Synthesizing AVO responses in visco-elastic media using fast one-way elastic propagators
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Summary
In this paper we first incorporate quality factors ($Q_p$ and $Q_s$) into elastic thin-slab propagators, which is a fast one-way elastic propagator, using complex velocities to specify intrinsic attenuation effects for elastic wave modeling in visco-elastic media. The accuracy of the Q-incorporated method at handling attenuation and scattering/reflection due to heterogeneities in both elastic parameters and intrinsic attenuation is numerically investigated. For models with Q contrasts only, the attenuation and reflection can be accurately calculated at wide incident angles up to 70 degrees. For general models with both Q contrasts and density and velocity perturbations, the use of complex velocities can improve the wide-angle capacity of the methods at large-angle scatterings. Second, we apply the method to AVO modeling in heterogeneous visco-elastic media and to frequency-dependent amplitude analysis.

Introduction
The realistic geologic models may contain arbitrary spatial variations (heterogeneities) in compressional- and shear-wave quality factors, as well as in density and compressional- and shear-wave velocities. These heterogeneities may have different effects on the AVO responses produced for homogeneous elastic models. Therefore, efficiently fast modeling the effects of these heterogeneities is desirable.

Dual-domain one-way propagators implement wave propagation in arbitrary heterogeneous media in mixed domains (space-wavenumber domains) using the fast Fourier transform (FFT) (Wu, 1994; Wu, 1996; Wild and Hudson, 1998). They are very efficient in fast computation and internal memory saving compared with the full wave finite-difference and finite-element methods. These methods have drawn broad interest in seismic wavefield migration, reflection seismic modeling, modeling crustal wave propagation in heterogeneous waveguides, and others.

In this study we further extend the dual-domain one-way propagators to deal with intrinsic attenuation using complex velocities. For dual-domain methods, the effect of intrinsic attenuation of background medium having a constant Q can be treated straightforward. However, for a spatially varying intrinsic attenuation that can cause not only wavefield attenuation but also scattering, the feasibility and accuracy of the method need to be investigated. Several numerical examples are conducted. Using the extended method, we synthesize AVO responses in heterogeneous visco-elastic media as well as frequency-dependent reflections.

Incorporating Q factors into thin-slab propagators
In this section we briefly describe the calculation of the perturbations of medium parameters after introducing complex velocities. The detailed derivation of the thin-slab methods can be seen in (Wu, 1996). Heterogeneities in elastic thin-slab propagators are expressed by $\delta \rho(x)$, $\delta \lambda(x)$, and $\delta \mu(x)$ based on the following decomposition of elastic parameters:

$$\rho(x) = \rho_0 + \delta \rho(x),$$
$$\lambda(x) = \lambda_0 + \delta \lambda(x),$$
$$\mu(x) = \mu_0 + \delta \mu(x),$$

where $\rho$, $\lambda$, and $\mu$ are density and Lamé constants of the model, $\rho_0$, $\lambda_0$, and $\mu_0$ are the corresponding parameters of background medium. For elastic, isotropic media, Lamé constants $\lambda$, $\mu$, $\lambda_0$, and $\mu_0$ are related with elastic parameters by

$$\lambda = \rho_0 \alpha^2 - 2 \rho_0 \beta^2,$$
$$\lambda_0 = \rho_0 \alpha_0^2 - 2 \rho_0 \beta_0^2,$$

where $\alpha_0$ and $\beta_0$ are compressional- and shear-wave velocities of background medium. We introduce complex velocities by performing the following transforms:

$$\alpha \rightarrow \alpha (1 - i/2Q_p), \quad \beta \rightarrow \beta (1 - i/2Q_s),$$
$$\alpha_0 \rightarrow \alpha_0 (1 - i/2Q^0_p), \quad \beta_0 \rightarrow \beta_0 (1 - i/2Q^0_s),$$

where $Q^0_p$ and $Q^0_s$ are compressional- and shear-wave quality factors of background medium. $i$ is imaginary unit. Once all parameters in equations (6-7) are known, we can calculate the perturbations $\delta \lambda$ and $\delta \mu$ using equations (1-5). Now $\lambda_0$ and $\mu_0$ become complex. As a result, the reference wavenumbers of $P$- and $S$-waves also become complex. The heterogeneities of quality factors ($Q_p$ and $Q_s$) have been included in the complex $\delta \lambda$ and $\delta \mu$. With the above extension, the dual-domain thin-slab propagators can be used to model visco-elastic seismic responses. The procedure of implementing the dual-domain thin-slab propagators can be seen in Wu and Wu (2001).

Numerical examples
The designed models for our investigations are sand reservoirs bearing gas, oil and brine, respectively, which are
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The elastic parameters of these formations are taken from Simmons and Backus (1994) and listed in Table 1. Each formation is extended to contain the compressional- and shear-wave quality factors. In Figure 3, a 2-D random field with exponential correlation functions is used to perturb the velocity, density, and quality factor parameters of the model. The correlation lengths are 100 m in horizontal direction and 40 m in depth. The rms values of perturbations used are 4% for elastic parameters and 25% for quality factors. For synthesizing seismograms, source time function used is a Ricker wavelet with dominant frequency of 30 Hz. Only vertical component of displacement (P-P) is displayed in all examples.

Table 1. Elastic parameters

<table>
<thead>
<tr>
<th></th>
<th>$V_p$</th>
<th>$V_s$</th>
<th>$\rho$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>shale</td>
<td>3170</td>
<td>1668</td>
<td>2.36</td>
<td>0.31</td>
</tr>
<tr>
<td>gas</td>
<td>3350</td>
<td>2231</td>
<td>2.02</td>
<td>0.10</td>
</tr>
<tr>
<td>oil</td>
<td>3527</td>
<td>2131</td>
<td>2.22</td>
<td>0.21</td>
</tr>
<tr>
<td>brine</td>
<td>3749</td>
<td>2262</td>
<td>2.31</td>
<td>0.21</td>
</tr>
</tbody>
</table>

First we give two numerical examples to show the feasibility and accuracy of the thin-slab methods at handling attenuation and reflections due to Q contrast existence. Then, using the extended propagators, AVO responses in heterogeneous visco-elastic media and frequency-dependent reflections are simulated, respectively.

Accuracy in attenuation and reflection due to $Q$

For simplicity, the model used consists of two homogeneous visco-acoustic halfspaces with a flat interface. Plane waves are incident on the interface at arbitrary angles from the upper halfspace. Two receivers are located at the lower halfspace and spaced 200 m apart in the vertical direction. Using the thin-slab propagators, we synthesize the transmitted wavefields received at the two receivers. Then we estimate the attenuation and the corresponding Q values. Figures 1a, 1b, and 1c correspond to the models without Q-contrast, with negative, and positive Q-contrasts crossing the interface, respectively. For each of the models, the velocity perturbations change from 0 to 30%. From Figure 1a we see that for $\delta\alpha/\alpha = 0$ the intrinsic attenuation of the background medium is exactly calculated in all angles. For the models with perturbations in either velocity or Q, quality factors are accurately estimated for small to moderate incident angles. The wide-angle capacity of the thin-slab methods at handling attenuation is dependent on not only Q contrasts but also density and velocity perturbations.

Figure 2 shows comparison of reflection coefficients (P-P) calculated by the thin-slab method (2d-2f) and by the Zoeppritz equations (2a-2c) for two-layered models with different intrinsic attenuation parameters. The results of both methods are in good agreement for small to mild incident angles. For this example, the upper halfspace is elastic shale and the lower one is elastic or anelastic sands (oil). Solid line in Figure 2 is the reflection coefficient corresponding to elastic sand. Figures 2a and 2l show that $Q_s$ has an additional contribution to P-P reflection for overall incident angles. While for those $Q_s$s that are greater than 50, $Q_P$ mainly affects wavefield attenuation. The contribution to P-P reflection can be neglected. Figures 2b and 2e show that $Q_s$ has an additional contribution to P-P reflection only for large incident angles ($>20^\circ$). When $Q_s$ is greater than 25, no additional contribution is generated to P-P reflection. Figures 2c and 2f show a combined effect of $Q_P$ and $Q_s$ on P-P reflection. It is interesting when incident angle is beyond 20° the contributions to the reflection from $Q_P$ and $Q_s$ can cancel each other. We did the same investigation for gas and brine sands listed in Table 1 and obtained similar results. That implies that generally, $Q_P$ and $Q_s$ mainly affect wavefield attenuation and only low $Q_P$ ($<30$) can generate strong P-P reflection at small incident angle.

AVO responses in heterogeneous visco-elastic media

In practice, the geologic models may contain arbitrary
spatial variations in compressional- and shear-wave quality factors, as well as density and compressional- and shear-wave velocities (see the top panel in Figure 3). In this subsection we model the effects of these heterogeneities on AVO responses for three kinds of interfaces: (3a) shale/gas, (3b) shale/oil, and (3c) shale/brine. For each kind of interface, three different averaged Qs (i.e., \( Q = 50, 150, \infty \)) are given to shale and sands have quality factors of \( Q_p = Q_s = 10 \). The correlation lengths of the random field for perturbing Q and elastic parameters are the same. The rms values are 4% for elastic parameters and 23% for Q. Source and receiver array are located in shale and 1200 m far from the interface. The dotted lines in Figure 3 correspond to homogeneous cases with constant Qs. We see that intrinsic attenuation mainly affect the absolute reflected amplitudes and heterogeneities in both Q and elastic parameters affect local amplitude fluctuation with offset. In other words, the AVO responses of the target subsurface have been significantly deformed due to the heterogeneities.

**Effects of frequency-dependent Qs on local reflections**

Since the thin-slabs propagators extrapolate wavefield in frequency domain, the frequency dependent information about the amplitudes and phases (the spectra) of the wavefield can be preserved. Therefore, using the Q- incorporated propagators, we can analyze the reflected spectra not only in angle-domain but also in frequency domain. Figure 4a shows frequency-dependent Q models where \( Q^{-1} = f/(a + b f^2) \). \( f \) is frequency and \( a \) and \( b \) are two constants. Figure 4b shows the spectra of the reflections by an elastic oil sand versus frequency and offset. Figure 4c shows the reflected spectra for the model with frequency-dependent Q where \( a=500 \) and \( b=5 \). Figure 4d shows a similar result to Figure 4c but for different Q model where \( a=900 \) and \( b=1 \). We see that for the models with frequency-dependent Qs, the local reflections may produce different variations with offset, compared with those produced for the models with frequency-independent Qs.

**Conclusions**

Numerical examples show that the extended thin-slab methods can not only handle attenuation and scattering/reflection due to quality factor variations but also enhance the stability of the thin-slab methods at handling large-angle scatterings, therefore, it can be applied to more realistic geologic models for forward modeling, true-amplitude migration, and frequency dependent amplitude analysis etc. Moreover, it is often several orders of magnitude faster than the grid methods such as finite-difference method for these applications.

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**References**


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Fig. 3: The top panel shows reservoir model bearing gas, oil and brine, respectively. The formation is anelastic and heterogeneous. The lower three panels show AVOs for various kinds of interfaces: (a) shale/gas, (b) shale/oil, and (c) shale/brine. For each kind of interface, three different constant Qs (Qs=∞, 150, 50) are given to shale. The sand has constant Qs = Qs = 10. The correlation lengths of the random field for perturbing Q and elastic parameters are the same. The rms values are 4% for elastic parameters and 20% for Q.

Fig. 4: Reflected amplitude versus offset and frequency for a oil sand. (a) Frequency-dependent Qs, (b) the reflected spectra by an elastic oil sand, (c) the reflected spectra for the frequency-dependent model for a=500 and b=5, (d) same as (c) but for a=900 and b=1.