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Elastic Envelope Inversion

J.R. Luo* (Xi'an Jiaotong University), R.S. Wu (UC Santa Cruz) & J.H. Gao (Xi'an Jiaotong University)

SUMMARY

We developed the elastic envelope inversion method. The envelope of the wavefields carries ultra low frequency information which can be used to recover the large-scale component of the model, and the initial model dependence of waveform inversion can be reduced. We derived the misfit function and the corresponding gradient operator for elastic envelope inversion. Numerical tests using synthetic data for the Marmousi II model proved the validity and feasibility of the proposed approach. The inverted p-wave velocity and s-wave velocity from the combined (EI+W1) envelope inversion plus waveform inversion indicated that it can deliver much improved results compared with regular elastic full waveform inversion.
Introduction

Elastic full waveform inversion (FWI) has been investigated by many authors (Mora, 1987; Crase et al., 1992; Sears et al., 2010). However, as in acoustic situation, good initial models are always needed in elastic full waveform inversion in order to get good inversion results.

To generate low-frequency signal below 5Hz is very expensive, so effort has been focused to the recovery of long wavelength background without very low frequencies. Travel time inversion and migration velocity analysis are two traditional methods in this area. In recent years, Shin and Cha (2008) has developed Laplace domain full waveform inversion which can give a smooth background model from an inaccurate initial model. Zhou et al. (1997) combined traveltime and waveform inversion. Biondi and Almomin (2014) combined full waveform inversion with wave equation migration velocity analysis. Liu et al. (2011) proposed the normalized integration method. Wu et al. (2013, 2014) developed envelope inversion (EI) method which can give a very smooth background structure.

Most of the approaches above are developed in the acoustic situation. However, the real earth is elastic. In this paper, we extend the envelope inversion method to elastic situation. We first introduce the elastic envelope inversion method and derive the gradient for the elastic envelope inversion, then we use the Marmousi II model to show the validity of this method.

Trace envelope and Hilbert transform

First we introduce what is the envelope of a signal. We extract envelope by taking the amplitude after the analytical signal transform using Hilbert transform. A signal having no negative-frequency components is called an analytic signal

\[ \tilde{f}(t) = f(t) + iH\{f(t)\}, \]

The envelope of \( f(t) \) can then be derived by,

\[ e(t) = \sqrt{\tilde{f}^2(t) + H\{f(t)\}^2}, \]

Envelope inversion for the elastic situation

The 2-D isotropic elastic wave equation can be expressed as,

\[ \rho \frac{\partial^2 u^i}{\partial t^2} - \frac{\partial \tau^{ij}}{\partial x^j} = f^i, \]

\[ \tau^{ij} = M^{ij} + \lambda \delta^{ij} u_k, + \mu (u_{i,j} + u_{j,i}). \]

Where \( u^i \) is the \( i \)th component of the displacement vector and \( \tau^{ij} \) is the \( ij \) component of the stress tensor. \( \rho \) is the density; \( \lambda \) and \( \mu \) are Lamé coefficients. \( f^i \) is the body force and \( M^{ij} \) is the traction. The p-wave velocity and s-wave velocity are given by,

\[ v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad v_s = \sqrt{\frac{\lambda}{\rho}}. \]

In the elastic envelope inversion, we define the misfit function as follow,

\[ \sigma(m) = \frac{1}{4} \sum_x \sum_i \int_0^T \left[ \left| e_{\text{obs}}^i(t) \right|^2 - \left| e_{\text{syn}}^i(t) \right|^2 \right] dt, \]

where \( m \) is the model parameter, \( e_{\text{obs}}^i \) and \( e_{\text{syn}}^i \) are the envelope of the \( i \)th component of the synthetic wavefield and the observed wavefield respectively. Using Hilbert transform, the above equation can be written as,

\[ \sigma(m) = \frac{1}{4} \sum_x \sum_i \int_0^T \left[ \left| \tilde{s}(t) \right|^2 + \left| s_{\text{obs}}^i(t) \right|^2 \right] - \left[ \left| u(t) \right|^2 + \left| u_{\text{obs}}^i(t) \right|^2 \right] \right| dt = \frac{1}{4} \sum_x \sum_i \int_0^T \tilde{E}_i^i dt \]
where \( s' \) and \( u' \) are the \( i \)-th component of the synthetic wavefield and the observed wavefield respectively, \( s''_i \) and \( u''_i \) are the corresponding Hilbert transforms. \( E_i \) is the instant envelope data residual. Here we applied squared envelope, because it has better performance in long-wavelength background recovery (Wu et al. (2014); Luo et al. (2014)).

**Gradient calculation for elastic envelope inversion**

We first assume the density is a constant. If we consider \( \lambda \) and \( \mu \) as the model parameters. Calculating the derivative of the misfit function \( \sigma \) with \( \lambda \) and \( \mu \), we get,

\[
\begin{align*}
\frac{\partial \sigma}{\partial \lambda} &= \sum_x \sum_j \int_i \left[ E_i s(t) - H \{ E_i s'_i(t) \} \right] \frac{\partial E_i S(t)}{\partial \lambda} dt = J_{\lambda}^T r_{env}, \\
\frac{\partial \sigma}{\partial \mu} &= \sum_x \sum_j \int_i \left[ E_i s(t) - H \{ E_i s'_i(t) \} \right] \frac{\partial E_i S(t)}{\partial \mu} dt = J_{\mu}^T r_{env}.
\end{align*}
\]

(8)

from equation (8) we see that the gradient for the elastic envelope inversion has exactly the same form as that in the conventional elastic full waveform inversion (Mora, 1987), except that the residual vector \( r_{env} \) is different. The gradient for the elastic envelope inversion can be obtained using the backpropagation method as follows,

\[
\begin{align*}
\frac{\partial \sigma}{\partial \lambda} &= -\int dt' s'_i j, \\
\frac{\partial \sigma}{\partial \mu} &= -\int dt' (s_j k + s_k j) i k.
\end{align*}
\]

(10)

where “\( \leftarrow \)” means backpropagation wavefields which is obtained using the envelope residual vector \( r_{env} \) as the source wavelet. Using equation (4) we can get the gradient with respect to the velocity as follows,

\[
\begin{align*}
\frac{\partial \sigma}{\partial v_p} &= -2 \rho v_p \int s_k k i i, \\
\frac{\partial \sigma}{\partial v_s} &= 4 \rho v_p \int s_j j i i - 2 \rho v_s \int (s_j j + s_k k) i k.
\end{align*}
\]

(11)

because \( r_{env} \) is related to the effective envelope residual not the seismic data residual, the low frequency information in the envelope can be lower than the lowest frequency in the source wavelet. This low frequency information can be used to construct the large-scale background structure. The low-frequency information coded in the envelope is not directly from the source wavelet.

**Examples**

We show the validity of this method using the Marmousi II model (Figure 1). The source wavelet we used is the Ricker wavelet with the dominant frequency of 10Hz, and the low frequencies below 5 Hz is cut off from the source wavelet, so there will be no low frequency information in the data. Figure 2 shows the initial model for inversion. We used 1-D linear initial model. Starting from these initial models, we invert the p-wave velocity and s-wave velocity using envelope inversion. Figure 3 shows the envelope inversion results.
From the results, we can see the large-scale component of the model. Using the results in Figure 3 as the new initial model, we perform elastic waveform inversion. Figure 4 shows the final inversion results (EI+FWI results). As comparison, we also performed the conventional elastic full waveform inversion starting directly from the linear initial model. Figure 5 shows the inversion results. From the comparison we can see that the combined EI+FWI can deliver much better results than the conventional FWI method.

**Figure 1** The Marmousi II model. (a) $vp$; (b) $vs$.

**Figure 2** Linear initial model. (a) $vp$; (b) $vs$.

**Figure 3** Envelope inversion results using the low-cut source. (a) $vp$; (b) $vs$.

**Figure 4** EI+FWI results using the low-cut source wavelet. (a)$vp$; (b) $vs$. 

**Figure 5** Conventional FWI results starting directly from the linear initial model.
Figure 5 The conventional FWI result using the low-cut source. (a) vp; (b) vs.

Conclusions

We extended the envelope inversion method to the elastic situation. Envelope carries ultra low frequency information which can be sued to estimate the large scale component of the model structure, so that the initial model dependence of waveform inversion can be reduced. This is demonstrated by the Marmousi II model tests, where we inverted the p-wave velocity and s-wave velocity and used 1-D linear gradient starting model. The elastic envelope inversion is independent of the frequency band of the source wavelet. The model parameter can be inverted while the low frequency is removed from the source wavelet.

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References


