Image amplitude compensations: propagator amplitude recovery and acquisition aperture correction
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Summary

Many factors may influence the amplitude of migrated image. The acquisition aperture correction and propagator geometrical spreading correction (propagator amplitude recovery) have shown to be important for improving the migrated image. Using local-angle domain one-way wave equation based migration, we investigate the influence of these two factors on the image amplitude to better understand their relative importance in true-reflection/amplitude imaging. We utilize the localized WKBJ correction in general heterogeneous media to recover the true geometrical spreading of one-way wave propagator. We then apply this localized WKBJ solution to migration and compare its effect with that of the acquisition aperture correction on the image amplitude. Numerical examples indicate that the effect of the propagator WKBJ compensation on the image amplitude is less significant than that of the acquisition aperture correction for the migration with limited acquisition aperture in general media.

Introduction

Seismic migration operators are not the inverse of the forward modeling operator due to many factors, which we will discuss in the following. As a result, images produced by migration have artifacts and biased amplitudes. True-reflection/amplitude imaging tries to give not only correct location but also correct image amplitude (or AVA) of the reflectors or direct estimation of the media property.

When waves propagate through the anelastic heterogeneous Earth, propagation effects (e.g., geometric spreading, transmission loss, intrinsic attenuation) change the amplitude of the wavefield. Due to limited data acquisition aperture in media with complex overburden structures, seismic migration/imaging often provides strongly non-uniform image amplitude of subsurface structures. To obtain a true-reflection/amplitude image, we should take all of these effects into consideration. True-reflection imaging has been and remains a challenge in seismic processing. We focus on wave-equation based methods here.

Geometrical spreading is automatically handled if the wavefield extrapolation uses full-wave equations based methods (e.g., Deng and McMechan, 2007); however the full-wave equations based propagator is computationally much more expensive than the one-way propagator. Conventional one-way propagators do not treat geometric spreading correctly. The asymptotic true-amplitude one-way propagator can provide accurate amplitude in the sense of high-frequency approximation (e.g., Zhang, 1993). With this propagator, better image amplitude is obtained in some smoothly varying models using the single-shot division-type imaging condition (Zhang et al., 2003; 2005); however only geometric spreading is handled in that method.

The limited data acquisition aperture in media with complex overburden structures often results in strongly non-uniform dip-dependant illumination and distorted image amplitude of subsurface structures. Note that U/D shot-profile imaging conditions based on deconvolving the source side downgoing wave (D) from the receiver side upgoing wave (U) (e.g., Zhang et al., 2005; Deng and McMechan, 2007) do not take into account the influence of finite recording geometry. Wu et al. (2004) propose an amplitude correction method in the local angle domain for acquisition aperture effects which include the acquisition configuration and propagation path effects through complex overburdens. It can significantly improve the image amplitude in prestack depth migration.

The methods discussed above use the back-propagation integral based imaging condition. There is another type of potential true-reflection imaging method based on minimizing the misfit function using the adjoint operator in the Hessian matrix (Lailly 1983; Tarantola 1984). We will discuss it in a separate paper.

The acquisition aperture correction and propagator geometrical spreading correction have shown to be important for improving the migrated image. Here we compare their effects on the image amplitude to understand their roles in the true-reflection imaging. First, we summarize the theory of localized WKBJ solution for one-way propagator geometrical spreading correction in heterogeneous media. Then we give the acquisition-aperture correction formulas for different imaging conditions. Finally, we apply the local WKBJ corrected propagator to the migration and compare the influence of the propagator amplitude correction with that of the acquisition aperture correction on image amplitude.

WKBJ solution in the local wavenumber domain for one-way wave propagators

Most of the amplitude compensation schemes for one-way propagators are formulated and implemented in the space domain (e.g., Zhang et al., 2005). The localized WKBJ correction for general heterogeneous media (Wu and Cao,
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The WKBJ solution in smoothly varying \( v(z) \) media (e.g., Stolt and Benson, 1986) can be derived from energy flux conservation (see Wu and Cao, 2005),

\[
P_s \frac{P_i}{\cos \theta_i \rho_i \nu_i} = \frac{P_s}{\cos \theta \rho \nu} \sqrt{\frac{k_i v_i}{\cos \theta_i \rho_i \nu_i}},
\]

where \( P \) and \( \rho \) are pressure and media density respectively; \( \theta_i \) is the propagation angle with respect to the vertical direction and \( k_i \) is the vertical wavenumber. We can generalize the WKBJ solution for smoothly varying \( v(z) \) media to a transparent (or energy-conserving) propagator for general \( v(z) \) media. For \( v(z) \) media with discontinuities, the WKBJ solution 1 corresponds to a transparent boundary condition at each interface and the propagator becomes a transparent propagator, which neglects all of the scattered/reflected energy loss when the waves cross the boundary. The energy flow is thus continuous and conserved in both the smoothly varying media and the media with sharp boundaries. Along this line, the concept of transparent propagator can be extended to general heterogeneous media in the local angle/wavenumber domain (Wu & Cao, 2005). It can be called the localized WKBJ correction. It can be implemented using a beamlet propagator (Luo et al., 2005). According to the above global WKBJ correction 1, the localized WKBJ correction in general heterogeneous media can be symbolically written utilizing the beamlet \( b_{nm} \) propagator (e.g. Wu et al., 2000) as

\[
\frac{u(\mathbf{x}_{nm}, z, +\Delta z, \omega)}{u(\mathbf{x}_{nm}, z, \omega)} = \sqrt{\frac{\rho(\mathbf{x}_{nm}, z + \Delta z)}{\rho(\mathbf{x}_{nm}, z)} \frac{k_z(\mathbf{x}_{nm}, z)}{k_z(\mathbf{x}_{nm}, z + \Delta z)}},
\]

(2)

with

\[
k_z(\mathbf{x}_{nm}, z) = \sqrt{\frac{\omega^2}{v_i^2} \frac{1}{v_i^2}(\mathbf{x}_{nm}, z) - \xi^2}
\]

(3)
as the window-position dependent local vertical wavenumber at depth \( z \), where \( u(\mathbf{x}_{nm}, z, \omega) \) is the beamlet domain wavefield, \( v_i(\mathbf{x}_{nm}, z) \) is the reference velocity for window at \( \mathbf{x}_{nm} \), and \( \xi \) is the wavenumber. The calculated impulse responses in general heterogeneous media demonstrate the effect of the localized WKBJ correction on the wavefield amplitude (Luo et al., 2005).

**Acquisition aperture correction**

In this part, we give the acquisition-aperture correction formulas for the crosscorrelation and division type imaging conditions. For detailed derivation, please refer to Wu et al. (2004).

(1) **For crosscorrelation type imaging condition**

The imaging condition in the local angle domain (e.g., Wu and Chen, 2002; Chen et al., 2006) can be written as,

\[
L(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{\bar{\rho}}) = 2 \sum_{i} G_i(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{x}_i) A(\mathbf{x}_i, \mathbf{x})
\]

(4)

\[
\int_{\Delta x_i, \Delta z} d\mathbf{x}_s \frac{\partial G_i(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{x}_s)}{\partial z} D(\mathbf{x}_i, \mathbf{x}_s)
\]

where \( G_i(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{x}_s) \) is the incident wavefield in the local angle domain, the integral is the back-propagated wavefield in the local angle domain from the recorded data \( D(\mathbf{x}_i, \mathbf{x}_s) \), \( A(\mathbf{x}_i, \mathbf{x}) \) is the receiver aperture for a given source at \( \mathbf{x}_i \), \( \Delta x_i \), \( \Delta z \) stands for complex conjugate, and \( \mathbf{\bar{\theta}} \) and \( \mathbf{\bar{\rho}} \) are the source and receiving angles respectively. Then the image obtained at each imaging point is no longer a scalar but a matrix, called the local image matrix \( L(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{\bar{\rho}}) \).

The local image matrix is a distorted estimate of the local scattering matrix due to the acquisition aperture limitation and the propagation path effects. Local scattering matrix is the intrinsic property of the scattering medium and contains information of the local structure and elastic properties (Wu et al., 2004). It is independent of the acquisition system and free from propagation effects. The task of true reflection/amplitude imaging is to restore the true local scattering matrix from the distorted local image matrix by applying image amplitude corrections. Wu et al. (2004) proposed an amplitude correction method in the local angle domain for the acquisition aperture effect. The relevant amplitude correction factor matrix, \( F_{nm} \) for above imaging condition 4 is,

\[
|F_{nm}(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{\bar{\rho}})| = \frac{\sum_{i} |G_i(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{x}_i)|^2 \left\{ \int_{\Delta x_i, \Delta z} d\mathbf{x}_s |G_i(\mathbf{x}, \mathbf{\bar{\theta}}, \mathbf{x}_s)|^2 \right\}^{1/2}.
\]

(5)

The final acquisition aperture corrected image can be obtained by applying the amplitude correction factor to the migration image in the local angle domain.

(2) **For division type imaging condition**

The U/D shot-profile imaging conditions based on deconvolving the source side downgoing wave (D) from the receiver side upgoing wave (U) are used to seek a true-amplitude image (e.g., Zhang et al., 2005; Deng and McMechan, 2007). Although these imaging condition do not take into account the influence of finite recording geometry, they might give image amplitude more close to the reflection strength than the crosscorrelation type imaging conditions. We will use this imaging condition for the following single shot migration in a simple model. However, we should keep in mind that the division imaging condition is less stable than the crosscorrelation type imaging condition since the correction is done before the stack over the shots; therefore for general case we still use the crosscorrelation type imaging condition. For the single shot problem, the division imaging condition to obtain the scattering strength in local angle domain can be written as,
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\[ L(x_i, \theta_i, g_s) = 2 \left| G_i(x_i, \theta_i, g_s) \right|^2 \]
\[ \int_{d(x_i, x_s)} G_i(x_i, \theta_i, g_s) \, dg_s \] (6)

By similar derivation to the crosscorrelation type imaging condition, we can obtain the corresponding acquisition aperture correction factor \( F_a \) for imaging condition 6,
\[ F_a(x_i, \theta_i, g_s) = \left\{ \int_{d(x_i, x_s)} G_i(x_i, \theta_i, g_s) \, dg_s \right\}^{1/2} \] (7)

Comparison of the effects of propagator amplitude correction and acquisition aperture correction

In this part, we investigate the influence of the propagator geometrical spreading correction and the acquisition aperture correction on the image amplitude using numerical examples in a smoothly varying \( v(z) \) and a general heterogeneous medium to understand their roles in the true-reflection imaging.

(1) \( v(z) \) model

First, a simplified example is designed to illustrate the general idea. A point scatterer is put in a smoothly varying \( v(z) \) medium. A split-spread survey covers the surface and the source is right above the scatterer. The velocity model used is \( v(z) = 1.5 + 0.3z \) (km/s). Since there is only one incident angle \( \theta_i = 0^\circ \) for the example here, we investigate the image only in the receiving/scattering angle \( \theta_s \) domain at the scattering point \( x_s \), \( L(x_s, \theta_s, g_s) \). Figure 1 shows the image amplitude with different corrections for data with 6 km aperture. For the migration with the WKBJ-corrected one-way propagator, the image amplitude curve within is flat, which agrees with the theoretical prediction. Without the WKBJ correction, the image amplitude is smaller and decreases faster with the increase of the scattering angle. The image amplitude after the aperture correction significantly improves for large scattering angles compared with that before the aperture correction for the conventional propagator. The aperture correction has a stronger compensation effect on the image amplitude than the propagator WKBJ correction. The image with both the WKBJ and aperture corrections shows no noticeable difference from that with only the aperture correction.

(2) 2D SEG/EAGE salt model

Here we utilize the local cosine basis beamlet propagator with the localized WKBJ correction in migration to compare the influence of the above two corrections on the image amplitude for the 2D SEG/EAGE salt model (Aminzadeh et al., 1994; Aminzadeh et al., 1995) (Figure 2).

Post-stack migration is a wave back-propagation process; therefore it can directly show the effect of the WKBJ correction on the propagator amplitude. The post-stack migration image amplitude after the localized WKBJ correction is stronger throughout the model (Figure 3), especially for the steep faults in the sediment, the salt boundary, and the subsalt structures. The reason is that the WKBJ solution represents a transparent propagation, which neglects all the reflection/scattering loss during extrapolation. The side effect is that the WKBJ correction might increase the amplitude of the artifacts, especially within and below the salt dome here.

Similar to the post-stack case, the pre-stack migration image amplitude after the localized WKBJ correction with the crosscorrelation imaging condition is stronger throughout the
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model and the image has stronger artifacts too (Figure 4). To make the image more closely represent the reflection strength, we normalize the image with the source side illumination. This correction assumes the receiver aperture is infinite. The normalized images with/without the localized WKBJ correction give very similar amplitude (Figure 5). The small-offset acquisition (maximum offset = 14000 ft) and relatively steep dipping structures in the SEG/EAGE salt model may be the reasons for the relatively weak effect of the WKBJ correction on the image amplitude. The small-offset acquisition can only acquire small angle reflections from the steeply dipping reflectors. This makes the paths of the incident and reflected waves quite similar. The top layer of the model is homogeneous water. Therefore the effects of the WKBJ correction for the incident and reflected waves are similar, which makes the image amplitude improvement after the propagator correction less significant.

The image after the acquisition aperture correction (Figure 6a) improves significantly throughout the model compared with the image before the correction (Figure 4a). The images of the steep faults in the sediment are sharper and more continuous. For subsalt structures, the images along the steep structures and the baseline are more uniformly distributed after the aperture correction. The artifacts in the subsalt region caused by salt body related multiples are also reduced relatively. The imaging with both the localized WKBJ and acquisition aperture corrections (Figure 6b) gives a result of similar structural image to that with only the aperture correction (Figure 6a), but stronger artifacts in the subsalt area compared to the latter. These results show that the acquisition aperture correction has a stronger effect on the image amplitude for the 2D SEG/EAGE salt model. This agrees with the result in the above v(z) medium.

As for the efficiency of the acquisition aperture correction, the original implementation (Wu et al., 2004) is rather time-consuming since the wavefield is decomposed into the local angle domain using local slant stack (e.g., Xie and Wu, 2002) which is computationally demanding. We (Cao and Wu, 2008) proposed a faster implementation using beamlet decomposition. The new method produces results similar to those by the original local slant stack based method. However, it can be more than twice as fast as the local slant stack based method and only cost about twice the computation time of traditional one-way wave-equation based migration.

Conclusions

We investigate the influences of the acquisition aperture correction and one-way propagator geometrical spreading correction on the image amplitude utilizing the local angle domain one-way wave equation based migration. Through numerical examples, we come to the following conclusions. First, the WKBJ correction for the propagator amplitude shows some improvements on image amplitudes, but not significant, and the WKBJ correction might bring a side effect, amplifying the artifact. Second, the effect of the acquisition aperture correction on the image amplitude is more significant than that of the propagator WKBJ compensation for the migration with limited acquisition aperture in general media.

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Figure 4: Prestack images before (a) and after (b) the localized WKBJ correction. Here, dx=dx=80 feet.

Figure 5: Prestack images normalized with the source side illumination before (a) and after (b) the localized WKBJ correction. Here, dx=dx=80 feet.

Figure 6: Prestack images with the acquisition aperture correction before (a) and after (b) the localized WKBJ correction. Here, dx=dx=80 feet.