True amplitude one-way propagators implemented with localized corrections on beamlets

Mingqiu Luo, Ru-Shan Wu, and Xiao-Bi Xie
Modeling and Imaging Laboratory, Institute of Geophysics and Planetary Physics, University of California, Santa Cruz, CA, 95064, USA

Summary

The transmission coefficient and WKBJ approximation can be adopted for constructing the true amplitude propagators. For vertically inhomogeneous media, the WKBJ approximation or transmission coefficients can be applied on the plane waves at each depth to get the true amplitudes. While for the media with lateral variation, the correction coefficients vary with horizontal locations, and cannot be applied on the global plane waves. However, the beamlet propagators decompose the wavefield into localized beamlets, and the laterally varying WKBJ or transmission correction coefficients can be conducted on a localized basis at each depth. In this paper, the theory for the localized WKBJ and transmission corrections are proposed and implemented with the local cosine basis (LCB) beamlet propagator. Numerical examples of the impulse responses are calculated to demonstrate the feasibility of the local WKBJ and local transmission corrections.

Introduction

Theoretically, the WKBJ solution is valid only for vertically smoothly varying media, in which velocity v(z) is a function of depth (e.g. Morse and Feshbach, 1953; Aki and Richards, 1981; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has been also obtained and generalized to complex media by introducing an extra amplitude term based on transport equation of high-frequency asymptotics to the traditional one-way wave equation that satisfy only the eikonal equations (Zhang, 1993; Zhang, et al., 2003). Based on the principle of energy conservations, it also can be derived as transparent propagator, by introducing the concept of transparent boundary condition (Wu and Cao, 2005).

While for the media with rapid velocity variation, the WKBJ approximation could be invalid. For one-way methods, the reflected waves are omitted and the transmitted waves may not be their true values. But for most cases, especially for the media with limited sharp velocity interfaces, the transmission coefficient for one-way wave provides a good approximation to the true transmitted waves.

For the vertical heterogeneous media, the wavenumber dependent WKBJ approximation and transmission coefficient can be implemented on each global wavenumber (plane wave) at each depth. While for the heterogeneous media, the correction coefficients vary with location and can't be applied on the global wavenumber directly. On the other hand, the beamlet propagation method decomposes the wave field into beamlets and each beamlet has a location and local wavenumber (Wu et al., 2000; Wang and Wu, 2002; Luo and Wu, 2003; Luo, et al, 2004; Luo and Wu, 2005), the location and wavenumber dependent WKBJ and transmission corrections can be applied on localized beamlets at each depth.

Global WKBJ and transmission correction

For heterogeneous media $v(x,z)$, the wave equation in Cartesian coordinates is

$$[\partial_{zz}+\partial_{xx}-1/v^2(x,z)\partial_{tt}]u(x,z,t)=0\ , \qquad (1)$$

where we are considering the 2-D wave propagation problem and $u(x,z,t)$ is the space domain wavefield. In frequency-space (f - x) domain, the wave equation can be written as

$$[\partial_{zz}+\partial_{xx}+\omega^2/v^2(x,z)]u(x,z,\omega)=0\ . \qquad (2)$$

We first consider a simple case where velocity is a function of depth z alone, i.e., $v=v(z)$. The wavefield at depth z can be decomposed to a superposition of plane waves

$$u(x,z,\omega)=\sum_{k_x}u(k_x,z,\omega)\exp(ik_x\cdot x)\ , \qquad (3)$$

where k_x is the horizontal wavenumber. Each plane wave satisfies

$$(\partial_{zz}+k_z^2)u(k_x,z,\omega)=0,\quad k_z^2=\omega^2/v^2(z)-k_x^2 \qquad (4)$$

where k_z is the vertical wavenumber and it is velocity dependent.

For each plane wave, the phase and the amplitude should change after one step of propagation. Fig.1 shows two plane waves propagating in the $v(z)$ model. The phase and amplitude relation between the incident plane waves and transmission plane waves depend on the change of velocities in the vertical direction.

If the velocities vary continually with the depth and the scattering (reflection) loss can be omitted, the WKBJ approximation can be applied, and the downward continuation from depth 0 to z can be written as

True amplitude one-way propagators implemented with localized corrections on beamlets

$$u(k_x,z,\omega)=u(k_x,0,\omega)\sqrt{\frac{k_z(0)}{k_z(z)}}\exp\{i\int_0^z k_z(z')dz'\}\,. \quad (5)$$

Equation (5) agrees with the WKBJ solution for smoothly varying media (e.g. Stolt and Benson, 1986).

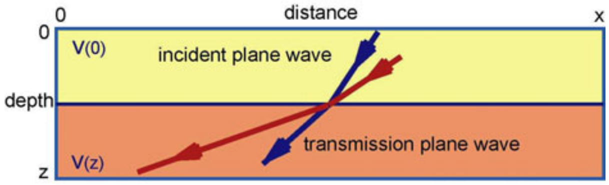


Fig. 1. Plane waves propagation in v(z) model.

If the velocity varies rapidly with depth and the transmission loss exists, the downward continuation from depth 0 to z can be written as

$$u(k_x,z,\omega)=u(k_x,0,\omega)\frac{2k_z(0)}{k_z(0)+k_z(z)}\exp\{i\int_0^z k_z(z')dz'\} \quad (6)$$

Equation (6) is a simplified transmission coefficient correction for plane waves, which omits the contribution from k_x .

Local WKBJ approximation and transmission correction

For laterally varying media $v(x,z)$, the relations (5) and (6) are no longer valid. However, instead of the global plane wave decomposition of the wavefield, the beamlet decomposition is applied to decompose the wavefield into beamlets (Wu et al., 2000; Wang and Wu, 2002; Wang, et al., 2003; Luo and Wu, 2003)

$$u(x,z,\omega)=\sum_n\sum_m u(\bar{x}_n,\bar{\xi}_m)b_{mn}(\bar{x}_n,\bar{\xi}_m,z,\omega), \quad (7)$$

where b_{mn} is the decomposition vector (beamlet), $u(\bar{x}_n,\bar{\xi}_m)$ are the coefficients of the decomposition beamlets located at $\bar{x}_n$ (space locus) and $\bar{\xi}_m$ (wavenumber locus), with

$$\bar{\xi}_m=m\Delta_\xi,\quad \bar{x}_n=n\Delta_x \quad (8)$$

For each beamlet with $\bar{x}_n$ and $\bar{\xi}_m$, it satisfies approximately,

$$[\partial_{zz}+k_z^2(z,\bar{x}_n)]b_{mn}(\bar{x}_n,\bar{\xi}_m,z,\omega)=0,$$
$$k_z^2(z,\bar{x}_n)=\omega^2/v^2(\bar{x}_n,z)-\bar{\xi}_m^{\,2} \quad (9)$$

where $k_z(z,\bar{x}_n)$ is the vertical wavenumber for the beamlet and depends on the velocity in the window.

Each beamlet is localized to its window, and in most cases, the horizontal velocity variation in a window can be very small. As an approximation, instead of the original heterogeneous media $v(x,z)$, a window-constant velocity model $v(\bar{x},z)$ is used to calculate the phase and amplitude change during the propagation (see Fig.2). The phase and amplitude between the incident beamlet and transmission beamlet depend on the velocity vertical variation in the window. Similar to the global plane wave propagation in $v(z)$ media, the WKBJ approximation and transmission coefficient can be applied to the beamlet propagation in windows.

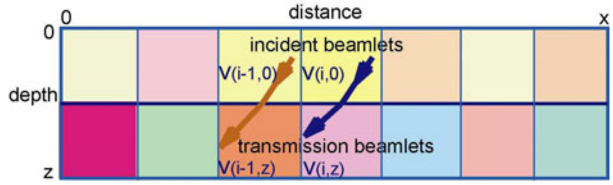


Fig. 2. Beamlet waves propagation in heterogeneous media

If the velocities vary continually with depth within the window and the scattering (reflection) loss can be omitted, the downward continuation beamlet with WKBJ approximation at depth 0 and z can be written as

$$b_{mn}(\bar{x}_n,\bar{\xi}_m,z,\omega)=b_{mn}(\bar{x}_n,\bar{\xi}_m,0,\omega)\cdot\sqrt{\frac{k_z(0,\bar{x}_n)}{k_z(z,\bar{x}_n)}}\exp\{i\int_0^z k_z(z',\bar{x}_n)dz'\} \quad (10)$$

Here vertical wavenumber k_z also depends on the beamlet window location. As shown in Fig.2, two incident beamlet have different window locations will result to different transmission corrections.

If the velocity varies rapidly with depth and the transmission loss cannot be neglected, the downward continuation beamlet with transmission approximation between depth 0 and z can be written as

$$b_{mn}(\bar{x}_n,\bar{\xi}_m,z,\omega)=b_{mn}(\bar{x}_n,\bar{\xi}_m,0,\omega)\cdot\frac{2k_z(0,\bar{x}_n)}{k_z(0,\bar{x}_n)+k_z(z,\bar{x}_n)}\exp\{i\int_0^z k_z(z',\bar{x}_n)dz'\} \quad (11)$$

Numerical examples

Here, the full wave FD method, one-way phase shift method and LCB method are applied to generate impulse responses for comparison. The full wave FD method is supposed to provide the true amplitude impulse responses in all cases. The phase shift method is convenient for applying the global transmission and WKBJ correction according to (5) and (6). For the LCB beamlet method, at each depth, the frequency space domain wave field is decomposed into local beamlets, and each beamlet has its location and wavenumber. The location and wavenumber related transmission or WKBJ correction coefficient, equation (10) and (11), can be applied to each beamlet separately. For the following calculations, a same Ricker wavelet with dominant frequency 15Hz is used for all the models and methods.

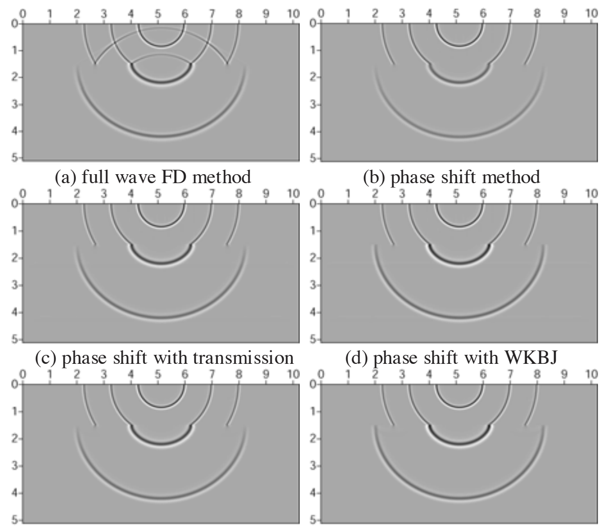
1. Vertically heterogeneous media

We first calculate the impulse responses in a two-layer model and a vertically linearly varying model to see the effects of the transmission and WKBJ approximation. The LCB method is also applied to these models to see the difference between

True amplitude one-way propagators implemented with localized corrections on beamlets

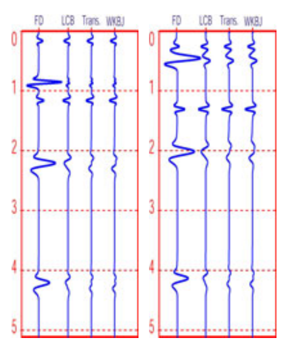
global and local transmission or WKBJ approximation.

The two-layer model has an upper velocity of 2km/s and a lower velocity of 4km/s, with the interface located at depth 1.5km. The full wave FD method, one-way phase shift method and LCB method are applied to generate impulse responses. As shown in Fig.3, the wavefields at 0.5s, 1.0s and 1.5s are added together for comparison. We also pick the wave fields at distance 5.11km and 4.39km for comparison. As shown in Fig.4, the first trace is the wave field by full wave FD methods. The second trace is the difference between the phase shift and FD method. The third trace is with transmission correction, and the fourth trace is with WKBJ correction.



(a) full wave FD method (b) phase shift method (c) phase shift with transmission (d) phase shift with WKBJ (e) LCB with local transmission (f) LCB with local WKBJ

Fig. 3. Comparison of impulse responses in two layer model.

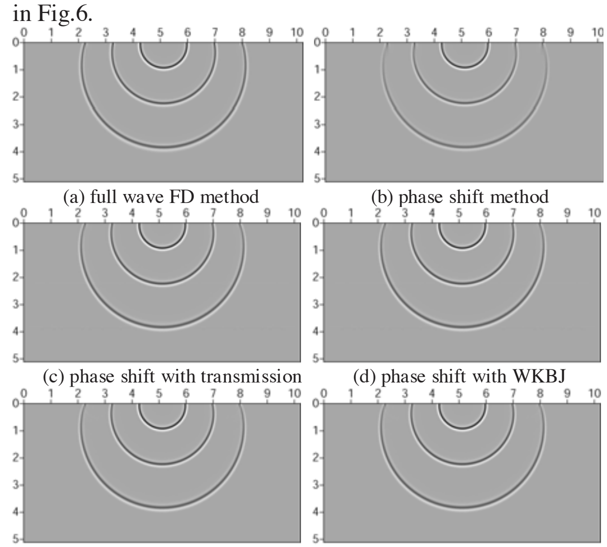


distance 5110m distance 4390m

Fig. 4. Comparison of the wave curves in two-layer model.

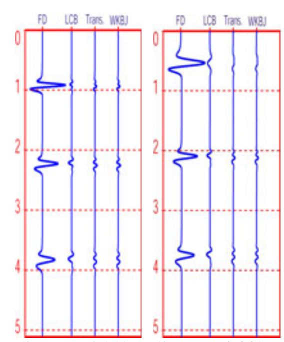
We can see from Figs.3 and 4 that: 1) The general amplitude difference between the one-way phase shift method and full wave FD method does exist, but not very large in the small angles, even after the sharp velocity interface; 2) The transmission or WKBJ correction can improve the wave field amplitude after crossing the velocity interface, especially for wide angles; 3) The WKBJ correction seems over corrected the wide angle amplitudes slightly. It is reasonable since it omits the transmission loss; 4) The LCB method with local transmission or WKBJ correction can provide the same results as the phase shift method with global correction; 5) the transmission and WKBJ methods provide nearly the same amplitude improvement in the near vertical angles. It is reasonable because the formula (5) and (6) are similar for small wavenumbers.

The vertically linearly varying model has a minimum velocity of 2km/s, and a linear varying parameter dv/dz of 0.4/s. The wave fields at 0.5s, 1.0s and 1.5s are picked and shown in Fig.5 for comparison. We also pick the wavefield curves at distance 5.11km and 4.39km for comparison, which is shown in Fig.6.



(a) full wave FD method (b) phase shift method (c) phase shift with transmission (d) phase shift with WKBJ (e) LCB with local transmission (f) LCB with local WKBJ

Fig. 5. Comparison of impulse responses in vertically linearly varying media.



distance 5.11km distance 4.39km

Fig. 6. Comparison of the wave curves in v(z) model.

Similarly we see from the Figs.5 and 6 that: 1) The amplitude errors by the one way phase shift method are generally small but increase with the angle and the distance to the source; 2) Global and local transmissions or WKBJ corrections can improve the amplitudes to their true values; 3) The transmission and WKBJ correction make nearly same amplitude improvements, especially for waves with small incident angles, but the WKBJ correction seems provide better amplitude improvements for wide-angle waves.

2. Heterogeneous media

We also applied the full wave FD method and LCB method to get the impulse response for two heterogeneous media, one is the model which has a high velocity lens, the other is depth and distance related linearly varying model. In these models, the one-way phase shift method and global transmission or WKBJ correction are not valid.

True amplitude one-way propagators implemented with localized corrections on beamlets

The lens model has a back ground velocity 2km/s and high lens velocity 4km/s. The wavefields at 0.5s, 1.0s and 1.5s are added together for comparison, as shown in Fig.7.

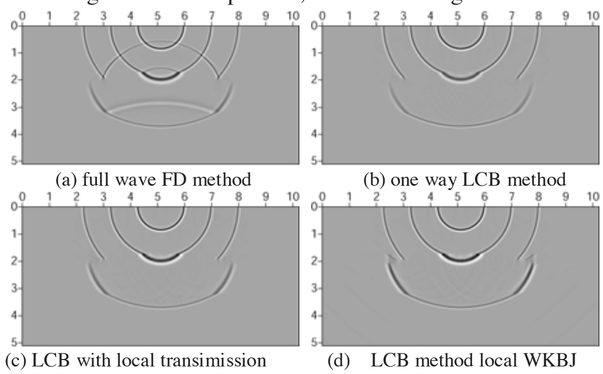


Fig. 7. Comparison of impulse responses in lens model.

We can see from Fig.7 that the amplitude error for LCB method exists but is small, except for the wide angle waves. With the local transmission or WKBJ corrections, the amplitudes of wide angle waves are improved. However, using the WKBJ method, they are slightly over corrected.

The model in which the velocity varying with depth and distance linearly have a minimum velocity of 2km/s, and linear varying parameters dv/dz of 0.25/s and dv/dx of 0.125/s. The wave fields at 0.5s, 1.0s and 1.5s are picked and added together for comparison, as shown in Fig.8.

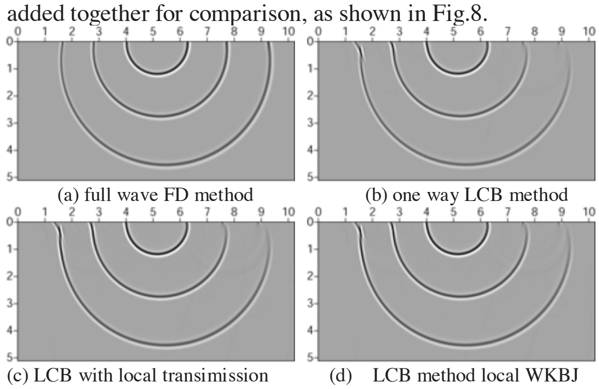


Fig. 8. Comparison of impulse responses in linearly heterogeneous media.

We can see from Fig.8 that both local transmission and WKBJ correction can improve the amplitudes, especially for wide angle waves.

Conclusion

The transmission coefficient and WKBJ approximation can be adopted for constructing the true amplitude propagators. The transmission correction is suitable for rapid velocity changes while the WKBJ approximation favors the smoothly varying media. For vertically heterogeneous media, the transmission and WKBJ corrections can be applied to global plane waves. Similarly, for laterally varying media, the localized transmission and WKBJ correction can be conducted using localized beamlets.

Acknowledgements:

The authors would like to acknowledge the support from the WTOPI (Wavelet Transform On Propagation and Imaging for seismic exploration) Project and the DOE/Basic Energy Sciences project at University of California, Santa Cruz. We wish to thank Jun Cao for helpful discussions.

References

Aki, K. and P. Richards, 1981, Quantitative Seismology, Vol.1.

Clayton, R.W. and Stolt, R.H., 1981, A Born-WKBJ inversion method for acoustic reflection data, Geophysics, 46, 1559-1567.

Morse, P.M. and Feshbach, H., 1953, Methods of Theoretical Physics, McGraw-Hill Book Company.

Stolt, R.H., and Benson, A.K., 1986, Seismic Migration: Theory and Practice, Handbook of Geophysical Exploration, Vol. 5, Geophysical Press.

Luo, M. and Wu, R.S., 2003, 3D beamlet prestack depth migration using the local cosine basis propagator, Expanded Abstracts, SEG 73th Annual Meeting, 985-988.

Luo, M., Wu, R.S., and Xie, X.B., 2004, Beamlet migration using local cosine basis with shifting windows, Expanded Abstracts, SEG 74th Annual Meeting, 945-948.

Luo, M. and Wu, R.S., 2005, Beamlet migration using local cosine basis with average window reference velocity, Expanded Abstracts, SEG 75th Annual Meeting, submitted.

Wang, Y. and Wu, R.S., 2002, Beamlet prestack depth migration using local cosine basis propagator, Expanded Abstracts, SEG 72th Annual Meeting, 1340-1343.

Wang, Y., Cook, R., and Wu, R.S., 2003, 3D local cosine beamlet propagator, Expanded Abstracts, SEG 73th Annual Meeting, 981-984.

Wu, R.S. and Cao, J., 2005, WKBJ solution and transparent propagators, EAGE 67th conference & exhibition, Expanded Abstracts.

Wu, R.S., Wang, Y. and Gao, J.H., 2000, Beamlet migration based on local perturbation theory, Expanded Abstracts, SEG 70th Annual Meeting, 1008-1011.

Zhang, G.Q., 1993, System of coupled equations for upgoing and downgoing waves, Acta Math. Appl. Sin., 16, 251-263.

Zhang, Y., Zhang, G.Q., and Bleistein, N., 2003, True amplitude wave equation migration arising from true amplitude one-way wave equation, Inverse Problems, 19, 1113-1138.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2005 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

True amplitude one-way propagators implemented with localized corrections on beamlets

References

Aki, K., and P. Richards, 1981, Quantitative seismology, **1**: University Science Books.

Clayton, R. W., and R. H. Stolt, 1981, A Born-WKBJ inversion method for acoustic reflection data: Geophysics, **46**, 1559–1567.

Morse, P. M., and H. Feshbach, 1953, Methods of theoretical physics: McGraw-Hill.

Luo, M., and R. S. Wu, 2003, 3D beamlet prestack depth migration using the local cosine basis propagator: 73rd Annual International Meeting, SEG, Expanded Abstracts, 985–988.

—, 2005, Beamlet migration using local cosine basis with average window reference velocity: Presented at the 75th Annual International Meeting, SEG.

Luo, M., R. S. Wu, and X. B. Xie, 2004, Beamlet migration using local cosine basis with shifting windows: 74th Annual International Meeting, SEG, Expanded Abstracts, 945–948.

Stolt, R. H., and A. K. Benson, 1986, Seismic migration: Theory and practice: Handbook of geophysical exploration, **5**: Geophysical Press.

Wu, R. S., and J. Cao, 2005, WKBJ solution and transparent propagators: Presented at the 67th Annual International Conference and Exhibition, EAGE.

Wang, Y., R. Cook, and R. S. Wu, 2003, 3D local cosine beamlet propagator: 73rd Annual International Meeting, SEG, Expanded Abstracts, 981–984.

Wu, R. S., Y. Wang, and J. H. Gao, 2000, Beamlet migration based on local perturbation theory: 70th Annual International Meeting, SEG, Expanded Abstracts, 1008–1011.

Wang, Y., and R. S. Wu, 2002, Beamlet prestack depth migration using local cosine basis propagator: 72nd Annual International Meeting, SEG, Expanded Abstracts, 1340–1343.

Zhang, G. Q., 1993, System of coupled equations for upgoing and downgoing waves: Acta Mathematicae Applicatae Sinica, **16**, 251–263.

Zhang, Y., G. Q. Zhang, and N. Bleistein, 2003, True amplitude wave equation migration arising from true amplitude one-way wave equation: Inverse Problems, **19**, 1113–1138.