Backscattered wave calculation using the De Wolf approximation and a phase-screen propagator

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SUMMARY

A new method of calculating synthetic seismograms of primary reflections or backscattering for surface reflection surveying is introduced using the De Wolf MFSB approximation and phase-screen one-way propagation. Two versions of algorithm are presented, both of which use the dual domain technique for computational efficiency. One is the direct use of MFSB approximation, in which the step length of calculating the backscattered waves uses the grid spacing, but the step length of forward propagation uses the screen interval which is much greater than the grid spacing. The other version is the screen approximation for both the forward and backward scattered waves and therefore both step lengths can adopt the screen interval, resulting in great computational efficiency. The screen-approximation involves small-angle approximation and may have some effects on large-angle backscattered waves. Numerical examples using both versions of algorithm show good results. Good agreements with finite difference calculation on the second example indicate that even though the screen approximation involves the small-angle approximation, satisfactory results can be obtained for many practical applications with a great computational efficiency.

INTRODUCTION

Fast modeling methods and algorithms in complex heterogeneous media, especially for 3-D media, are crucial to the development of imaging and inversion methods, interpretations and applications of seismic methods for complex structures. Finite difference and finite element methods, which in principle can model wave propagation in arbitrarily heterogeneous media, are time consuming, even formidable in the case of large 3-D elastic wave problems. In this study we develop a new method based on multiple-forescattering single-backscattering (MFSB) approximation, i.e. the De Wolf approximation for calculating the backscattered field in the configuration of surface reflection surveying. A dual domain formulation is derived for fast implementation of the method. When the scales of heterogeneities are greater than the dominant wavelength, the theory can be further approximated by the thin-slab scattering approximation and the screen scattering approximation. It is shown that the screen approximation can substantially reduce the computation time. Finally two numerical examples are given to demonstrate the validity of the method.

MULTIPLE FORESCATTERING SINGLE BACKSCATTERING (MFSB) APPROXIMATION - DE WOLF APPROXIMATION

When discontinuities inside a medium are not very sharp or parameter perturbations of heterogeneities are not very strong, reverberations between heterogeneities or resonance scattering usually can be neglected. However, accumulated effect of forward scattering, refer as forescattering hereafter, can not be always neglected. In fact for large volume heterogeneous media or long propagation distance multiple forescattering is very important for both forward modeling and inverse problems. In such cases, the Born approximation is not valid and the De Wolf approximation can be applied.

Lipmann-Schwinger equation:

Start from the scalar wave equation,

$$(\nabla^2 + \frac{\omega^2}{c^2(\mathbf{x})})p(\mathbf{x}) = 0 \qquad (1)$$

Define

$$F(\mathbf{x}) = \frac{c^2_0}{c^2(\mathbf{x})} - 1 = \frac{s^2 - s_0^2}{s_0^2} \qquad (2)$$

resulting in

$$(\nabla^2 + k^2)p(\mathbf{x}) = -k^2F(\mathbf{x})p(\mathbf{x}) \qquad (3)$$

Let

$$p(\mathbf{x}) = p^0(\mathbf{x}) + P(\mathbf{x}) \qquad (4)$$

Then

$$p(\mathbf{x}) = p^0(\mathbf{x}) + k^2\int_v d^3\mathbf{x}' g(\mathbf{x};\mathbf{x}')F(\mathbf{x}')p(\mathbf{x}') \qquad (5)$$

where $g(\mathbf{x};\mathbf{x}')$ is the Green's function in the background medium. This is the Lipmann-Schwinger equation.

Renormalization of scattering series and the De Wolf approximation

The Lipmann-Schwinger equation can have a formal solution of multiple scattering Born- series. The widely used Born approximation is the leading term of the series. The Born approximation is only valid when the heterogeneities are weak and the propagation distance is short. After renormalization of the multiple scattering series, De Wolf (1971, 1985) derived a MFSB approximation:

$$p(x) = p^f(x) + k^2\int_V d^3\mathbf{x}' g^f(\mathbf{x};\mathbf{x}')F(\mathbf{x}')p^f(x') \qquad (6)$$

where pf and g^f are the renormalized, multiple forescattered field and Green's function respectively. In this paper p^f and g^f will be calculated using phase-screen propagator. Note that the De Wolf approximation replaces both the total exact field and the free space Green's function with the renormalized, multiple forescattering approximations, and is superior to the approximations made by Wu and Huang (1992) in which only the total exact field is approximated but the free space Green's function is left intact. This MFSB approximation is valid whenever the backscattered field is much smaller than the forescattered field.

Implementation of MFSB synthetics using phase-screen propagator

Backscattered field can be calculated using (6) as

$$P^b(z,\mathbf{x}_T) = k^2\int d^3\mathbf{x}' g^f(z,\mathbf{x}_T;\mathbf{x}')F(\mathbf{x}')p^f(x') \qquad (7)$$

where $\mathbf{x}_T$ is the horizontal position in the receiver surface at depth z. Equation (7) can be numerically implemented using a phase-screen propagator (Thomson and Chapman, 1983; Stoffa et al., 1990; Wu and Huang, 1992; Wu, 1994). To speed up calculation of backscattered field, local Born approximation can be used within a thin-slab. This means that the forescattered field p^f can be kept unperturbed and g^f can be replaced by a constant medium Green's function within the slab. Fourier-transforming equation (7) with respect to $\mathbf{x}_T$, we have

$$P^b(z,\mathbf{K}_T) = k^2\int dz'\iint d\mathbf{x}'_T\, g^0(z,\mathbf{K}_T;\mathbf{x}')F(\mathbf{x}')p^f(\mathbf{x}') \quad (8)$$

where

$$g^0(z,\mathbf{K}_T;z',\mathbf{x}'_T) = \frac{i}{2\gamma}e^{i\gamma|z-z'|}e^{-i\mathbf{K}_T\cdot\mathbf{x}'_T} \quad (9)$$

with

$$\gamma = \sqrt{k^2 - K_t^2} \quad (10)$$

Therefore (8) becomes

$$P^b(z,\mathbf{K}_T) = \frac{i}{2\gamma}k^2\int_z^{z_1} dz' e^{i\gamma|z-z'|}$$

$$\times\iint d^2\mathbf{x}'_T\, e^{-i\mathbf{K}_T\cdot\mathbf{x}'_T}[F(z',\mathbf{x}'_T)p^f(z',\mathbf{x}'_T)] \quad (11)$$

where z_1 is the bottom of the slab. Note that the two dimensional inner integral is a 2-D Fourier transform. Therefore, the dual domain technique can be used for backscattering calculation. The procedure of calculating backscattered field can be summarized as follows.

1. Fourier transform the incident field at the entrance of each thin-slab into wavenumber domain.

2. Free propagation in wavenumber domain: calculate the primary field within the slab.

3. At each depth within the slab, inverse FT the primary field into space domain, and then interact with the medium: calculate the backscattered field.

4. FT the backscattered field into wavenumber domain and multiply it with a weighting factor $\frac{i}{2\gamma}$, and then free propagates to the entrance of the slab. The total backscattered field by the thin-slab can be propagated to the surface using phase-screen propagator.

5. Calculate the forescattered field at the slab exit and add it to the primary field to form the total field as the incident field at the entrance of the next thin-slab.

6. Continue the procedure iteratively.

6. Sum up all the backscattered waves to form the total scattered field at the surface.

However, in order to accelerate further the computation, approximations of thin-slab interaction or screen interaction can be applied.

THIN-SLAB SCATTERING APPROXIMATION

Under the MFSB approximation we can update the total filed with a marching algorithm in the forward direction. We can slice the whole medium into thin-slabs perpendicular to the propagation direction. Weak scattering condition holds for each thin-slab and therefore, the Born approximation can be applied to each thin-slab. For each step forward, the forward scattered field by a thin-slab between z' and z_1 (see Fig. 1) is added to the incident field and then the updated field becomes the incident field for the next thin-slab. The scattered field by a thin-slab can be calculated as

$$P(z^*,\mathbf{K}_T) = i\frac{k^2}{8\pi^2\gamma}e^{ik_z z^*}$$

$$\times\iint d\mathbf{K}'_T\tilde{F}(k_z-\gamma,\mathbf{K}_T-\mathbf{K}'_T)p^0(z',\mathbf{K}'_T)e^{-i\gamma z'} \quad (12)$$

where z^* is the depth of the receiving plane, z' is the top of the thin-slab, k_z is the vertical wavenumber with $k_z = \gamma$ for forescattering and $k_z = -\gamma$ for backscattering, and $\tilde{F}(\mathbf{K}_T, K_z)$ is the 3-D spectrum of the thin-slab. When the receiving level is at the bottom of the thin-slab (forescattering), $z^* = z_1$; while for $z^* = z'$ is for backscattered field at the top of the thin-slab. The total transmitted filed at the slab bottom can be calculated as the sum of the primary filed and the forescattered field.

Note that the wave-slab interaction in wavenumber domain is not a convolution and hence the operation in space domain is not local. Therefore, the wave-slab interaction involves matrix multiplication and is the major computation burden of the method.

SCREEN SCATTERING APPROXIMATION

For some special applications, the medium varies slow in the vertical direction and the synthetics only involve small-angle backscattering. In this case the screen approximation can be applied to circumvent the matrix multiplication and greatly speed up the computation.

Under small-angle scattering approximation, we can compress the thin-slab into an equivalent screen and therefore change the 3-D spectrum into a 2-D spectrum, making equation (12) into a convolution integral:

$$P(z^*,\mathbf{K}_T) \approx i\frac{k^2}{8\pi^2\gamma}e^{ik_z z^*}\iint d\mathbf{K}'_T\tilde{S}(\mathbf{K}_T-\mathbf{K}'_T)p^0(\mathbf{K}'_T) \quad (13)$$

where we put $z' = 0$ and $\tilde{S}(\mathbf{K}_T)$ is the 2-D screen spectrum, i.e. the Fourier transform of the screen function $S(\mathbf{x}_T)$ with

$$S(\mathbf{x}_T) = S^f(\mathbf{x}_T) = \int_{z'}^{z_1} dz F(\mathbf{x}_T, z) \quad (14)$$

for forescattering, and

$$S(\mathbf{x}_T) = S^b(\mathbf{x}_T) = \int_{z'}^{z_1} dz e^{i2kz}F(\mathbf{x}_T, z) \quad (15)$$

for backscattering. The above equation is a convolution integral in wavenumber domain and the corresponding operation in space domain is a local. The dual domain technique can be used to implement the computation:

$$P(z^*,\mathbf{K}_T) \approx i\frac{k^2}{2\gamma}e^{ik_z z^*}\iint d\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T}S(\mathbf{x}_T)p^0(\mathbf{x}_T) \quad (16)$$

The total transmitted field is

$$p(z_1,\mathbf{K}_T) = p^0(z_1,\mathbf{K}_T) + P^f(z_1,\mathbf{K}_T)$$

$$= e^{ik_z z_1}\iint d\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T}p^0(\mathbf{x}_T)\left\{1 + i\frac{k}{2}S(\mathbf{x}_T)\right\}$$

$$= e^{ik_z z_1}\iint d\mathbf{x}_T e^{-i\mathbf{K}_T\cdot\mathbf{x}_T}p^0(\mathbf{x}_T)\exp[ikS^f(\mathbf{x}_T)/2] \quad (17)$$

This is the dual domain implementation of phase-screen propagation.

Procedure of MFSB using the screen approximation:

1. Fourier transform the incident field into wavenumber domain and free propagate to the screen.

2. Inverse Fourier transform the incident field into space domain. Interact with the backscattering screen to get the backscattered field and interact with the phase-screen to get the transmitted field.

3. Propagate the backscattered field back to the surface

using the phase-screen propagator.

4. Fourier transform the transmitted field into wavenumber domain and free propagate to the next screen.

5. Repeat the propagation & interaction screen-by-screen to the bottom of the model space.

NUMERICAL EXPERIMENTS

For the purpose of testing the theory and method, in this paper we present only the 2-D examples. First we test the use of the De Wolf approximation for synthetic seismograms using phase-screen as propagator. In Fig.1 shown is the model of a thick high velocity layer with $v = 2200m/s$ and thickness of $1250m$, located in a homogeneous background with $v_0 = 2000m/s$. A point source is situated on the surface and receivers are spread to both sides the source to the extent of 1280m with an interval of *5m*. A Ricker wavelet with dominant frequency 1OHz is used as the source time function. Fig. 2 shows the recorded seismograms using the De Wolf MFSB approximation. The grid space is 2048×360 with a spacing of *5m*. In this approximation, the one-way propagation using phase-screen method has a step length of 10 grid points (the screen interval), but the backscattering calculation has the original step length of 1 grid spacing. From Fig. 2 we can see the reflected signals from both interfaces of the high velocity layer. The results agree well with the theoretical predictions.

The second example is for the test of screen approximation for synthetic seismograms. In this approximation, the backscattering calculation uses the same step length as the one-way propagation, the screen interval and therefore a much higher computation efficiency can be gained compared to the previous algorithm. Fig. 3 shows the model space. A high velocity cylinder with v = 3850 m/s and a radius of 150 m is situated at the middle of the model space. The center of the cylinder is located at the depth of 800 m. The background velocity is 3500 m/s. So the velocity perturbation of the cylinder is 10%. A plane wave is incident vertically down to the model space. The source time function is a Gaussian first derivative with the dominant frequency of 3OHz. The time sampling interval is 0.002 sec. and the record length is 512 time steps. There are total 26 receivers on the surface with an interval of 100 m. The screen interval is 10 m inside the cylinder; while the horizontal grid spacing is 3.125 m. Fig. 4 shows the comparison of synthetic seismograms calculated by the screen approximation and by the finite difference method based on the full wave equation. In the figure only the transversal components are shown. We see good agreement between the results of finite difference calculation and this method.

CONCLUSIONS

A new method of calculating synthetic seismograms of primary reflections or backscattering for surface reflection surveying is introduced using the De Wolf MFSB approximation and phase-screen one-way propagation. Two versions of algorithm are presented, both of which use the dual domain technique for computational efficiency. One is the direct use of MFSB approximation, in which the step length of calculating the backscattered waves uses the grid spacing, but the step length of forward propagation uses the screen interval which is much greater than the grid spacing. The other version is the screen approximation for both the forward and backward scattered waves and therefore both step lengths can adopt the screen interval, resulting in great computational efficiency. The screen-approximation involves small-angle approximation and can be expected to have some deterioration on large-angle scattered waves. Numerical examples using both versions of algorithm show good results. The accuracy and limitation of the method, especially the version using screen approximation for backscattering calculation, will be detailed in future publications.

ACKNOWLEDGEMENTS

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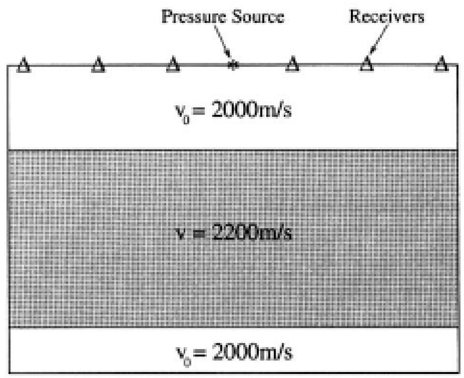

Figure 1: The schematic illustration of a 2-D layered model.

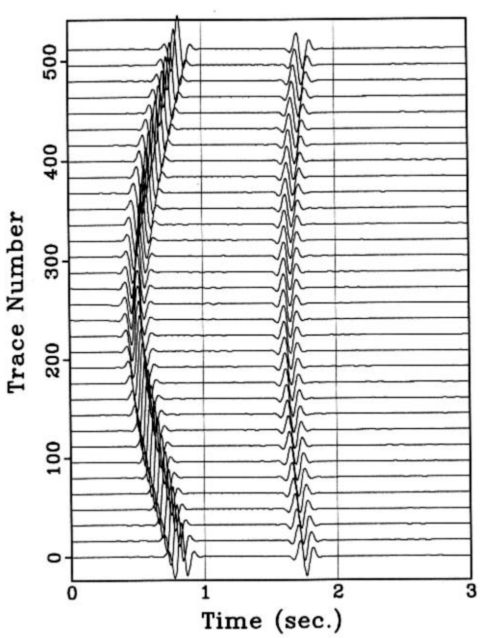

Figure 2: The seismograms recorded during the calculation of backscattered waves in the layered model using the De Wolf approximation and a phase-screen propagator.

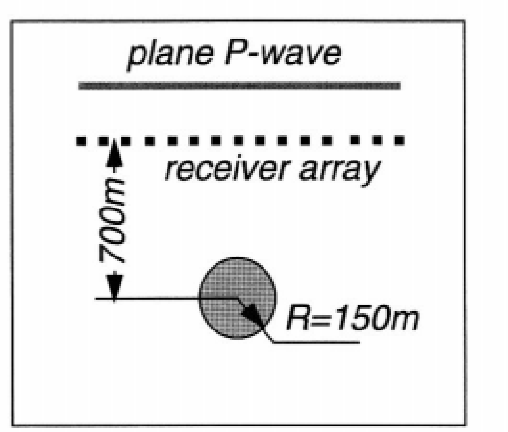

Figure 3. A two dimensional model used to compare the results from screen-approximation method and finite-difference method. The model is composed of a solid cylinder buried in a homogeneous background. The velocity for the background medium is 3500m/s. and the cylinder has a velocity of 3850m/s. A plane wave illuminates the cylinder. The receiver array is indicated by solid squares.

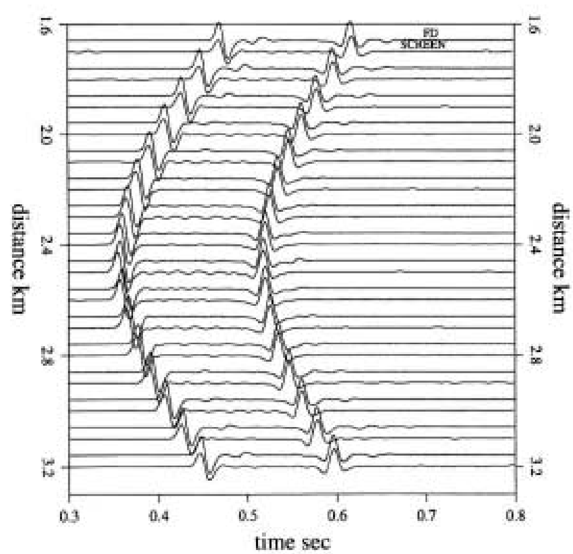

Figure 4. Comparison between results from the two methods. The synthetics marked with SCREEN is by screen-approximation method and the ones marked with FD is by a finite-difference method. Shown in the figure are two arrivals which are reflections from both the upper and lower boundaries of the cylinder. The results show general consistency in bath amplitude and arrival time.