging using exploding reflector data generated by a full-wave finite difference algorithm, in a random medium (Figure 2). We use both the ray-Kirchhoff migration method and the dual-domain one-way wave propagator method as the migration operator. Since the medium has random heterogeneities with scale comparable to the wavelength of the central frequency, the ray approximation of the Green's function can produce large errors for long range propagation. This error has severe consequences on the resolution matrix of the imaging system. As shown in Figure 2, both the multi-arrival and first-arrival ray-Kirchhoff migrations give distorted point spreading functions, and the horizontal resolutions are also degenerated (bottom panel of Figure 2).

Conclusions

The resolution of an imaging system defined in this paper based on the inverse theory includes both the effects of acquisition system and migration process. It represents and quantifies the actual resolution we observed in the migrated images. If we consider only the influence of the acquisition system, the resolution obtained is the resolution limit of the system. Theoretical analysis and numerical examples have shown the importance of propagator accuracy to the evaluation of resolution in complex media.

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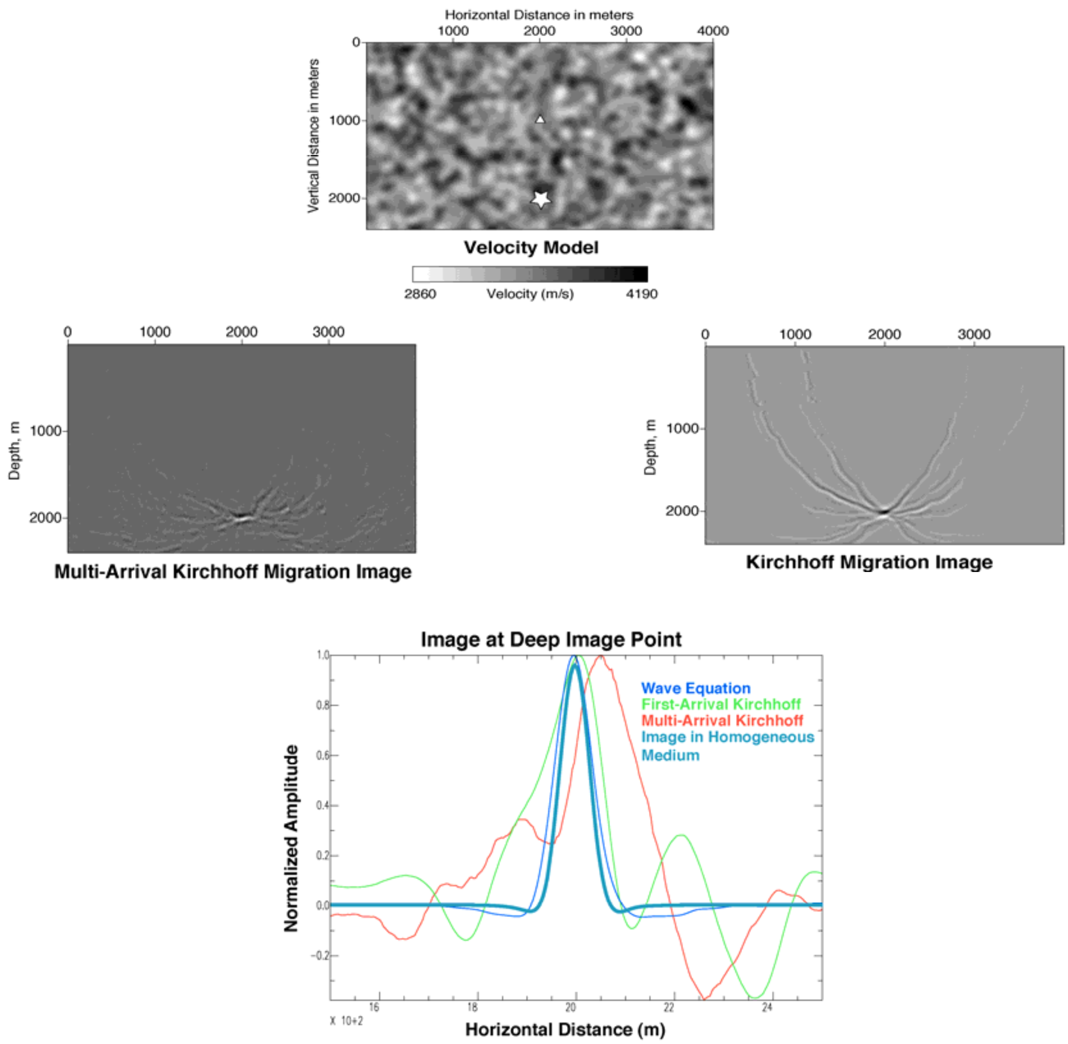

Figure 2: *Top*: velocity model of point scatterers in a random medium; *Middle*: point spreading functions using the multi-arrival and first-arrival ray-Kirchhoff migration operators; *Bottom*: horizontal resolution curves of the same acquisition system but different migration operators