Abstract

Employing Gabor-Daubechies frame bases, beamlet domain decomposition and propagation provide localizations in both space and direction of the wave fields. First directional illumination is discussed and then the acquisition-aperture efficacy matrix and dip-response function are derived. As numerical examples, we calculate the directional illumination maps and acquisition efficacy maps for a lens model and the SEG-EAGE salt model. We investigate the influences of acquisition geometry and overlaying structures on the prestack migration image quality using the acquisition dip-response.

Introduction

Illumination analysis in the target area is a powerful tool to study the influences of acquisition aperture and overlaying structures. In the past, most techniques predicting illumination intensity distributions under certain acquisition geometries are based on ray tracing modeling (Berkhout, 1997; Muerdter et al., 2001a; 2001b; 2001c; Schneider, 1999; Bear et al., 2000). Some authors used FD (finite-difference) method to model the wave phenomena. Although ray tracing is inexpensive, the resulted illumination map may bear large errors in complex areas due to the high frequency approximation involved and the singularity problem of ray theory in complex regions. This may severely limit its accuracy (Hoffmann, 2001). On the other hand, FD modeling is limited by its high computation cost. As a spatial domain method, it can provide only the total illumination but the directional information of the wave propagation is lost. The ray method seemingly can provide angle-dependent illumination at any point. However, this “over-precise” directional illumination map does not reflect the real behavior of the wave field since it violates the Heisenberg uncertainty principle. In order to have reliable directional illumination, we need to have a wave-theory based method which possesses both space and direction localizations satisfying the uncertainty principle.

Recently, a wave theory-based beamlet wave propagation and migration method has been developed (Wu et al., 2000; Wu and Chen, 2001). Instead of global FT (Fourier transform), Wu et al. used efficient decomposition schemes with Gabor-Daubechies frame (G-D frame) or local Cosine bases for the wave field and derived the corresponding propagators in beamlet domain. In this study, illumination variations and reflector dip-response for the given acquisition geometry and overlaying structures are evaluated through beamlet wave field decomposition and propagation using G-D frame atoms. We also tested using the GSP (generalized screen propagator) to propagate the waveform and using beamlet decomposition at each step for illumination analysis. A lens model with high velocity contrasts and the 2D SEG-EAGE salt model are used as examples to demonstrate the feasibilities of the approach. The influences of these factors on the final prestack image quality are also investigated for the 2D SEG-EAGE salt model. An example of 3D illumination analysis is given for the 3D SEG-EAGE salt model.

G-D beamlet decomposition and propagation

For 2D case, G-D beamlet decomposition of wave field at depth $z$ can be expressed as (Wu et al., 2000; Wu and Chen, 2001):

$$u(x,z,\omega) = \sum_m \sum_n \left( \hat{u}_m(n,\xi_m) g_{mn}(x) \right) = \sum_m \sum_n \hat{u}(\bar{\xi}_n,\bar{\xi}_m,\omega) g_{mn}(x)$$

(1)

where $\hat{u}(\bar{\xi}_n,\bar{\xi}_m,\omega)$ are beamlet coefficients, $\omega$ is the circular frequency, $g_{mn}(x)$ and $\hat{g}_{mn}(x)$ are G-D frame atoms and dual frame atoms, $\bar{\xi}_n = n\Delta_{\xi}$, $\bar{\xi}_m = m\Delta_{\xi}$, with $\Delta_{\xi} < 2\pi$ are the $n$th window location and the $m$th local wavenumber position, respectively. We see that each beamlet (in this case a G-D frame atom) is a windowed plane wave, which has both space localization $\bar{\xi}_n$ and direction localization $\bar{\xi}_m$. Beamlets can be propagated using beamlet domain propagators (Wu et al., 2000; Wu and Chen, 2001).
\[ \tilde{u}_{x,\omega}(\tilde{x}, \tilde{\omega}, \omega) = \sum_{n} p(\tilde{x}, \tilde{\omega}; \tilde{x}_n, \tilde{\omega}_n, \omega) \tilde{u}_n(\tilde{x}_n, \tilde{\omega}_n, \omega) \]  

To form the local plane waves, we can also have a partial reconstruction (mixed domain wave field: local phase space):

\[ u(x, z, \tilde{\omega}, \omega) = e^{i\tilde{\omega} / \lambda} \sum_{j} g(x - \tilde{x}_j) \tilde{u}_j(\tilde{x}_j, \tilde{\omega}, \omega) \]  

### Directional illumination map

For the illumination problem, only the intensity or amplitude is concerned. From the point of view of computation efficiency, we consider the main energy of the source field around the dominant frequency \( \omega_0 \). The directional illumination (DI) map of a single source is defined as

\[ D_s(x, z, \tilde{\omega}; x_0, \omega_0) = |G(x, z, \tilde{\omega}; x_0, \omega_0)| \]  

where \( x_0 \) is the source position on the surface, and \( a \) is the window width. By summing up the DI-maps of individual sources, the DI-map of group sources or all the sources of an acquisition system can be calculated as well. Taking point sources as an example:

\[ D_s(x, z, \tilde{\omega}) = \sum_{S=1}^{N_s} D_s(x, z, \tilde{\omega}; x_0) \]  

Here we consider a model consisting a high velocity lens embedded in a homogeneous background medium. The velocity inside the lens is 4000\( \text{m/s} \), and the velocity for the surrounding medium is 2000\( \text{m/s} \). An acquisition system consisting of 257 shots with 176 left-side receivers per shot is used to calculate the directional illuminations and dip-responses. A dominant frequency of 15 Hz is used.

### Local AE (Acquisition Efficacy) matrix and ADR (Acquisition Dip-Response) vector

In order to evaluate the effects of the acquisition geometry for a specific area (target area), including the aperture and propagation effects, we put unit impulses at both the source and receiver points for the whole acquisition configuration. Following the procedure of DI mapping, we neglect the detailed wave interference pattern in the illumination and system efficacy problem, and consider only the energy distribution pattern of the acquisition configuration. We define

\[ E(\tilde{x}, \tilde{\omega}, x, z, \omega) = \left[ \sum_{S} |G(x, z, \tilde{x}_S; x_0, \omega_0)| \sum_{r} |G(x, z, \tilde{x_r}; x_0, \omega_0)| \right]^{1/2} \]  

as the Local Acquisition Efficacy matrix, where \( G(x, z, \tilde{x}; x_0, \omega_0) \) and \( G(x, z, \tilde{x_r}; x_0, \omega_0) \) are the local plane waves of the impulse responses or Green’s functions. We can further reduce the acquisition efficacy matrix to a function of reflector dip by summing up all the reflected energy for a reflector

\[ E(\tilde{\omega}, x, z, \omega) = \sum_{\tilde{\omega}} E(\tilde{\omega}, x, z, \omega) \]  

where \( \tilde{\omega} \) is the angle of the norm and \( \tilde{\theta} \) is the reflection angle. Fig. 1 shows the DI and ADR maps for the lens model, in which the left column is for DI maps and the right column is for ADR maps. Shown in the top row are DI in vertical direction and ADR for horizontal reflectors. Clear difference between the illumination map and acquisition dip-response (ADR) can be seen for the case of horizontal reflector. Since the dip-response of a reflector is the sum of received reflected energy, horizontal reflectors are more sensitive to the influence of receiver aperture. However, the ADR
albums for other dip-angles are not too different from the corresponding DI maps. This indicates that perpendicularly incident illumination plays an important role for large dip-angle reflectors.

**DI and ADR analysis for the 2D SEG-EAGE salt model**

We use the 2D SEG-EAGE salt model as an example for the application of DI and ADR analysis. The acquisition system consists of 325 shots with 176 left-side receivers for each shot. For simplicity, only the dominant frequency $f_0 = 15$Hz is considered for the calculations. The results are shown in Fig. 2. On the top is the prestack depth migration image. The three subsalt steep faults are poorly imaged. Many authors suggested that this is caused by poor subsalt illumination. However, if we consider only the total illumination and total ADR, the puzzle still cannot be solved. We see that in the steep fault region, neither total illumination nor total dip-response is very weak. On the other hand, if we check the DI and ADR for different angles, the correlation between image quality and the ADR response becomes quite clear. The $0^\circ$ and $45^\circ$ ADR’s show excellent correspondence with the related reflector images. Note that the steep fault in the middle has very weak dip-response, especially for the upper half. This explains the weak image of the fault and the total absence of the upper half. The left steep fault has even weaker dip-response. However, its reflection coefficient is 2.5 times larger than the middle one, so its image is stronger than the steep fault in the middle.

![Migrated image](image1.jpg)

![Total illumination](image2.jpg)

![Horizontal ADR](image3.jpg)

![Total ADR](image4.jpg)

![45 degree ADR](image5.jpg)

Figure 2. Correlation between the image quality of prestack depth migration and the ADR. On the top is migrated image. On the bottom are DIs and ADRs. In the left the total illumination and total dip-response; in the right is the horizontal and 45 deg dep-responses. We can see the clear correlations between the $0^\circ$ and $45^\circ$ ADR’s and the image quality of the corresponding faults.

**DI analysis for the 3D SEG-EAGE salt model**

Similar calculations are also applied to 3D SEG-EAGE salt model for obtaining directional illumination. Figure 3 shows a vertical slice for the 3D velocity model and some directional illuminations. Figure 3A is the velocity model. From B to F are DIs for vertical direction, 15 degree right, 15 degree left, 45 degree right and 45 degree left. The shadow zones in the illumination map indicate these regions are hard to penetrate for seismic energy.

**Conclusions**

Directional illumination and acquisition-aperture efficacy analysis based on beamlet wave field decomposition and propagation are proposed. Beamlet decomposition provides localizations in both space and direction of the wave field, and is more flexible and accurate compared with the traditional illumination analysis methods. The DI maps and the acquisition-aperture efficacy maps are calculated for the lens model and the SEG-EAGE salt model, showing clearly the directional features of source illumination and dip responses of local reflectors to the acquisition systems used. For the salt model,
acquisition geometry influences on the image quality of prestack migration are studied through analysis of the DI-maps and the ADR-maps.

Figure 3. Directional illumination for the 3D SEG/EAGE salt model. Shown here are velocity model and the directional illumination for vertical, ±15 degree and ±45 degree.

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References