Orthogonal dreamlet decomposition and its application to seismic imaging

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Summary

Dreamlets have localizations in both time and space. In this paper, we present the orthogonal dreamlet atoms generated by the tensor products of local cosine basis, namely applying the local cosine transforms to both spatial and temporal decompositions. These dreamlet atoms form an orthogonal basis with good localization in time-frequency and space-wavenumber. Both the wavefield and the one-way propagator are decomposed by the orthogonal dreamlets and the wavefield is extrapolated directly in the dreamlet domain. Since the space-time records obtained from the acquisition system are real serials, the wave propagation in the dreamlet domain using the multi-dimensional local cosine basis will be kept in real number calculation through the process. This outstanding feature can save computer memory and accelerate the propagation/imaging process significantly. We present the formulation of the wavefield and propagator decomposition using the orthogonal dreamlets. Its applications to a 5-layer model and SEG/EAGE A-A’ salt model demonstrate the validity of the formulation and good imaging quality of this prestack depth migration approach.

Introduction

One interesting and important approach in seismic data processing is to study seismic data compression and migration operator compression using wavelet transforms and then processing the compressed data directly in the wavelet domain. The key is selecting suitable basis to achieve maximum compression while still preserving useful information. Due to the properties of the seismic data, the complete space-time localized decomposition method can achieve high compression ratio, such as the 2-dimensional local cosine/sine transform (Wang, Wu 1998, 2000) and the curvelet transform (Candès & Donoho., 2002; Candès & Demanet, 2005). Working directly on the compressed seismic data leads to the localized propagators. Many efforts have been made in the past to study this method (Steinberg, 1993; Steinberg and McCoy, 1993; Wu et al., 2000; Wu and Chen, 2002; Chen et al., 2006; Wu et al., 2008). However, these local propagators have only space-wavenumber localization, without time-frequency localization. Recently, curvelet transforms have been applied to wave propagation and seismic imaging using a map migration method (Douma and De Hoop, 2007; Chauris and Nguyen, 2008). The dreamlet method further localizes the frequency domain beamlet method to a local time-space domain wave propagation using time-space wavelets (Wu et al., 2008, 2009). In the above study the $t - \omega$ localization adopts the local exponential frames (Auscher, 1994; Mao et al., 2007), which have redundancy 2. In this work, we apply the orthogonal local cosine basis on the time-space localization to construct the orthogonal dreamlets. This not only reduces the amount of the dreamlet coefficients and propagator size but also changes the complex data type to real. All the propagation processes, migration and imaging are implemented in the real data domain, which is an outstanding feature of the orthogonal dreamlet method.

The local cosine basis is introduced and the dreamlet propagator, based on the tensor product of two one-dimensional LCB, is presented. As numerical examples, a 5-layer model and SEG/EAGE salt model are used to demonstrate the validity and imaging quality of this approach.

Local Cosine Bases and orthogonal dreamlet decomposition

In the space domain, the local cosine basis element can be characterized by position $\mathbf{x}_n$, the interval (the nominal length of the window) $L_m = \mathbf{x}_n - \mathbf{x}_i$, and wavenumber index $m$ as follows ($m = 0, \ldots, M-1, M$ denotes the total sample points of the interval)

$$b_{\omega m}(x) = \frac{1}{\sqrt{L_m}} B_{\omega m}(x) \cos \left( \pi m \frac{x - \mathbf{x}_i}{L_m} \right)$$

Where $B_{\omega m}(x)$ is a bell function which is smooth and supported in the compact interval $[\mathbf{x}_i - \varepsilon, \mathbf{x}_i + \varepsilon']$, where $\varepsilon, \varepsilon'$ as the left and right overlapping radius.

Similarly in time domain, the local cosine basis can be characterized by time location $t_j$, time interval $T_j = t_{j+1} - t_j$, and local frequency index $i$ as follows

$$b_{\omega j}(t) = \frac{1}{\sqrt{T_j}} B_{\omega j}(t) \cos \left( \pi i \frac{t - \mathbf{t}_j}{T_j} \right)$$

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The orthogonal dreamlet basis can be generated by the tensor products of \( b_\omega(x) \) and \( b_\omega(x) \).

\[
d_{\omega,t} \triangleq d_{\omega,t}(t,x) = b_\omega(t)b_\omega(x)
\]

\[
= \left[ \frac{2}{T} \right]^{2} B(t)B(x) \cos\left(\frac{\omega}{2} (t-t)\right) \cos\left(\frac{\omega}{2} (x-x)\right)
\]

(3)

where \( \omega_n = \pi(m+1)/L_x \), \( \omega_k = \pi(i+1)/T \). The nominal supports of the dreamlet basis are the Cartesian product rectangles of the nominal supports of the time and space.

A waveform in time-space domain can be represented as

\[
u(t,x) = \sum_{m,n} \sum_{i,j} \hat{u}_{mj} d_{\omega,y}^{m,j}(t,x) = \sum_{m,n} \sum_{i,j} u_{ij} d_{\omega,y}^{m,j}(t,x)
\]

(4)

with \( \mu = (t_0, \omega_0, x_0, \omega_0) \).

As a numerical example, we decompose the 175th shot profile from SEG/EAGE A-A’ salt model (Figure (1a)) using the orthogonal dreamlets. Both the nominal length in time and space are 16 sampling points. The dreamlet coefficients are shown in Figure (1b).

(a) (b)

Figure (1) The time space seismogram (a) and its orthogonal dreamlet decomposition coefficients (b).

**One-way propagator in the dreamlet domain**

The evolution of dreamlets must observe the wave equation. After propagation, a dreamlet is no longer a dreamlet, and is spreading into other cells in the localized phase-space. The redecomposition of the distorted dreamlet into new dreamlets forms the propagator matrix elements:

\[
\mathcal{P}_p^\omega = \left\{ \hat{d}_p \right\} \mathcal{P} \left\{ \hat{d}_p \right\}
\]

(5)

\( \mathcal{P} \) stands for the one-way propagator.

In the following, we use the analytic solution of the wave equation in the frequency-wavenumber domain in a homogeneous background for the derivation of dreamlet propagator in the background media under the local perturbation theory.

\[
\mathcal{P}_p^\omega = \text{real} \left\{ \frac{1}{2\pi} \int d\omega \overline{b_\omega^*(\omega) b_\omega(\omega)} \right\}
\]

(6)

Where \( b_\omega(\omega) \) is the Fourier transform of \( b_\omega(t) \) and

\[
\mathcal{P}_p^\omega = \frac{1}{2\pi} \int d\omega \overline{b_\omega^*(\omega) b_\omega(\omega)} e^{i(\omega \omega - \omega_0 \omega - \omega_0 \omega)}
\]

(7)

is the beamlet propagator. \( \omega = \omega_0 \omega - \omega_0^2 \) is the vertical wavenumber and \( p \) is the horizontal slowness.

We keep the bell shape the same for all time windows, we have

\[
b_\omega(\omega) = \int b_\omega(t) e^{\omega dt}
\]

(8)

\[
\mathcal{P}_p^\omega = \frac{1}{2\pi} \int d\omega \overline{b_\omega^*(\omega) b_\omega(\omega)} e^{i(\omega \omega - \omega_0 \omega - \omega_0 \omega)}
\]

and \( b_\omega(\omega) \) is the time window at \( T = 0 \).

After some simple derivation, equation (6) gives

\[
\mathcal{P}_p^\omega = \text{real} \left\{ \frac{1}{2\pi} \int d\omega \overline{b_\omega^*(\omega) b_\omega(\omega)} \right\}
\]

(9)

For a fixed \( (\omega, \omega) \) pair, the dreamlet background propagator is a matrix because after interaction with the media, one dreamlet coefficient will spread to other time and space cells. In order to visualize the coefficient distribution in the propagator matrix, we rearrange the sub-matrix and space cells. In order to visualize the coefficient distribution in the propagator matrix, we rearrange the sub-matrix and space cells. In order to visualize the coefficient distribution in the propagator matrix, we rearrange the sub-matrix and space cells. In order to visualize the coefficient distribution in the propagator matrix, we rearrange the sub-matrix and space cells.

**Some computational aspects**

The orthogonal dreamlet algorithm has some advantages over the LEF-LCB dreamlet method. First of all, it is orthogonal. The number of orthogonal dreamlet coefficients of the wavefield is half the number of coefficients as the LEF-LCB dreamlet coefficients. This feature can dramatically reduce the computation of wavefield decomposition and reconstruction. Another advantage of this method is the data type used in the data process, migration, imaging, etc. is kept real. Unlike the complex dreamlet atom implemented by the tensor product of LEF and LCB, the 2 dimensional LCB dreamlet atoms are real. As a result, the dreamlet coefficients of the wavefield at every depth are real, and therefore the dreamlet propagator also stays real.
Figure (2) Coefficient distribution in the propagator matrix of the Dreamlet propagator. In the figure, each small sub-matrix represents the cross-coupling between different \((\omega, \xi)\) components at that time-space location \((T, \tau)\), as can be seen in the three zoomed-in sub-matrices.
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As the formulation shown in equation (9), we just take the real part of the propagator deduced from the complex frequency-wavenumber domain one-way propagator. Numerical test in the following section will show the validity of this presumption. Preliminary numerical tests on the 5-layer model and SEG/EAGE salt model demonstrate that the orthogonal dreamlet method is at least 3 times faster than the LEF-LCB dreamlet method and uses 75 percent less memory.

Numerical examples

We use a 5-layer velocity model to investigate the dreamlet imaging method (shown in Figure 3). The synthetic data consists of 201 shot gathers with 561 receivers per shot. The lateral and vertical sampling intervals are 25 and 10 m respectively. The shot interval is 50 m, and the largest offset is 7 km. A ricker wavelet is used as the source time function and the dominant frequency is 17.5 Hz. The time sampling interval is 8 milliseconds and the total recording time is 3.5 seconds.

Figure (3) 5-layer velocity model.

Figure (4) shows the prestack depth migration results of the dreamlet propagator based on the velocity model in Figure 3. The image result shows that all four of the interfaces are imaged quite well and have similar image quality compared with the LEF-LCB dreamlet method.

Next, we apply the method to the 2D SEG/EAGE A-A’ salt model. The acquisition has 325 shots with left-hand-side receivers and the maximum number of receivers for one shot is 176. The original velocity model has 1200 samples in the horizontal extend and 150 samples in depth, both the sample intervals are 80 feet, as shown in Figure (5). The prestack depth migration result using dreamlet is shown in Figure (6). We apply the automatic gain control on the time series, that is to say, the recorded seismic data is multiplied by the square root of time. The number of the velocity in the background propagator is 50 and is equally spaced from the minimum to the maximum velocity in the SEG/EAGE model. From the image result we can see that the boundary of the salt body and sharp edges are clearly and correctly imaged. Most of the subsalt structures and the base straight line are also well imaged. This demonstrates the validity of theory and method of the orthogonal dreamlet migration.

Figure (5) 2D SEG/EAGE A-A’ salt model.

Figure (6) Dreamlet prestack depth migration result.

Conclusions

We presented the orthogonal dreamlet method using the tensor product of the Local Cosine Bases to achieve both the time and space localization. Numerical examples demonstrated the validity of the approach and showed its imaging ability on the complex salt model. One outstanding feature of this method is the real-number calculations throughout the whole propagation and imaging process, which saves significantly memory and computation time. We also analyzed the computational aspects of this method compared with the LEF-LCB dreamlet method. Both the theory and the numerical tests demonstrated the advantages of the orthogonal dreamlet algorithm over the LEF-LCB dreamlet method.

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