Imaging diffraction points using the local image matrix in prestack migration
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Summary

The energy angle-distribution in the local image matrix (LIM) for a planar reflector and for a discontinuous point are different with the former exhibiting a linear energy concentration along certain dip direction while the latter showing a scattered energy distribution. Therefore the cross-correlation value of the local image matrix between adjacent image points can be used to distinguish these two situations. The seismic images of these diffraction points may provide important information about geological discontinuities.

Introduction

Due to under- or overmigration, migration artifacts, which often manifest themselves as apparent faults or edges and are mistakenly interpreted as structural details. Diffraction points contain valuable information about the subsurface structures, such as faults, pinchouts, rough edges, fractures, channels, salt bodies, small-sized scatterers and any sudden changes of facies. They can be used for imaging, inversion and interpretation of geological discontinuities and providing clearer identifications of geological discontinuities.

The significance of diffracted waves has been recognized more than fifty years ago (Krey, 1952). The signal amplitude from diffraction points can be extracted from the seismic section to detect local heterogeneities. A correlation procedure can be used to enhance the amplitude of the seismic signal at the location of the diffractions on the common-diffraction-point section (D-section) (Landa et al., 1987, Landa and Keydar, 1998). However, the method needs the seismic survey data at different times for the same survey area. The difference between the reflected event from a plane specular reflector and that from a point diffractor can be used to separate specular reflections and diffraction events (Taner et al., 2006). Hence we can use plane-wave destruction filters to suppress specular events to get plane-wave sections of diffractions (Fomel, 2002). Several workflows are tested to enhance diffraction-like signals and to remove ordinary reflections (Bansal and Imhof, 2005). The result shows that the eigenvector filter is the most efficient one for both 2D and 3D data sets. The coherence of seismic data measured by the cross-correlation between each seismic trace and its neighboring traces has proven to be an effective method for imaging geological discontinuities. The seismic coherence makes a clear interpretation of subtle features which may not be readily apparent in the seismic data. The coherence algorithms have developed from using only three traces (Bahorich and Farmer, 1995) to multitrace coherence measurement which are based on the eigenstructure of the covariance matrix formed from the traces in the analysis cube (Gersztenkorn and Marfurt 1999, Marfurt et al., 2000).

Instead of using the global Fourier transform in the seismic imaging, Wu et al. (2000) used the Gabor-Daubechies frame (G-D frame) (Daubechies, 1990; 1992) and the local cosine bases (Wu et al., 2000; Wu and Chen, 2001; 2002a; 2002b) to decompose the wavefield locally and derived the corresponding local propagator in the beamlet domain. A beamlet is a windowed harmonics along spatial axes. In the case of the G-D frame, translated and modulated Gaussian window functions are used to construct frame atoms. The beamlet decomposition provides localizations in both space and wavenumber domains for the wavefield. By the beamlet imaging condition, we can obtain the local image matrix $L(\overrightarrow{g}_i, \overrightarrow{g}_s)$ for each image point during the migration process, where $\overrightarrow{g}_i$ and $\overrightarrow{g}_s$ are local incident and receiving angles, respectively.

Due to different energy distribution in the local image matrix for a diffraction point and a planar reflector, in this study, we apply the singular value decomposition (SVD) technique and the cross-correlation method to identify and image these diffraction points in the model. Two simple models and the SEG-EAGE salt model are used as examples to demonstrate the accessible of our approach.

Method

The energy distribution of local image matrix. The local image matrix is defined as the matrix of diffracting amplitude for incident-receiving angle pairs, hence it is the intrinsic property of the diffracting medium and is independent of the acquisition system and free from propagation effects, and it also contains the information of the local structure and the elastic properties revealed by the image experiments at a local heterogeneity point. For a planar reflector, most of the energy in local image matrix is distributed linearly along certain dip direction (Figure 1a), but for a diffraction point, the energy in the local image matrix scatters widely in the entire matrix (Figure 1b) because it does not have a well-defined normal direction.
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-80 -40 0 40 80
Receiving angle (degree)
-80 -40 0 40 80
Incident angle (degree)

Figure 1. Two local image matrices. (a) planar reflector and (b) diffraction point.

(a)                                         (b)

Figure 2. Two different representations of the local image matrix. (a) the LIM as a function of the incident angle \( i \) and receiver angle \( g \); (b) the LIM as a function of the dip \( \nu \) and the reflection angle \( r \), where \( \nu = (\bar{\theta} + \bar{\theta}_r) / 2 \) and \( r = (\bar{\theta}_s - \bar{\theta}_r) / 2 \).

Data processing flow. Recognizing that the energy of a planar reflector in the LIM distributes along a certain direction, we transform the LIM from the form of \( L(\bar{\theta}_s, \bar{\theta}_r) \) (Figure 2a) to the form of \( L(\nu, r) \) (Figure 2).

In the \( (\nu, r) \) representation of the LIM, the energy distribution along the same dip angle will become horizontal. The singular values of the matrix \( L(\nu, r) \) can be viewed as energy values along the set of dip directions. Hence, some neighboring image of planar reflectors will have similar singular values and the correlation coefficient between these sets of singular values should be large. Lack of this correlation indicates that the image point is a diffraction point. Therefore, we can use such a criterion to separate diffraction points from planar reflectors and finally get the image of diffraction points. The image amplitude of the diffraction point results from summing all the values in the corresponding LIM.

In summary, the flow for separating the energy of diffraction points from that of reflect waves is:

1. Retrieve the LIMs using the beamlet imaging technique and incorporate the acquisition aperture correction in the local angle domain;
2. Transform the local image matrix from representation \( L(\bar{\theta}_s, \bar{\theta}_r) \) to \( L(\nu, r) \);
3. Compute singular values of the LIM for each point using singular value decomposition;
4. Select a point as a reference point \( O \) (Figure 3) and compute the cross-correlation coefficient \( C_{OO} \) (and \( C_{OQ} \)) of sets of singular values between the reference point and neighboring points \( Q \) (and \( Q' \)) in all possible directions (Figure 3). These cross-correlation coefficients are to be compared with the auto-correlation coefficient \( C_{OO} \) of the reference point in the next step.
5. Make a judgment by formula (1). By setting a proper threshold \( \alpha \), if the left side is smaller than the right side, the reference point can be taken as a planar reflector, otherwise it will be taken as a diffraction point;
6. Take each point in original model as a reference point and do the same processes as described in steps (4) and (5) and then sum up all values of local image matrix of each diffraction point and take them as the final image amplitude.

\[
|C_{OQ} - C_{OO'}| \leq \alpha \times C_{OO} \quad \text{planar reflector}
\]
\[
|C_{OQ} - C_{OO'}| > \alpha \times C_{OO} \quad \text{diffraction point}
\]

Where \( 0 < \alpha < 0.3 \) and \( Q \) can be points A, B, C, D, ... (Figure 3).

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Examples

Model 1. We construct a model with two high velocity (4700 km/s) horizontal reflectors in a background velocity model, which has a constant vertical gradient (Figure 4). We generated the data using a finite difference method with a Ricker wavelet of dominant frequency 15Hz. We form images of the two reflectors using different frequency bands and window lengths for the local cosine decomposition (Figure 5, Table 1).

Figure 5(a) and (b) are two images of full waves after doing prestack beamlet migration by using the background velocity model 1. The images of diffraction points of the two types (dominant frequency \( f_a = 10 \)Hz for type A and \( f_a = 100 \)Hz for type B) are shown in Figure 5. The images of diffraction points of the two types (dominant frequency \( f_a = 10 \)Hz for type A and \( f_a = 100 \)Hz for type B) are shown in Figure 5.
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\( f_0 = 15\text{Hz} \) for type B) with different thresholds \( \alpha = 0.10 \) (Figure 5c) and \( \alpha = 0.12 \) (Figure 5d) for all directions respectively show that the method is effective.

Figure 4. The velocity model 1.

Figure 5. The images of full waves and diffraction points in model 1. (a), (c) from type A (dominant frequency \( f_0 = 10\text{Hz} \)) and (b), (d) from type B (\( f_0 = 15\text{Hz} \)) (Table 1).

Figure 6. The velocity model 2.

Table 1. The computational parameters of the three models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>dominant frequency</th>
<th>frequency bandwidth</th>
<th>window length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 2</td>
<td>A</td>
<td>10 Hz</td>
<td>1–20 Hz</td>
<td>810 m</td>
</tr>
<tr>
<td>SEG</td>
<td>B</td>
<td>15 Hz</td>
<td>1–30 Hz</td>
<td>510 m</td>
</tr>
<tr>
<td>Model</td>
<td>A</td>
<td>10 Hz</td>
<td>1–20 Hz</td>
<td>1585 m</td>
</tr>
<tr>
<td>Model</td>
<td>B</td>
<td>15 Hz</td>
<td>1–32 Hz</td>
<td>805 m</td>
</tr>
</tbody>
</table>

Model 2. This model is same with Model 1 except the two high velocity (4700 km/s) reflectors are dipping (Figure 6). The computational parameters are listed in Table 1 for type A and B, respectively. By using the same workflow as the one used for model 1, the images of diffraction points of the two types (type A and B) with different thresholds (\( \alpha = 0.10 \) for horizontal and vertical directions and \( \alpha = 0.15 \) for the rest of directions in Figure 7c, and
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$\alpha = 0.15$ for all directions in Figure 7d) demonstrate the validity of the method even for a finite dipping reflector. SEG-EAGE salt model. The acquisition system of this model consists 325 shots and each shot has 176 left-hand-side receivers. The LIM at each point is generated by using shot migration and a smoothed velocity model (Figure 8). The computational parameters are listed in Table 1. By Comparing the images of diffraction points ($\alpha = 0.15$ for horizontal and vertical directions and $\alpha = 0.20$ for the rest of directions in Figure 9c and $\alpha = 0.20$ for all directions in Figure 9d respectively) with those of full waves (Figure 9a, 9b), it is clear that most of the energy of reflect wave has been removed, though there is still some reflected energy that cannot be completely removed due to the roughness of some planes. Hence the boundary of the salt body can be imaged by only diffraction points and the image is sharper with broader frequency bandwidth (Figure 9c,9d).

Conclusions

This paper proposes a method to separating diffraction points from planar reflectors based on different energy distribution in the LIM. Through three numerical examples, we have demonstrated that the method is effective in obtaining images of diffraction points. Separating and imaging diffraction points from planar reflectors can provide valuable information about geological discontinuities, such as faults, pinchouts, rough edges, fractures, salt bodies and any sudden changes of facies.

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