Fast modeling of 2D/3D elastic reflections using thin-slab method
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Summary

In this paper we first incorporate reflectivity method into elastic thin-slab propagator to make the thin-slab method more efficient for thick elastic layer with large perturbations. With reflectivity method, the background medium parameters can be changed during wave propagation. This can not only increase the computation speed but also reduce the accumulated errors generated in forward propagation. The capability of the method in handling large-angle scatterings and strong perturbation cases can be significantly enhanced.

For the 2D and 3D French models, we calculated synthetic seismograms using both finite-difference and the thin-slab methods, respectively. The results of two methods are in excellent agreements. For 2D, thin-slab method is 130 times faster than finite-difference method; for 3D, the factor is about 10.

Introduction

For one-way elastic wave propagators (Wu, 1994; Wu, 1996), forward marching of the wavefield and the calculation of reflection are based on the Wolf approximation or multiple-forescattering-single-backscattering (MFSB) approximation, which overcomes the limitations of the Born and Rayov approximations in long-range forward propagation and backscattering calculations. Further applying small-angle approximation to the elastic thin-slab propagators, a very computationally efficient propagator "complex screen" propagator was obtained. Using the complex screen propagator, Xie and Wu (1996; 2001) calculated the reflection synthetic seismograms for both 2D and 3D complex models. For small-angle reflections in 2D model, the complex screen results are in excellent agreement with the finite-difference method, while for 3D case, the complex screen results do not match finite-differences well. In complex screen approximation, the effect of scattering patterns was neglected, Wu and Wu (1998) retained the first-order approximation of the scattering patterns in the formulation of complex screen propagator. Numerical tests show that its wide-angle capability for modeling reflections is generally improved. Wu and Wu (1999) developed a method to directly implement the thin-slab propagator in which the small-angle approximation can be avoided. For 2D model, the wide-angle capability of the thin-slab propagator is significantly improved compared with the complex screen propagator. Wu and Wu (2002) applied the wide-angle thin-slab propagator to AVO modeling in complicated reservoir models including the effects of thin-beds and random heterogeneities.

For all one-way elastic propagators mentioned above, the background medium parameters used do not change even in homogeneous thick layer region. This can produce large accumulated errors for high-contrast thick layers. In this paper we first incorporate reflectivity method into thin-slab propagator so that the background medium parameters can be changed as soon as the waves enter a laterally homogeneous region. This can not only increase computation speed but also reduce accumulated errors in forward propagation. Using the thin-slab propagator incorporated with reflectivity method, we recalculated the reflection seismograms for 2D French model (French, 1974). It has shown a great improvement in computation speed and accuracy. Second, we applied the thin-slab propagator to 3D elastic wave modeling and compared the reflection seismograms with finite-difference results. They show excellent agreements even for large offset receivers.

Elastic thin-slab propagators

Dual-domain thin-slab propagators are implemented in mixed domains, i.e., wave propagates in wavenumber domain using a constant background velocity but interacts with heterogeneities in space domain. The key part of the method is the formulae describing the interaction between incident fields and heterogeneities at each step. It can be summarized by the following two equations (scattered displacement field, Wu and Wu, 2001):

\[
\begin{align*}
\mathbf{U}^p(K_T, z) &= \frac{i k_a^a}{2 \gamma_a^a} e^{\gamma_a^a \Delta z / 2} \Delta z \mathbf{k}_a \begin{bmatrix} \hat{k}_a \\ \end{bmatrix} \\
&\quad \int d^2 \chi_T e^{-i k_T \cdot \chi_T} \frac{\delta \rho(x)}{\rho} \left[ \eta^p_{\varphi} u^p_{\varphi}(x) + \eta^p_{\rho} u^p_{\rho}(x) \right] \\
&\quad - \int d^2 \chi_T e^{-i k_T \cdot \chi_T} \frac{\delta \lambda(x)}{\lambda + 2 \mu} \frac{1}{i k_a} \nabla \cdot \left[ \eta^p_{\varphi} u^p_{\varphi}(x) \right] \\
&\quad - (\hat{k}_a \hat{k}_a) : \int d^2 \chi_T e^{-i k_T \cdot \chi_T} \frac{\delta \mu(x)}{\lambda + 2 \mu} \frac{1}{i k_a} \left[ \eta^p_{\varphi} \varepsilon^p_{\varphi}(x) + \eta^p_{\rho} \varepsilon^p_{\rho}(x) \right] \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{U}^i(K_T, z) &= \frac{i k_b^b}{2 \gamma_b^b} e^{\gamma_b^b \Delta z / 2} \Delta z (1 - \hat{k}_b \hat{k}_b) \cdot \begin{bmatrix} 1 \\ \end{bmatrix} \\
&\quad \int d^2 \chi_T e^{-i k_T \cdot \chi_T} \frac{\delta \rho(x)}{\rho} \left[ \eta^i_{\varphi} u^i_{\varphi}(x) + \eta^i_{\rho} u^i_{\rho}(x) \right] \\
&\quad - \hat{k}_b \cdot \int d^2 \chi_T e^{-i k_T \cdot \chi_T} \frac{2 \delta \mu(x)}{\mu} \\
\end{align*}
\]
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\[ \frac{1}{ik_0} \left[ \eta^P \varepsilon'_a(x) + \eta^S \varepsilon'_b(x) \right] \]

where \( K_r \) is the transversal component of wavenumber and \( x_r \) is the transverse position in \( z \)-plane. \( x \) stands for \((x, z)\). \( \gamma_a = (k_0^2 - k_r^2)^{1/2} \) and \( \gamma_b = (k_0^2 - k_r^2)^{1/2} \) are vertical components of wavenumbers of scattered \( P \)- and \( S \)-waves. \( I \) is unit dyadic, and \( u_a' \) and \( u_b' \) are the incident displacement, divergence, and strain fields and can be obtained by performing appropriate operations to incident displacement fields. \( \lambda, \mu \) and \( \rho \) are Lamé constants and density, and \( \delta \lambda, \delta \mu \) and \( \delta \rho \) are the corresponding perturbations of the thin-slab. \( \tilde{k}_a \) and \( \tilde{k}_b \) are unit wavenumber vectors. \( \Delta z \) is thin-slab thickness. The modulation factors \( \eta^P, \eta^S_P \) and \( \eta^SS \) can be found in Wu and Wu (2001). Using equations (1-2), we can calculate forescattering at the thin-slab exit and backscattering at the thin-slab entrance and add the forescattering to incident field to form new incident for next thin-slab. Iteratively, we can propagate the initial incident field from the first thin-slab to the last one and obtain the transmitted field. Similarly, we can propagate the all local backscattering fields from where they are produced to the top of the model and obtain total reflections. The detail implementation of the thin-slab propagator can be found in Wu and Wu (2001).

Thin-slab propagator is a dual-domain (space-wavenumber domains) approach. In wavenumber domain, the wavefield can be expressed by a superposition of plane waves. Therefore, incorporating reflectivity approach into thin-slab propagator is straightforward. Suppose that \( u_a'(x) \) and \( u_b'(x) \) are incident \( P \) and \( S \)-waves. In wavenumber domain, they can be expressed by

\[ u_a(K_T) = \int d^2 \mathbf{x} e^{-iK_T \cdot x} u_a'(x_T), \]

\[ u_b(K_T) = \int d^2 \mathbf{x} e^{-iK_T \cdot x} u_b'(x_T). \]

Let \( \mathbf{e}_z \) be normal direction of the reflection plane. Using \( \tilde{k}_a \) and \( \tilde{k}_b \), the incident angles relative to \( \mathbf{e}_z \) and azimuthal angles relative to positive \( x \)-axis can be determined for \( P \) and \( S \)-waves. The spectra of incident fields can be decomposed into \( P \) and \( SV \)-waves and \( SH \)-components as

\[ A^P(K_T) = u_a(K_T) \cdot \tilde{k}_a \]

\[ A^{SV}(K_T) = u_b(K_T) \cdot (\tilde{k}_b - \mathbf{e}_z), \]

\[ A^{SH}(K_T) = u_b(K_T) \cdot (\tilde{k}_b + \mathbf{e}_z) \]

Combining the reflection and transmission/conversion coefficients at the interface, the reflected and transmit-
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Fig. 1: A 2-D heterogeneous model (French, 1974). Background medium is taken as $v_0=3.6\text{km/s}$, $\beta_0=2.08\text{km/s}$ and $\rho_0=2.2\text{g/cm}^3$. The intermediate layer (the dark structure) has a perturbation of -20\% for both P and S wave velocities.

Especially for the reflected/converted waves generated at the lower interface of the model, the use of reflectivity method makes those events to be well matched with those by finite-difference method.

Conclusions

An incorporation of the thin-slab method with reflectivity method has been developed for changing the used background medium parameters during wave propagation in thick elastic solid. 2D and 3D numerical examples show that the change of the background medium parameters can not only increase computation speed but also reduce the accumulated errors generated in forward propagation, resulting in accurate reflection seismograms. The numerical comparisons with finite-difference method have shown great potential of the thin-slab propagator method in fast modeling of seismic wave propagation in 3D complicated geologic environments.

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References


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Fig. 2: Comparison of synthetic seismograms calculated by finite-difference method (solid) and by the improved thin-slab method (dotted) for the model shown in Figure 1.

Fig. 3: 3D French model. Background medium is taken as $v_0=3.6\text{km/s}$, $\beta_0=2.08\text{km/s}$ and $\rho_0=2.2\text{g/cm}^3$. The intermediate layer (the grey structure) has a perturbation of -10\% for both P and S wave velocities.
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Fig. 4: Comparison of synthetic seismograms are calculated by finite-difference method (solid) and by the improved thin-slab method (dotted).