Efficient surface-aware ray tracing based on implicit model representation
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Summary
We propose a novel surface-aware ray tracing method based on implicit model representation. Each horizon is represented by a signed distance field and the spatial relationships between horizons are described by a stratigraphic binary tree. With this kind of implicit representation, complex structural model (e.g., heavily faulted, intrusions and unconformities) are easily handled, and the velocity discontinuities across interfaces can be represented accurately. Moreover, the ray-intersection with the interfaces can be accomplished very efficiently because only finite number of local computations is involved. The proposed method has many potential applications such as ray-based seismic modeling, ray-based seismic migration and tomographic migration velocity analysis, etc.

Introduction
To realize reflection and refraction at interface in the ray tracing procedure, a detailed velocity-depth model which includes the structural surfaces and velocity variation should be loaded. The common used model representation is as follows. The interfaces are specified with smooth triangular meshes or some analytic expressions; Blocks are defined as the areas enclosed by smooth interfaces. The distribution of the velocity in each block may be specified by analytic expressions or just be taken from the finely sampled Cartesian velocity grids (Červený et al at 1988; Bulant 2004). As it is difficult to maintain the internally consistent relationship between volumes and interfaces, Vinje et al (1999) present the concept of open model to consistent relationship between volumes and interfaces, Bulant (2004). As it is difficult to maintain the internally consistent relationship between volumes and interfaces, Vinje et al (1999) present the concept of open model to consistent relationship between volumes and interfaces, Bulant (2004). As it is difficult to maintain the internally consistent relationship between volumes and interfaces, Vinje et al (1999) present the concept of open model to consistent relationship between volumes and interfaces, Bulant (2004).

Representing velocity model and interfaces separately may greatly affect the efficiency of calculating intersection point between a ray and interfaces, even if certain intersection-detection techniques are applied (e.g. the bounding box, Octree searching). Moreover, to reduce the spurious numerical artifacts due to mismatch of interface and velocity, volume averaging of the material properties (e.g. Moczo et al. 2002) should be applied. However, accurately representing velocity discontinuity across interfaces is important to guarantee acceptable result (Peltries et al., 2010), and has the potential to improve the fidelity of ray-based modeling, and consequentially to bring improvements to the migrated image (Hobro et al., 2008; Lecomte et al., 2009).

Unstructured mesh has the ability to incorporate interface directly into velocity models, and to represent the velocity discontinuity across interface accurately. Moreover, when the ray moves forward, we only need to check the interfaces in the current grid cell for hitting. Several authors investigated various mesh-based velocity model representations for ray tracing (Wiggins et al, 1993; Stankovic and Albertin, 1995) and tomographic inversion (e.g., Cox and Verschuur, 2001, 2004; Bohm, 2000). Ruger et al. (2006) generated meshes of subsurface velocity structures from finely sampled uniform velocity grids without providing external additional constraints such as horizons and faults. They approximated the velocity discontinuities or the horizons by roughly refining the tetrahedral cells crossing discontinuities area.

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Implicit Structural Model Representation
We have extended level set method (Zhao et al., 2001; Osher and Fedkiw 2003) to non-manifold surface reconstruction and geological structural modeling. By constructing a signed distance field for each horizon with level set method and the horizon is represented by the zero isosurface of the field. All the signed distance fields have the common underlying mesh topology, which is called the computational mesh in the following.

We show how the surface-aware ray tracing is performed in a newly developed geometric model representation, the implicit model. The implicit model provides a convenient way to represent very complex structures and to determine the stratum that a given point locates at. It also greatly facilitates the ray-interfaces intersection calculation. Based on the implicit model, the velocity discontinuity across interfaces can be represented accurately.

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faults. On the other hand, conforming to the fault-network is a relaxed mesh constraint, which means the conforming Delaunay triangulation is feasible for practical implementation. In the case of without fault-network, the computational mesh could simply take the form of structured grid.

There are some attractive properties of the proposed implicit representation for ray tracing. The signed distance field, as a solution to level set equation, is typically continuous except along the faults; the gradient of the signed distance field is perpendicular to its isosurface, thus the unit normal vector to the reconstructed horizon can be directly obtained; Because the field value of a point denoting its signed distance to the corresponding reconstructed horizon, thus it is easy to determine the relationship between the point and the horizon, i.e., if the sign of the value is positive, the point must lie above the horizon; if the sign of the value is negative, the point must lie below the horizon; otherwise, the point must lie on the horizon; Moreover, implicit representation makes simple Boolean operations and more advanced constructive solid geometry (CSG) operations easy to apply, thus, if the proper macro topological relationships of the horizons are defined, complex geological model can be constructed by applying constructive solid geometry (CSG) operations.

Efficiently Determining the Stratum That a Given Point Locates at

For example, the structural model shown in Figure 1a consists of one fault and eight horizons \( h_i \) \((i=1, \ldots, 8)\), which further divide the model into seven stratums \( S_i \) \((i=1, \ldots, 7)\). There exists a salt intruding up into overlying younger strata. A binary tree like the one shown in Figure 1b can be constructed to describe the spatial relationships of the horizons. Each non-leaf node represents a stratigraphic horizon, and each leaf node represents a stratum divided by the horizons. The non-leaf node always divides the spatial regions into two parts which are represented by its two sub-trees (the left sub-tree represents the region above the horizon, while the right sub-tree represents the below one).

Determining the stratum that a given point locates in is the fundamental and frequently called subroutine in the procedure of seismic ray tracing. By traversing the stratigraphic binary tree like the one shown in Figure 1b, the stratum that a given point is located in can be fast reached after a finite number of binary decisions. The signed distance field of each horizon provides a simple and efficient way for the binary decision, i.e., if the given point’s value of the signed distance field corresponding to the considered horizon is positive, the traversing will enter the left sub-tree which represents the region located above the considered horizon; otherwise, the traversing will enter the right sub-tree. Such an algorithm is called spatial decision algorithm below.

Figure 1: using binary tree to describe the spatial topology of the subsurfaces. a) internal view of a structural model, which consists of one fault and eight horizons \( h_i \) \((i=1, \ldots, 8)\) that further divide the model into seven stratums \( S_i \) \((i=1, \ldots, 7)\). Interface \( h_7 \) is the top boundary of the salt \( S_7 \), and \( h_8 \) is the last horizon. b) Each non-leaf node represents a stratigraphic horizon, and each leaf node represents a stratum divided by the horizons. \( S_0 \) and \( S_8 \) represent the regions that are above the first horizon and below the last horizon respectively. The non-leaf node always divides the spatial regions into two parts which are represented by its two sub-trees (the left sub-tree represents the region above the horizon, while the right sub-tree represents the below one).

Accurately Representing Velocity Discontinuities

Because the generated implicit horizon usually cannot be represented by the facets from tetrahedral mesh, thus, if we directly assign the velocity to the vertices of the tetrahedral mesh, we would get a misleading velocity with simple interpolation for the point lying in a tetrahedron across horizons (i.e., the velocity discontinuity) and make ray tracing inaccurate.

Different from the conventional methods dealing with velocity discontinuity, e.g., subdividing the related tetrahedral cells (Ruger et al. 2006), volume averaging of the material properties (Moczo et al. 2002; Bargteil et al. 2007), we solve this problem by duplication of tetrahedral cells across interface. Specifically, two replicas of this kind of tetrahedral cells were used during the velocity modeling...
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process. One replica is deemed to be located logically in the upper stratum, and the other one is logically located in the lower stratum. For any point located in the vicinity of horizons, its velocity is always obtained by linear interpolating the replica which belongs to the same stratum as the point, and thus the discontinuity of velocity is accurately represented.

Let's consider the replication strategy from a different perspective. That is, for each stratum, we need to find out all tetrahedral cells that intersect the stratum (i.e., each of the qualified tetrahedral cells either intersects the boundaries of the stratum, or is fully contained in the stratum.) Thus velocity on vertex of those tetrahedral cells can be assigned according to the velocity distribution principle of the stratum. In fact, determining whether a tetrahedral cell intersects the stratum can be efficiently accomplished by traversing from the leaf node which represents the stratum to the root node of the stratigraphic binary tree, i.e., unless the values on the four vertices, which are obtained from the signed distance field corresponding to a intermediate tree node in the traversing process, indicate that the tetrahedral cell is completely outside the stratum, the tetrahedral cell is a qualified cell.

Intersection of Ray Step with Horizons

With the kinematic ray tracing equations, the ray path is computed step by step using linear segment increments. For the sake of accuracy, as a mesh-based velocity model, it is better to confine each ray step in one mesh cell.

Representing horizon with signed distance field greatly facilitates with the implementation of ray-interface intersection. For a given horizon (i.e., the signed distance field), the value on either endpoint of ray step can be computed by linear interpolation. It is obvious that only when those two values have different signs, can the ray step intersect with the horizon. Furthermore, the intersection point can be directly obtained with the appropriate position along the ray step by linearly interpolating those two scalar values.

Determining the Stratum the Refracted Ray Entered

If the actual end point of the ray step is from a horizon/fault, the reflection ray will still begin at the current stratum but with different ray direction. However, the refracted ray may begin the new ray step in another stratum.

If the actual end point of the ray step is from a fault, it means the end point is actually located on a tetrahedral cell's facet lying on a fault. Thus, the refracted ray will enter into the opposite tetrahedral cell of the fault from the current tetrahedral cell. Therefore, we can use the spatial decision algorithm on the newly entered tetrahedral cell to find out the stratum that the new starting point belongs to.

If the actual end point of the ray step is from a horizon, we can also use the spatial decision algorithm to find out the stratum that the new starting point belongs to. Because the new starting point is actually lying on a horizon, we need a slight modification to the spatial decision algorithm, i.e., when traversing reaches the non-leaf node which represents the current intersected horizon, we use a definite indication to decide the sub-tree that it should enter into.

If the new starting point is located at a different stratum, the new starting point will be assigned with the velocity interpolated using the corresponding tetrahedral replica belonging to the newly entered stratum.

Examples

For the subsurface shown in Figure 2a, with the seismic interpretation data, we represented each horizon with a signed distance field by using the level set method, and the underlying computational mesh was generated by Delaunay tetrahedralization only conforming to the fault. The stratigraphic binary tree was defined as shown in Figure 1b.

The easiest way of defining an initial velocity model (e.g., for the inversion) is to specify the velocities for each stratum with the velocity gradient method. Thus, for each stratum, the vertices of tetrahedral cells that intersect the stratum are assigned with the value computed by the constant gradient velocity formula: $v(P)=v_0(P)-g\cdot h(P)$ ($v(P)$ is the velocity at a certain tetrahedral vertex $P$; $v_0(P)$ is the base velocity at the bottom interface of the stratum; $g$ is the magnitude of velocity gradient specified in the stratum; $h(P)$ is the normal distance of point $P$ to the bottom interface of the stratum). This velocity calculation method can greatly benefit from the signed distance field of the bottom interface because the value of $h(P)$ is usually equal to the value of $P$ from the signed distance field. Moreover, marching from $P$ to the bottom interface along the negative gradient direction of the signed distance field, the base velocity $v_0(P)$ can be obtained. In this example, a constant velocity 4.5 km/s was used in the salt. Above the salt, a constant velocity gradient of -0.1 km/s/km was used in each stratum, and the bottom interface of each stratum was assigned with a constant value, varying from 3.5 km/s to 2.3 km/s from the bottom up.

To verify the velocity field thus built, we create a finely sampled velocity profile with value at each grid point obtained by interpolating the tetrahedral replica that contains the grid point and belongs to the same stratum as the grid point. The result demonstrates that the velocity discontinuities across interface are accurately represented.
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Moreover, because the value of \( h(P) \) is directly obtained from \( P \)'s value of the signed distance field corresponding to the bottom interface of the stratum, thus the constructed velocity field in each stratum brought out a transverse media with tilted axis of symmetry.

Figure 2: ray tracing on the implicit model representation. (a) representing each horizon with a signed distance field, zero isosurface is extracted as triangular meshes from the field for visualization. (b) A velocity profile sampled from the constructed velocity field demonstrates that the velocity discontinuities across interface are accurately represented. (c) The propagation of ray paths initially launched in a profile (refraction only). (d) The propagation of ray paths initially launched in a profile (permission for reflection on the boundary of salt). (e) Wavefront ray tracing in a cone region. (f) Using dynamic ray tracing for calculating Gaussian beam of a ray. Plot the real part of Gaussian beam on a profile that almost contains the ray path.

Based on such an implicit structural model combined with complicated velocity property, because all operations related to ray intersection with interfaces are actually local computations, and the velocity discontinuity across interfaces are accurately represented, ray tracing was efficiently implemented as illustrated in (Figure 2c-f).

Conclusions

The significant capabilities of the proposed implicit-model based ray tracing method mainly lie in two aspects. First, complex geological interfaces and the velocity discontinuities across interfaces can be represented accurately; second, the reflection and refraction in the ray tracing can be implemented very fast and robustly with the implicit representation of model. Thus, the proposed method has many potential applications such as ray-based seismic modeling, ray-based seismic migration and tomographic migration velocity analysis, etc.

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EDITED REFERENCES

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