ACQUISITION APERTURE CORRECTION IN ANGLE-DOMAIN TOWARDS THE TRUE-REFLECTION RTM
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Summary
Due to the limitation of acquisition aperture, RTM, though as a true-amplitude propagator, cannot provide a true-reflection image. To relate the image amplitude with the target reflectivity, we propose an amplitude correction in the dip angle domain for RTM. The stacked migration image is decomposed into dip-angle dependent partial images, which are compensated individually by the corresponding acquisition dip response (ADR). Then, corrected images are summed up to form a new image consistent with the target impedance contrast. ADR is formulated as the energy contributed from all possible combinations of incident and scattered plane waves to the image with specific dip angle. 2D SEG/EAGE model is used to demonstrate the improvement in the image amplitude obtained by applying the dip-angle domain amplitude correction.

Introduction
In addition to provide correct geometrical locations of subsurface structures, the true-reflection imaging strives to give correct image amplitude, which reveals the reflection/scattering strength of the subsurface structures. However, even with accurate wave propagators, the limited acquisition aperture and the complex overburden structure usually prevent us from obtaining correct image amplitude. The combined effects from these two factors can cause irregular illuminations to subsurface structures, leading to biased image amplitude. To tackle this problem, Tarantola (1987) treated the imaging process as an inverse problem and solve it in an iterative way. However, the resulted approach usually is too expensive to practice. In addition, the instability and non-uniqueness are other unattractive features of the general inverse approach. Another approach lies between the conventional migration and general inversion. It operates on (filter) image field and makes the amplitude proportional to the true-reflectivity there. In order to distinguish the imaging provide correct reflectivity from the imaging using true-amplitude propagators, we name the former as “true-reflection imaging” (Wu et al., 2004) or “true-reflectivity migration”.

Along this direction, Gelius et al. (2002) formulated the wavenumber-domain resolution function and used it to correct the migration image. This approach has been applied to the ray-based migration (Gelius et al., 2003, 2008) and one-way wave migration (Xie et al., 2005; Wu et al., 2006). Wu et al. (2004) proposed to compensate the image amplitude in local image matrix (LIM), which is in reflection and dipping angle domain, and generated by one-way beamlet propagator. Wu and Luo (2005) tested different approaches for correction and demonstrated that the correction in dip angle domain has the greatest improvement to eliminate the acquisition effect. Later, based on the LEF (local exponential frame) decomposition, several authors (Jin et al., 2006; Cao and Wu, 2009; Mao and Wu, 2010) worked to speed up this technique.

In this paper, we develop an amplitude correction algorithm implemented in the dip angle domain to remove the acquisition configuration effect and propagation path effects through complex overburdens. We will start from the theory of amplitude correction in dip angle domain. The method used to calculate the dip-angle domain partial image will be provided in the second part. Third, we formulate aperture dip response (ADR), which serves as the amplitude correction factor. Both the ADR and dip-angle domain partial images are obtained in RTM scheme. Finally, we use 2D synthetic examples to demonstrate the impact of ADR correction on the image amplitude.

Theory
Consider a survey system composed of a shot located at \( r_s \) and a geophone located at \( r_g \) to image the subsurface target region \( V \) surrounding \( r \). The wave reflected from \( V \) reaches to the receiver can be expressed as

\[
D(r_s, r, \omega) = \int_{V} G(r, r', \omega)m(r')G(r', r_g, \omega)dr',
\]

where \( \omega \) is frequency, \( m(r') = \delta c(r')/c(r') \) is the velocity perturbation; \( k_0 = \omega/c_0 \) is the background wavenumber; \( c_0 \) is the local background velocity. \( G(r', r_g, \omega) \) is the Green’s function from source location \( r_s \) to the target location \( r' \); \( G(r', r, \omega) \) is the Green’s function from geophone location \( r_g \) to target location \( r' \). The reciprocity \( G(r', r_g, \omega) = G(r_g, r', \omega) \) is used here.

For a system composed of multiple shots and geophones, the image can be formed by

\[
I(r, \omega) = \sum_{s} \sum_{g} D(r_s, r, \omega) \frac{\partial G(r, r_g, \omega)}{\partial \omega} G(r, r_g, \omega).
\]
TRUE-REFLECTION RTM

Substituting $D(x, y, z)$ with equation 1 will result in

$$I(x, y) = \int m(x') R(x', y') dx',$$

where

$$R(x, y) = k_0^2 \sum_{r, r'} G(r, r', \omega) G'(r', r'', \omega) \times G'(r'; r', \omega) \frac{\partial}{\partial r} G(r, r', \omega),$$

is the resolving kernel. In equation 4, two Green’s functions come from the data and two are from the migration process. The image can be considered as the convolution of the resolving kernel and the model parameter. In another word, the resolving kernel maps a scattering point in model space into the image space. It includes the effects of both the acquisition (modeling) and imaging (inversion) (Wu et al., 2006).

The convolution in space domain corresponds to the multiplication in wavenumber domain, so equation 3 can be expressed as (Xie et al., 2005; Xie et al., 2006)

$$I(k_x, k_y, \omega) = m(k_x, k_y) R(k_x, k_y, \omega),$$

where $I(k_x, k_y, \omega)$, $R(k_x, k_y, \omega)$ and $m(k_x, k_y)$ are wavenumber-domain representations of the image, resolving kernel and model.

Equation 5 can be transformed from the wavenumber domain to dip angle domain

$$I(\theta_x, \theta_y, \omega) = \frac{1}{k_0} m(\theta_x, \theta_y) R(\theta_x, \theta_y, \omega),$$

followed by integrate over $\omega$ to obtain

$$I(\theta_x, \theta_y) = m(\theta_x, \theta_y) R(\theta_x, \theta_y),$$

where

$$I(\theta_x, r) = \int I(\theta_x, \theta_y, \omega) d\theta_y,$$

$$R(\theta_x, r) = \frac{1}{k_0} \int R(\theta_x, \theta_y, \omega) d\omega,$$

where $\theta_x$ is the dip angle; $I(\theta_x, r)$ and $R(\theta_x, r)$ are 2D dip-angle domain partial image and acquisition dip response (ADR), respectively.

By summing up all the dip-angle components, we will get the space-domain model parameter.

$$m(r) = \int I(\theta_x, r) d\theta_y,$$ (10)

Next, we describe how to calculate $I(\theta_x, r)$ and $R(\theta_x, r)$.

**Dip-angle domain partial image**

The dip-angle domain partial image $I(\theta_x, r)$ can be obtained by decomposing the migration image with local slant stacks:

$$I(k_x, \theta_x, r) = \int W(r' - r) I(r') e^{i k_x (r' - r)} dr',$$

followed by partial reconstruction

$$I(\theta_x, r) = \int I(k_x, \theta_x, r) k_x dk_x,$$ (12)

where $W(r' - r)$ is the spatial sampling window centered at $r’$; $I(r)$ is the final stacked migration image; $k_x$ and $\theta_x$ are the length and the polar angle of wavenumber vector $k_x$.

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Figure 2. (a) The 2D SEG/EAGE salt model. (b) The conventional RTM image.

Figure 3. (a) Illumination which only considers source energy and (b) image compensated by the source illumination.
Figure 4. Partial images for different dip angles, with (a) -15°, (b) 15°, (c) -45° and (d) 45°.

Figure 5. ADRs for selected dip angles, with (a) -15°, (b) 15°, (c) -45° and (d) 45°.

Figure 6. Partial images compensated by ADRs for selected dip angles, with (a) -15°, (b) 15°, (c) -45° and (d) 45°.
TRUE-REFLECTION RTM

Acquisition dip response (ADR)
We transform equation 4 to incident angle $\theta_i$ and scattering angle $\theta_s$ (Figure 1):

$$ R(\theta_i, \theta_s, \mathbf{r}, \omega) = J_{s}^G J_{s}^R k_0^2 \sum_{s} \sum_{g} G(\theta_i, \mathbf{r}, \theta_s, \omega) \times G^*(\theta_s, \mathbf{r}, \theta_s, \omega) \frac{\partial}{\partial \theta_s} G(\theta_s, \mathbf{r}, \theta_s, \omega), \quad (13) $$

The above equation gives the target illumination contributed from a local incident wave propagating along $\theta_i$ and a local scattered wave propagating along $\theta_s$; coefficients $J_{s}^G$ and $J_{s}^R$ are resulted from source- and receiver-side Jacobians when we change the integration from slowness domain to angle domain. It is $k_0$ for 2D case. $G(\theta, \mathbf{r}, \omega)$ is the plane wave component of Green’s function, and can be obtained by local slant stacks (Xie and Wu, 2002; Xie and Yang, 2008; Yan and Xie, 2012).

The representation of $R(\theta_i, \theta_s, \mathbf{r}, \omega)$ can be transformed to

$$ R(\theta_i, \theta_s, \mathbf{r}, \omega) $$

through coordinate transform

$$ \theta_t = (\theta_i - \theta_s)/2, \quad (14) $$

$$ \theta_d = (\theta_i + \theta_s)/2, \quad (15) $$

where $\theta_d$ is the reflection angle. Summing up all the reflection angles for a local reflector with dip angle $\theta_d$ will result in the target illumination, called acquisition dip response (ADR)

$$ R(\theta_d, \mathbf{r}, \omega) = \int R(\theta_i, \theta_s, \mathbf{r}, \omega) d\theta_s. \quad (16) $$

For commonly used seismic source functions such as the Ricker wavelet, the major energy is carried by waves near the dominant frequency. Therefore, the acquisition dip response at the dominant frequency is a good approximation of amplitude correction factor.

$$ R(\theta_d, \mathbf{r}) = \frac{1}{k_0} R(\theta_d, \mathbf{r}, \omega_0), \quad (17) $$

where $\omega_0$ is the dominant frequency of the source.

Numerical example
As an example, we conduct the amplitude correction on 2D SEG/EAGE salt model (Figure 2a). The observation system for the model is composed of 325 shots with an interval of 160 feet. Each shot is matched with 176 left-side receivers separated by 80 feet. Figure 2b shows the raw RTM image. Figure 3 shows the illumination which only considers source energy and the image corrected by the source illumination. Though the amplitudes of the image are more balanced than the conventional image, it ignores the propagation effects from the target to the geophones as well as the aperture effects. We decompose the stacked migration image into dip-angle domain and show four sampled partial images in Figure 4. Figure 5 displays the corresponding ADR. By comparing the partial image and ADR, we notice that the amplitudes of the images are consistent with the ADR strength. We correct individual partial images with the ADRs and the results are shown in Figure 6. Then all the corrected images are summed up to form the final image which is shown in Figure 7. After the correction, the image amplitudes of reflectors with different dipping angles become more balanced and the structures appear more continuous. The subsalt structures are greatly enhanced, particularly for steep structures. Overlapped on the image are the wiggle display of the image amplitude and the true reflectivity. They match with each other very well.

Conclusions
Under the RTM framework, we formulate a dip-angle domain amplitude correction method for true reflection imaging. We decompose the migration image and Green’s function to the dip angle domain. The decomposed Green’s function is used to construct the ADR. In the dip-angle domain, the partial images are corrected with the ADR and corrected images are stacked to obtain the final image. Compared to the correction only consider the source energy, correction with the ADR shows significant improvement in image amplitude.

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Figure 7: The migration image compensated by ADR. Overlapped wiggles are reflectivity (red) and the image amplitude (blue).
EDITED REFERENCES

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