Velocity analysis using shot-indexed sensitivity kernels: application to a complex geological model
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Summary

We investigate the capability of inverting large scale velocity errors through wave equation migration velocity analysis using shot-indexed sensitivity kernels. We find that the global shift of the image (absolute residual moveout) is crucial for constraining the large-scale velocity errors in the model when the shift of the image is more than a number of wavelengths. We estimate the shift through fitting the curvature of RMOs on CIGs and add this information into the inversion system. Although the extraction of the shift information is based on a simple model, including this information into RMO data considerably improves the inversion result. We apply our method to Sigsbee benchmark model and demonstrate that the velocity analysis based on sensitivity kernels can handle both large-scale and small-scale velocity errors in a complex geological environment.

Introduction

Velocity model building remains a challenge as migration/imaging techniques are facing increasingly complex structures. Although the ray based method has dominated the velocity tomography for a long time, many authors suggested that the wave equation based method may provide better velocity models in transmitted wave tomography (Luo and Schuster, 1991; Woodward, 1992; Dahlen et al., 2000; Zhao et al., 2000; Jocker et al., 2006). In the past decade, wave equation based method has also been adopted into migration velocity analysis (Biondi and Sava, 1999; Sava and Biondi, 2004; Xie and Yang, 2007; Shen and Symes, 2008; Symes, 2008; Xie and Yang, 2008a) because it naturally embraces the characteristics of seismic waves and is consistent with the wave equation based migration scheme. In this method, the residual moveouts (RMO) in migrated common image gathers (CIG) are back-projected into model space, through the wavepaths (or sensitivity kernels), to update the velocity (Vasco et al., 1995; Fliedner et al., 2007; Fliedner and Beve, 2008; Xie and Yang, 2008a).

Various types of sensitivity kernels were derived for different types of CIGs extracted from prestack depth migration. Shot-indexed sensitivity kernels have been formulated by Xie and Yang (2007, 2008a) for testing migration velocity model building. Yang et al. (2009) derived the sensitivity kernels for plane-wave source migration and obtained the similar results as from shot-indexed kernels. He et al. (2009) derived the angle-domain sensitivity kernels for local angle domain CIGs. The inversion using angle-domain sensitivity kernels was also tested for single shot data. In the above mentioned works, relative RMOs combined with differential sensitivity kernels were used to build the inversion system. Application of these sensitivity-kernel based tomography methods achieved high resolution and acceptable convergence speed. However, these methods have not been tested for migration models with large scale velocity errors.

In this article we conduct sensitivity-kernel based velocity analysis for models with large-scale velocity errors. The velocity errors can extend to large areas and/or have high perturbations, which not only causes the events on CIG curved up or down extensively, but often causes the entire CIG shifted for a number of wavelengths. The shift of the entire CIG contains important information regarding the large-scale velocity errors in the migration velocity model. This information has been eliminated in the relative RMO data. Without this information, the inversion system is lack of constraints on large-scale velocity errors in the model. Under this circumstance, we find that the inversion based on relative RMO and differential kernels sometimes cannot provide correct macro velocity model. In fact, the information of the absolute shift is contained in the curvature of the CIGs and can be recovered and added to the data for better constraining the large-scale errors in the model.

In the rest of this paper, we discuss the velocity scanning method for recovering the global shift of the CIG, followed by the demonstration on how to combine this information into the absolute RMO data for inversion. As an example, we apply this method to the migration velocity analysis of the Sigsbee velocity model. The results show that, combining the absolute moveout with the relative moveout data, the inversion system has better constraints on both small-scale and large-scale velocity errors.

Shot-index sensitivity kernel

Let us first briefly review the theory on the shot-index sensitivity kernel. Xie and Yang (2007, 2008a) derived the sensitivity kernels which link the RMOs in shot-indexed CIGs to velocity model errors \( \delta v(x) \) through an integral equation,

\[
\int_k K^B(x, x_1, x_2) \cdot \delta v(x) \, dx = R(x_1, x_2),
\]

where \( R(x_1, x_2) \) is the absolute residual moveout along reflector normal direction and \( K^B(x, x_1, x_2) \) is the
Migration velocity analysis using shot-indexed sensitivity kernels

broadband sensitivity kernel for the source at \( x_s \) and common image point at \( x_i \). In previous works (Xie and Yang, 2007, 2008b, a; He et al., 2009; Yang et al., 2009), relative RMOs combined with differential sensitivity kernels were used in the inversion, while the information regarding the absolute moveout is neglected from the data,

\[
\int K^B(x, x_1, x_2) \delta v(x) dx = \delta R(x, x_1, x_2),
\]

(2)

where

\[
\delta R(x, x_1, x_2) = R(x, x_2) - R(x, x_1)
\]

(3)

is the relative RMO, and

\[
K^B(x, x_1, x_2) = K^B(x, x_1, x_2) - K^B(x, x_1, x_2)
\]

(4)

is the differential sensitivity kernel. The explicit form of the broadband sensitivity kernels can be found in Xie and Yang 2008a. In equations (2)-(4), the information of global shift of the image has been eliminated from the data. As we mentioned above, the lack of this information considerably weakens the constraints on large-scale errors in migration velocity model.

Recovering the absolute moveout in the RMO data

We first demonstrate how to estimate the absolute moveout based on the curvature in a CIG. Simply considering a horizontal reflector embedded at depth \( h_0 \) in a homogeneous background model with velocity \( v \) (Figure 1a), a layer of medium with velocity error \( \delta v \) can cause RMO in the migrated image at the midpoint

\[
\delta h = \frac{\delta v}{v} \left( x^2 + h_0^2 \right). 
\]

(5)

The absolute moveout at the zero offset is \( \delta h_0 = (\delta v/v) h_0 \), which is a function of the velocity perturbation and the thickness of the overburden that contains \( \delta v \). The image will shift to a shallower depth if migration velocity is lower than the true velocity and the CIGs will curve upward, and vice versa. Figure 1 shows the comparison between the theoretical curve (blue) and measured vertical shift (red) for a homogeneous velocity model and for different velocity perturbations. The vertical shift of the image can be obtained by fitting the RMO curve for each common image point. For actual measurement, a method similar to velocity scan is used and the results are averaged within a horizontal range. Equation (5) and its variations have been widely used for 1D velocity scanning (Al-Yahya, 1989; Liu and Bleistein, 1992; Stork, 1992; Ji, 1997; Meng et al., 1999; Mosher et al., 2001; Jiao et al., 2002; Xia et al., 2006; Schneider, 2008), and have been modified to adapt to dipping reflectors. Although equation (5) is based on a very simple model, it provides a good estimate of the global shift and will considerably improve the inversion result. More accurate estimation method could be considered for future work.

Figure 1 Comparison between theoretical (blue) and measured (red) vertical shift of the image for different velocity perturbations in a homogeneous model with a velocity of 3 km/s. The reflector is located at a depth of 2.5 km.

Figure 2 (a) Sigsbee benchmark model; (b) True velocity model; (c) Initial velocity model; and (d) velocity difference between (b) and (c).

Numerical examples

As a preliminary test, we conduct migration velocity analysis to part of the Sigsbee benchmark model (Figure 2a), excluding the salt body’s effect. We reduce the velocity in original model between grid points 650 and 1050 in depth, by multiplying it with a Gaussian shaped function along depth, and use the resulted model as the initial model (Figure 2c). The maximum perturbation is -10% of the true velocity model. We obtain the prestack
Migration velocity analysis using shot-indexed sensitivity kernels

depth images using a local-cosine-basis-beamlet migration method (Wu et al., 2008) for both the true and initial velocity models, shown in Figures 3a and 3b, respectively. The red lines are used as references to visualize the image quality.

We select 51 common image points located from 200 to 800 along horizontal axis to extract shot-indexed CIGs (Figure 4). By comparing the image from the initial model to that from the true velocity model, we see that the baseline reflector is shifted from the depth 1180 (true position) to a shallow depth 1140 grid points. The four scattering points at depth around 1000 are not focused. The CIGs related to the baseline reflector are curved upward, indicating a macro lower velocity model is used for migration. At each common image point, we use a cross-correlation method to measure the RMOs between the stacked trace and individual traces.

In the second step, we calculate the shot-indexed sensitivity kernels for each pair of source and common image points using the initial velocity model combined with the picked reflector from the initial image. One-way generalized screen propagator (GSP) (Xie and Wu, 1998) is used to calculate sensitivity kernels along the source side, and one-return propagator is used to calculate sensitivity kernels along the receiver side. Shown in Figure 5 is an example of these sensitivity kernels. The source side kernel has the well-known banana-donut pattern and the receiver side kernel bears a fan shape.

![Figure 4](image1.png)  
**Figure 4** Shot-indexed CIGs using initial velocity model.

![Figure 5](image2.png)  
**Figure 5** The sensitivity kernel for a shot at \( x_s = 371 \) and an image point at \((x, z) = (500, 1140)\). The center frequency is 20 Hz.

We first estimate the absolute moveout using the method introduced in the previous section. Velocity scan reveals that the CIGs above 600 are generally flat which leaves the target area between 600 and 1200 in depth. Figure 6 shows...
Migration velocity analysis using shot-indexed sensitivity kernels

our fitting curves and measured RMOs. We choose $h_0 = 600$ and obtain an average vertical shift of -24 grid points, with the negative sign indicating the image shifts upward.

(a)

Figure 7 (a) Velocity model difference between true velocity model and updated velocity model and (b) velocity along depth for selected profiles.

We include the global shift information into our RMO data to form absolute RMOs. Thus we can directly use equation (1) and the shot-indexed sensitivity kernels to form the inversion system. The difference between the true velocity model and the updated velocity model is shown in Figure 7a. Comparing with initial velocity model difference shown in Figure 2d, we can see the residuals become smaller. Shown in Figure 7b are selected velocity profiles. Comparing to the initial velocity model (black line), the updated velocity model (red dash line) is approaching the true velocity model (blue line). Note the high vertical resolution is the footprint from the initial velocity model.

Prestack image using the updated velocity model is shown in Figure 8. The corresponding shot-indexed CIGs (Figure 9) for the same common image points are extracted. We see that the reflector image is migrated towards the right position and related CIGs are flattened. The images of point scatter at depth 1000 become better focused. There are still unexplained moveouts in CIGs, implying further iterations are required. As a preliminary test, we only use the RMOs on the baseline reflector. Better inversion results are expected if more events on other reflectors are included in the inversion.

Figure 8 Prestack image using updated velocity model.

Figure 9 Shot-indexed CIGs using updated velocity model.

Conclusions

We applied migration velocity analysis to Sigsbee benchmark model using shot-indexed sensitivity kernels. The capability of inverting large scale velocity anomaly is investigated. We find that the information of absolute moveout is crucial to provide a stable constraint on large-scale errors in velocity models. Even with a simple technique, the extracted absolute moveout helps improve the result considerably. The numerical example demonstrates the potential of MVA using sensitivity kernels on complex geological structure.

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REFERENCES


