Dreamlet transform applied to seismic data compression and its effects on migration
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Summary
Local cosine/sine basis is a localized version of cosine/sine basis with a smooth window function. It has orthogonality and good time and frequency localization properties together with a fast algorithm. In this paper, we present a new method combining the main idea of local cosine/sine bases, multi-scale decomposition and the dispersion relation to form a multi-scale, multi-dimensional self-similar dreamlet transform. Meanwhile, a storage scheme based on the dreamlet decomposition is proposed by using zig-zag sequence. We apply this method to the SEG-EAGE salt model synthetic poststack data set for data compression. From the result, almost all the important features of the data set can be well kept, even in high compression ratio. Using the reconstructed data for migration, we can still obtain a high quality image of the structure. Through comparing with other decomposition and compression schemes, we believe that our scheme is more closely related to the physics of wavefield and has a better performance for seismic data compression and migration.

Introduction
For seismic data compression, the most important consideration is how to represent seismic signals efficiently (Donoho et al., 1995; Vassiliou & Wickerhauser, 1997; Averbuch et al., 2001; etc), that is to say, using few coefficients to faithfully represent the signals, and therefore preserve the useful information after maximally possible compression. It is easy to understand that compression effectiveness varies for different expansion bases. Recently curvelets as a multi-scale, anisotropic multi-dimensional transform were introduced (Candes & Guo, 2002; Candès & Donoho, 1999, 2004) and were quickly applied for seismic data processing (Hennenfent & Herrmann, 2006, Lin & Herrmann, 2007) and migration using a map migration method (Douma & Maarten, 2007; Chauris & Nguyen, 2008). Curvelets can build the local slopes information into the representation of the seismic data, and were proved to be an efficient and sparse decomposition of both seismic data and wave propagation operator (Candès & Demanet, 2005). However, curvelets are developed in the field of image processing and do not take into account of the physical features of wavefield. In the former work of Wang and Wu (1999), they applied the local cosine/sine bases to 2D seismic data compression, which is proved to be efficient and sparse to represent seismic data. Wu et al (2008, 2009) proposed a dreamlet time-space-localized transform for migration. In this paper, we will use local cosine bases and dispersion relation together with multi-scale idea to form a multi-scale dreamlet decomposition, which can be seen as an effective way to represent seismic data while keeping wave properties of the seismic data. It gives sparse coefficients matrix and the migration images using the compressed data sets can reserve most of the important features of the model. Meanwhile, by using the zig-zag sequence (such as JPEG), an efficient way of storing the compressed data is also discussed for further application.

Dreamlet (Drumbeat-beamlet) decomposition of wavefield
A dreamlet (drumbeat-beamlet) atom $d_{\tau \bar{\tau} \bar{m}}(x,t)$ can be formed by the tensor products of two 1D local harmonic atoms $h_{\tau \bar{m}}(x)g_{\tau \bar{m}}(t)$, where $h_{\tau \bar{m}}(x)$ refers to beamlet and $g_{\tau \bar{m}}(t)$ to drumbeat. Here the local harmonic atom refers to either local cosine/sine basis (Coifman & Meyer, 1991, Pascal, et al, 1992) or local exponential frame vector (Wu and Mao, 2007; Mao and Wu, 2007). Based on the wave theory, the time-space atom needs to satisfy the wave equation, and has a localized phase factor similar to $\exp[i(\tau \bar{\tau} - \bar{m} \theta)]$ resembling a localized beamlike wave-packet (or local plane-wave packet). So the $(\bar{\tau} - \bar{\theta})$ parameters and $(\tau - \bar{\theta})$ parameters are not independent but related with dispersion relation (or causality) which comes out from the wave equation. In this paper, we use the local cosine basis as the drumbeat atom, and it can be seen that since

$$\cos(\theta t) = \frac{e^{i\theta t} + e^{-i\theta t}}{2}$$

either of the components will satisfy the causality relation.

Thus, a wavefield can be represented as superposition of the phase space atoms (dreamlets)

$$u(x,t) = \sum_{\tau,\bar{\tau},\bar{m}} c_n(x,t) d_{\tau \bar{\tau} \bar{m}}(x,t) = d_{\tau \bar{\tau} \bar{m}}(x,t)$$

where a wavefield in time-space domain is decomposed into a representation in phase space of local time-frequency-space-wavenumber $(\tau - \bar{m}, \bar{\tau} - \bar{\theta})$.

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Self-similar Dreamlet transform

Local cosine/sine bases or local exponential bases were widely used in beamlet decomposition and propagation (Wu et al, 2000, 2008, Wu & Chen, 2006, Chen et al, 2006) with a fix nominal window length. Taking the multi-scale into consideration, we can find a more efficient representation of broadband seismic wave records. By using the dispersion relation, we can define that

\[
\sin \theta = \nu_{z} / \omega, \quad \cos \theta = \nu_{x} / \omega
\]

where \( \nu_{x} \) and \( \nu_{z} \) is the horizontal and vertical wavenumber, respectively. After using drumbeat for time-frequency decomposition along \( t \) direction, the nominal length of window \( L_{x} \) along \( x \) direction can be controlled by

\[
L_{x} = 2\lambda = 4\pi / k = 4\pi v / \omega
\]

in \( x \) direction with different \( \omega \). Thus, we can obtain a new dreamlet decomposition method. Since the wave packet will keep the same shape for different frequency band, we can also call these atoms as 'self-similar' dreamlets. It can preserve localizations in both space and time domain, which in the seismic data situation is actually local time-frequency-space-wavenumber \((T - \sigma, x - \xi)\) domain.

Figure 1 shows the 2D SEG-EAGE salt model poststack data as \( u(x,t) \) in our numerical test, which is amplitude balanced in time domain by vertical auto-gain control, and has zero-extension on both sides along space direction. The coefficients in \((T - \sigma, x - \xi)\) domain of dreamlet and self-similar dreamlet transform are shown in Figure 2 (a) and (b), respectively.

Data compression

Since the introduction of time-axis localization in dreamlet, the most important decomposition coefficients in dreamlet domain will tend to be concentrated in the upper-left corner of each \((T - \sigma, x - \xi)\) block, while very few of the most important coefficients may sparsely distribute in the other region. As a result, it can provide a sparse representation and also the possibility of obtaining high compression ratio after applying dreamlet and self-similar dreamlet decomposition to wavefield data.

Taking a block in the dreamlet decomposition coefficients matrix as example, shown in Figure 3, we can easily found that in the area circled by the dash line, coefficients tends to be concentrated together, while outside this region, coefficients distribute very sparsely. That is to say, by selecting a proper threshold, this sparse coefficient matrix can give a high compression ratio as well as a good reconstruction result.

Here, the compression ratio (CR) is defined as

\[
CR = \frac{\text{Uncompressed Data Size}}{\text{Compressed Coefficients Size}}
\]

Figure 1: Poststack data of 2D SEG-EAGE salt model.

Figure 2: Decomposition coefficients of dreamlet (a) and self-similar dreamlet transform (b) for the seismic data.

By defining the threshold as different percentages of the maximal coefficient of the coefficients matrix, we use dreamlet, self-similar dreamlet, Curvelet, and beamlet to compress the poststack data for 2D SEG-EAGE salt model.
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(shown in Figure 1). Figure 4 shows the comparison of compression ratios by different decomposition methods under different thresholds. As shown in the figure, the threshold varies from 0.1% to 5% of the maximal coefficient in each decomposition method, and the solid-circle line, dashed-square line, dash-dot-diamond line and dotted-asterisk line stand for the CR of self-similar dreamlet, dreamlet, curvelet and 1D beamlet method, respectively. We can see that the beamlet decomposition is the least efficient one, since there is no \( f-\omega \) localization along the \( \omega \)-axis. The dreamlet decomposition shows high compression ratios in different thresholds just like we discussed before. The error between reconstructed data using 3% of maximal coefficient of dreamlet compression (CR=73.6) as threshold and original data are shown in Figure 5 (a). Figure 5 (b) shows the error between reconstructed data using 3% of maximal coefficient of self-similar dreamlet compression (CR=78.7) as threshold and original data.

Figure 5: Error between reconstructed data using compressed coefficients (threshold is 3% of the maximal coefficient) and original data. (a), dreamlet compression, (b), self-similar dreamlet compression.

the given data. Therefore, to store the data is to store the compressed coefficients. If we simply store the position and the coefficient value, the storage used is at least twice as much as the number of coefficients. Therefore, we need some better ways to store the coefficients.

In each \((\tau-\varphi, x-\xi)\) block, the useful coefficients tend to be concentrated in the low wavenumber-frequency part and sparsely distribute in the other region (shown in Figure 3). Therefore, we can probability apply zig-zag sequence (an certain order for a \( n \times n \) matrix) for the storing of each block, which helps to facilitating further coding by placing low wavenumber-frequency coefficients (more likely nonzero part) before high wavenumber-frequency coefficients (more likely zero part).

Considering the sparsely distributed coefficients in high wavenumber-frequency region, after converting each \((\tau-\varphi, x-\xi)\) block into the zig-zag sequence, the number of a group zero coefficients between two nonzero coefficients could be embedded into the nonzero coefficient before it. Therefore, each block is stored into a much shorter 1D array, in which each atom is the nonzero coefficient value. By defining an ending value, each sequence can be able to be recognized when doing decompression. Therefore, the storage needed will be almost the same as the number of nonzero coefficients during compression. Figure 6 shows an example of 8x8 zig-

Figure 3: Dreamlet coefficients distribution in one \((\tau-\varphi, x-\xi)\) block.

Figure 4: Compression Ratio of different methods (SEG/EAGE salt model poststack data).

Storage of the compressed data

Data storage is also an important issue in application. Using the dreamlet transform and compression we discussed above, we can easily do compression and decompression of...
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zag sequence for one block and the storage scheme of each block.

Figure 6: zig-zag sequence for converting an 8x8 block. After embedding numbers of zero coefficients into nonzero coefficients, the compressed sequence is stored one after another; red squares stand for an ending value of a block.

Imaging with compressed data

The image quality using the different methods with same threshold is another important criterion to consider. In this section, the migration results of the compressed data will be shown to demonstrate the effect of compression. Different percentages of maximal coefficient in different decomposition methods are used as threshold for comparison.

For calibration, the depth migration result of the original data is shown in Figure 7 (a). Images of migration results under threshold of 1% and 3% of maximal coefficient are shown for both dreamlet and self-similar dreamlet decomposition. From the imaging results, we can see that the important structures are well imaged, even the faults under salt body. Therefore, we can conclude that the both dreamlet and self-similar dreamlet method can provide high CR as well as good migration result under the same condition, as we discussed above.

Conclusions

We develop a new method of data compression using dreamlet and self-similar dreamlet, which is faithful to the property of wavefield. This method shows great promise in terms of sparse and efficient representation of seismic data. Even in the case of high compression rate, the main feature of the migrated image is well preserved. We also discuss the storage scheme under dreamlet compressed method to save the compressed coefficients.

Figure 7: Depth migration results of different thresholds. (a), migration result using original data. (b), (c), migration result using dreamlet compression under threshold as 1%, 3% (CR=29.6, 73.6, respectively) of the maximal coefficient. (d), (e), migration result using self-similar dreamlet compression under threshold as 1%, 3% (CR=30.3, 78.7, respectively) of the maximal coefficient.

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