Influence of Propagator and Acquisition Aperture on Image Amplitude
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Summary
We apply the local angle domain migration method using one-way wave equations to study the influence of different propagator (with/without WKBJ correction) and different acquisition aperture on the image amplitude. An example of a point scatterer is used to demonstrate the concept and methods in a $c(z)$ media. The results indicate that the propagator and acquisition aperture both influence the imaging amplitude, however aperture correction has much stronger effect on image amplitude than the WKBJ correction for migration with limited aperture acquisition.

Introduction
Traditional wave equation based migration can provide a reflector map consistent with the real velocity model, but provides unreliable amplitude information of the reflectors. True-amplitude imaging tries to give both the correct position and amplitude of the reflectors. The factors influencing the amplitude of the imaging include focusing and defocusing by heterogeneity, geometrical spreading, path absorption and path scattering loss, numerical dispersion and numerical anisotropy, propagator errors and acquisition aperture effect. Among these factors, the amplitude errors caused by one-way wave propagators have been studied extensively in recent years (e.g. Zhang et al., 2003; Zhang et al., 2004). In this paper we will compare this effect with the acquisition aperture effect to understand the nature and magnitudes of these two effects.

The one-way wave equation based propagators provide powerful and fast tools for forward modeling and migration, but the original one-way wave equations cannot provide accurate amplitude (Zhang et al., 2003). With the true-amplitude one-way wave equations, better image amplitude is got in common-shot migration (Zhang et al., 2003) and common-angle gathers (Zhang et al., 2004). Most true-amplitude propagators are formulated and implemented in the space-domain. Wu & Cao (2005) proposed a method of amplitude correction based on WKBJ solution in local angle-domain.

Due to the limited data acquisition aperture in reality, the inverse-propagated waves cannot completely recover the scattered wave field, which will influence the amplitude of the image. Wu et al. (2004) proposed an amplitude correction method in angle domain with acquisition aperture correction. Their numerical examples showed significant improvement in both the total strength images and the angle-dependent reflection amplitudes, which demonstrated the significance of aperture correction in true-amplitude imaging.

The true reflectivity (or scattering coefficient) depends on the angle, but usually there is no local angle information in the wave equation based migration methods. The recently developed methods, local plane wave analysis based on window Fourier Frame theory (Wu & Chen, 2002) or local slant stack (Xie & Wu, 2002), can decompose the wave field into localized beamlets carrying angle information, which make it possible to get the image in local angle domain.

In this paper, we will first briefly discuss the WKBJ correction in local angle domain. Then we apply different propagator to get the migration image and study the influence of the propagator and acquisition aperture on the image amplitude in scattering angle domain for a point scatterer in a smooth $c(z)$ media.

True amplitude one-way propagator

(1) WKBJ solution
Traditionally WKBJ solution is derived by asymptotic approximation for smoothly varying $c(z)$ media, where $c(z)$ is the wave speed at level $z$ (e.g. Morse and Feshbach, 1953; Aki and Richards, 1980; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has been also obtained by introducing an extra amplitude term based on transport equation of high-frequency asymptotics to the traditional one-way wave equations that satisfy only the eikonal equations (Zhang, 1993; Zhang et al., 2003). In a recent paper by Wu & Cao (2005), WKBJ solution is also derived from the conservation of power flux,

$$
\frac{P_2}{P_1} = \frac{\cos \theta_1 \rho_2 c_2}{\cos \theta_2 \rho_1 c_1} = \frac{\rho_2}{\rho_1} \sqrt{\frac{k_z(c_2)}{k_z(c_1)}},
$$

where $P, \rho, c, \theta, k_z$ is pressure, density, velocity, propagation angle and vertical wavenumber, respectively (see Figure 1). In this way, we can generalize the WKBJ solution for smooth $c(z)$ media to a transparent propagator (or energy-conservative Green’s function) for general heterogeneous $c(x, z)$ media. For $c(z)$ media with discontinuities, we introduce the concept the transparent boundary condition, which implies the neglect of all the scattering and reflection loss during the propagation. Therefore the energy flow is continuous in $z$-direction and conserved in both the slowly varying media or across sharp
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Figure 1: Diagram for WKBJ correction.

Figure 2: Diagram for extracting the amplitude along the radial direction for different emergency angles \( \theta \). The dots represent the receivers.

shows that the one-way propagator without WKBJ correction cannot give the correct amplitudes. The larger the emergency angle is, the larger the error of the amplitude is. For the emergency angle \( \theta = 75^\circ \), the amplitude got from one-way propagator without WKBJ correction is only about half of the true amplitude. However, with WKBJ correction, the one-way propagator can give almost the same amplitude as the full wave FD method does. Even for large angle (e.g., \( \theta = 75^\circ \)), their difference is very small. It should be noted that all of our above implementations include the contribution from evanescent waves, which are usually discarded in the literature. In our previous research (Wu & Cao, 2005), we found that the evanescent waves have a significant influence on the wave amplitude, especially for the near-field waves.

Imaging with one-way propagator

In previous part, we have demonstrated that the one-way propagator with WKBJ correction can give almost the same amplitude as the full wave FD method does even for large angles. In this part, we will use the one-way wave propagator to study the influence of the WKBJ correction and aperture correction on image amplitude. Because the true reflectivity (or scattering coefficient) depends on the angle, we use the imaging condition in local angle domain (Wu and Chen, 2002, 2004) to get the local image matrix (LIM) \( L(\mathbf{T}, \mathbf{\theta}, \mathbf{\beta}) \), which is distorted from the local scattering matrix (LSM) due to the acquisition aperture limitation and the propagation path effects. In \( L(\mathbf{T}, \mathbf{\theta}, \mathbf{\beta}) \), \( \mathbf{T} = (\mathbf{r}, \mathbf{z}) \) is the window position at depth \( \mathbf{z} \), \( \mathbf{\theta} \) and \( \mathbf{\beta} \) are the source and receiving angles, respectively (Wu, et al., 2004). For a single shot (point source) the imaging condition to obtain the scattering strength (for a single frequency) in local angle domain can be written as,

\[
L(\mathbf{T}, \mathbf{\theta}, \mathbf{\beta}) = 2 \frac{G(\mathbf{T}, \mathbf{\theta}, \mathbf{\beta}, \mathbf{z})}{G(\mathbf{T}, \mathbf{\theta}, \mathbf{\beta}, \mathbf{z})}
\]

Before WKBJ correction (\( v_0 = 3 \text{km/s}, \frac{dv}{dz} = 0.36 \))

After WKBJ correction (\( v_0 = 3 \text{km/s}, \frac{dv}{dz} = 0.36 \))

Figure 3: Curves of amplitude vs. distance from the source for \( \theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ \). The solid lines are the results from FD. The dashed lines are results from phase shift method. (a) is for the propagator without WKBJ correction and (b) is for the true-amplitude propagator with WKBJ correction.
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\[
\int_{c(z)} dx \frac{\partial G(x, \theta; x, x', \mu, \nu)}{\partial \theta} \nabla^2 \psi(x, x', \mu, \nu) u(x, x, \mu, \nu),
\]

where \(G\) is Green’s function used in the imaging process, which could be different from the Green’s function of forward modeling; “*” stands for complex conjugate; \(G(x, \theta; x, x', \mu, \nu)\) is the incident field in the local angle domain at the image point \(\vec{x}\); and the integral is a back propagation Rayleigh integral, \(A(x)\) is the spatial receiver aperture and \(u(x, x, \mu, \nu)\) is the recorded scattered waves at receiver \(x\) from the source at \(x\) on the surface.

To illustrate the influence of WKBJ and aperture correction on image amplitude, we design a simplified problem. A point scatterer is put in a smooth \(c(z)\) media and the data is recorded on surface, which theoretically should have the same scattering coefficient in all direction. To avoid the numerical errors during generating the dataset by directly putting the source on surface, which may cause the anisotropy of the scattering coefficient, we generate the dataset by firstly generating a data by a point source at the position of the point scatterer with full wave FD method and then give the data a time delay by which the waves travel from the real source to the imaginary source (see Figure 4). Because there is only one incident angle \(\theta^0\) for the example here, we can investigate the image at the scattering point \(x\) in receiving angle \(\theta\) domain, \(I(x, \theta, x, \theta)\), to compare it with the theoretical prediction.

Here, we take the velocity \(c(z)=1.5+0.36z\) (km/s) and the shot is in the center of the section and the receivers cover the surface in an aperture of 5000m on both sides with a 25m interval. The point scatterer is 2km below the source. The source time function is Ricker wavelet with dominant frequency 30Hz. The final input data for the migration is shown in Figure 5.

First, we will study the influence of WKBJ correction on the image amplitude. Figure 6 shows the image amplitude at the scattering point in the local receiving angle domain for the peak frequency. For the migration with true-amplitude propagator and with full 10km-aperture data (solid line in Figure 6a), the amplitude curve within ±30° is almost flat, which agrees well with the theoretical prediction. With the same aperture data but without WKBJ correction (dashed line in Figure 6a), the image is smaller than that from true-amplitude propagator. With smaller 6km aperture data, the results are similar (Figure 6b).

From Figure 6b, we can see that the acquisition aperture also has significant influence on the image amplitude. Here we will use the amplitude correction method in local angle domain with acquisition aperture correction proposed by Wu et al. (2004) to eliminate the aperture effect. Since we only have the image for one local incident angle (0°) here, we can do the aperture correction in the local receiving angle domain with amplitude correction factor.

We will see first the effect of aperture correction for the original one-way wave propagator without WKBJ correction. Figure 7 shows the image amplitude at the scattering point in receiving angle domain before and after aperture correction for the 6km-aperture data with receivers on both sides. The amplitude after aperture correction (dotted line) is greatly improved for the large scattering angle compared with that before aperture correction (dash-dot line). Figure 8 shows the similar results for the case with 3km-aperture on the right side.

Finally, we compare the influence of WKBJ correction and aperture correction. The dashed line and thin solid line in Figure 7 are the same as the dash-dot line and dotted line respectively except that they use the true-amplitude one-way wave propagator with WKBJ correction. Figure 7 shows that the result only with aperture correction improves the amplitude more than that only with WKBJ correction, and the result with both WKBJ and aperture correction gives the best amplitude distribution with scattering angle. These results demonstrate that aperture correction has stronger effect on image amplitude than the WKBJ correction for the example shown here. The results are similar for the one-side receiver aperture case (see Figure 8).

Conclusion

We apply the local angle domain migration method using one-way wave equations to study the influence of different propagator (with/without WKBJ correction) and different acquisition aperture in migration on the image amplitude in the local scattering angle domain for a point scattering problem in a \(c(z)\) media. The results show that the propagator and acquisition aperture both influence the imaging amplitude, however the result only with aperture correction improves the amplitude more than that only with...
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Figure 6: Comparison of image amplitude for true-amplitude and original one-way propagator. The solid lines are results with true-amplitude propagator, and the dashed lines are results without WKBJ correction. (a) Full 10km-aperture with receivers on both sides; (b) 6km-aperture with receivers on both sides (The thick solid line is the result for the full 10km-aperture with true-amplitude propagator as the reference curve). The amplitudes are normalized with the maximum among all images.

Figure 7: Comparison of image amplitude for one-way propagator before and after aperture correction for the 6km-aperture data with receivers on both sides. The dash-dot line is the result before aperture correction; the dotted line is result after aperture correction; and the thick solid line is the result for the full-10km aperture with true-amplitude one-way propagator as reference curve. The dashed line is the result with WKBJ correction only; and the solid line is with both WKBJ and aperture correction. The amplitude after aperture correction is normalized with its maximum. All the other amplitudes are normalized with their maximum.

Figure 8: Same as Figure 7 except that the data is 3km-aperture on the right side.

WKBJ correction, and the result with both WKBJ and aperture correction gives the best amplitude distribution with scattering angle. These results demonstrate that aperture correction has much stronger effect on image amplitude than the WKBJ correction for migration with limited acquisition aperture.

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EDITED REFERENCES

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