Summary

A 2D one-way wave equation based local cosine basis beamlet propagator was proposed and developed recently (Wu, Wang and Gao 2000, Wang and Wu 2002). It has been successfully applied to 2D poststack and prestack depth migration for media with large lateral velocity variations, such as the SEG-EAGE salt model, Marmousi model, and also real field data. In this abstract, the 2D algorithm is extended to the 3D case. We first formulate the 3D beamlet propagator, and then apply it to the 3D SEG-EAGE C3 exploding-reflector dataset and then to the AMO (Azimuthal Moveout) processed zero-offset dataset. The migrated images of both datasets, including subsalt structures, are improved. These encouraging results and its special features demonstrate the great potential of this approach for practical use in seismic exploration.

Introduction

Seismic migration using dual-domain wide-angle one-way wave propagators, especially the Generalized Screen Propagators (GSP), has been recognized as an efficient imaging method for complex, high-contrast media (Wu 1994, Xie and Wu 1998, Jin et al. 1999, Huang et al. 1999, De Hoop et al. 2000). In such a method, the medium is decomposed at each depth level into a background (reference) medium and perturbations to account for lateral velocity variations. However, in the approach, the background velocity is global. For strong contrast media, the perturbations can be very large, leading to difficulties in correctly propagating large-angle waves. To try to overcome this weakness, Steinberg (1993), Steinberg and Birman (1995) derived the phase-space propagators using the Windowed Fourier Transform (WFT) and a perturbation approach. Their studies represent the attempt to develop localized propagators instead of the traditional global propagator methods. Similarly, a windowed-screen method has been introduced (Wu and Jin 1997) to improve the global screen propagators. In that method, local background velocity and local perturbations are introduced through WFT. Since the perfect WFT reconstruction is formidably expensive, the method mostly relies on broadly overlapped windows and empirical interpolations, and therefore, is only applicable to the case where a few distinctive material boundaries exist.

Recently, a beamlet migration method based on local perturbation theory was proposed (Wu, Wang and Gao 2000). Then, it was successfully applied to synthetic datasets, for example, SEG-EAGE salt model and Marmousi model (Wang and Wu 2002, Wu and Chen 2001, Wu, Chen and Wang 2002a), and also to real datasets from the Gulf of Mexico and South Africa. Both the synthetic and real datasets are very complex with salt bodies complicating the subsurface.

To further improve the efficiency of the beamlet propagator, plane-wave beamlet migration was proposed and tested (Wu, Chen and Wang 2002a, b). For the SEG-EAGE salt model and Marmousi model, the computational efficiency was improved 5 to 10 times. Besides the benefit from the computational efficiency, by inspecting independent depth sections for different plane-wave illumination angles, the alignment in the z-p gathers can determine the validity of the velocity model, and furthermore, these z-p gathers can be used as an input for AVO or AVA analyses.

In this abstract, we extend the beamlet propagator to 3D. First, a 3D local free beamlet propagator is given semi-analytically; Then the 3D propagator is applied to the 3D SEG-EAGE C3 synthetic true exploding-reflector zero-offset dataset (generated by ARCO using a finite-difference method) and another dataset that is AMO processed zero-offset data provided by Biondo Biondi and the Stanford Exploration Project (Biondo et al. 1998, Aminzadeh et al. 1996). The migration results are encouraging. Also the computational speed is acceptable for practical use. These characteristics demonstrate the great potential of 3D beamlet propagators.

3D local cosine beamlet propagator

For 3D, the wave equation in heterogeneous media having a constant density but a spatially varying velocity can be written in frequency–space domain as

\[
\left[ \partial^2_x + \partial^2_y + \partial^2_z + \frac{\omega^2}{\gamma^2(x, y, z)} \right] u(x, y, z; \omega) = -f(\omega)\delta(x - x_0)\delta(y - y_0)
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where \( u(x, y, z; \omega) \) is the wavefield, \( \omega \) is the circular frequency, \( V \) is the velocity, and \( f(\omega) \) is the source function. The field at depth \( z \) can be decomposed into beamlets with windows along the \( x \)-axis and \( y \)-axis:

\[
\begin{align*}
  u(x, y, z) &= u_z(x, y) \\
  &= \sum_{n_1} \sum_{m_1} \sum_{n_2} \sum_{m_2} [u, b_{m_1 m_2 n_1 n_2}] b_{m_1 m_2 n_1 n_2} (x, y, z) \\
  &= \sum_{n_1} \sum_{m_1} \sum_{n_2} \sum_{m_2} \hat{u}_z (\bar{x}_{n_1}, \bar{y}_{n_2}, \bar{\xi}_{m_1}, \bar{\xi}_{m_2}) b_{m_1 m_2 n_1 n_2} (x, y, z)
\end{align*}
\]

After the introduction of local perturbations to reduce the strength of perturbation (similar to the manipulations in the 2D case, see Wang and Wu 2002) and the use of one-way approximation, the 3D wave propagation in beamlet domain can be derived as

\[
\begin{align*}
  \hat{u}_z + \Delta z (\bar{x}_{n_1}, \bar{y}_{n_2}, \bar{\xi}_{m_1}, \bar{\xi}_{m_2}) \\
  &= \sum_{n_1} \sum_{m_1} \sum_{n_2} \sum_{m_2} P(\bar{x}_{n_1}, \bar{y}_{n_2}, \bar{\xi}_{m_1}, \bar{\xi}_{m_2}) \cdot \hat{u}_z (\bar{x}_{n_1}, \bar{y}_{n_2}, \bar{\xi}_{m_1}, \bar{\xi}_{m_2})
\end{align*}
\]

where \( \hat{u}_z + \Delta z \) is the amplitude of local cosine beamlet at depth \( z + \Delta z \) and \( P(\cdot) \) is the 3D beamlet propagator, which handles the local free propagation and cross-coupling of beamlets.

Under the general screen approximation (Wu 1994, De Hoop et al. 2000), the square-root operator can be decomposed into background part and perturbation part. Similarly, the beamlet propagator can also be decomposed into a local free propagator and a phase-correction operator, which takes care of the lateral velocity variations. Specifically, if we use the uniform local cosine basis and keep all of the window shapes the same, in the 3D case, the local free beamlet propagator can be derived as follows

\[
P^{(0)} (\bar{x}_{n_1}, \bar{y}_{n_2}, \bar{\xi}_{m_1}, \bar{\xi}_{m_2}) \\
= \frac{1}{4L_{n_1} L_{n_2}} \cdot \frac{1}{(2\pi)^2} \int \int d\bar{\xi}_1 d\bar{\xi}_2 \\
\cdot \left[ e^{i\bar{\xi}_1 \cdot \bar{n}_{n_1}} \hat{b}_0 (\bar{\xi}_x + \bar{\xi}_j) + e^{-i\bar{\xi}_1 \cdot \bar{n}_{n_2}} \hat{b}_0 (\bar{\xi}_x - \bar{\xi}_j) \right] \\
\cdot \left[ e^{i\bar{\xi}_2 \cdot \bar{n}_{n_2}} \hat{b}_0 (\bar{\xi}_y + \bar{\xi}_j) + e^{-i\bar{\xi}_2 \cdot \bar{n}_{n_1}} \hat{b}_0 (\bar{\xi}_y - \bar{\xi}_j) \right]
\]

where \( \hat{b}_0 (\bar{\xi}_x) \), \( \hat{b}_0 (\bar{\xi}_y) \) is the Fourier transform of the bell (window) function \( b_0 (x) \), \( b_0 (y) \), respectively, located at \( (\bar{x}_0 = -L_{n_1} / 2, \bar{y}_0 = -L_{n_2} / 2) \). \( (L_{n_1}, L_{n_2}) \) is the nominal length of window \( (n_1, n_2) \) along \( x \)-axis and \( y \)-axis, respectively.

For other part of beamlet propagator, \( i.e. \), phase-correction operator, in practical applications, only the 1st order expansion, \( i.e. \), the split-step Fourier correction (Stoffa et al. 1990) is used.

Numerical tests

In this section, we present our results of the 3D local cosine beamlet migrations on two synthetic datasets. One is the SEG-EAGE C3 salt model true exploding-reflector poststack dataset, and the other is the SEG-EAGE C3 AMO processed zero-offset dataset.

Example One This exploding-reflector dataset is generated by ARCO using a finite difference algorithm. The size of this dataset is \( N_x=250, \ dx=40m, \ N_y=250, \ dy=40m, \ N_z=201, \ dz=20m \). The time sampling interval is 8ms with 501 samples per trace.

In our migration implementation, we select the frequency range from 0 to 30 Hz. Fig. 1(a) is the velocity model of a vertical section at line \( X=120 \). This model is very complex with 3 times velocity contrast between the salt body and surrounding media. This creates challenges to migration methods in correctly imaging the subsalt structures. Fig. 1(b) is the poststack migrated image of the model in Fig. 1(a) using 3D local cosine beamlet
propagator. From the result, we can see that, the salt top boundary and lower irregular boundary are imaged correctly, most of the subsalt structures are migrated well except for two steeply dipping events. The base line is also reconstructed and positioned correctly. Fig. 2(a) is the velocity model for a depth slice \(Z=40\), and Fig. 2(b) is the poststack migrated result by the 3D beamlet propagator. Generally speaking, the image quality is good.

**Example Two** This is an AMO processed zero-offset dataset. As opposed to exploding-reflector dataset in example one, here the data may include multi-paths. The grid size and spacing are as follows: \(N_x=400\), \(d_x=20m\), \(N_y=400\), \(d_y=20m\), \(N_z=201\), \(d_z=20m\). The time sampling interval is 8ms with 555 samples per trace. The useful migration frequencies are 0 ~ 28.5 Hz. Note that the grid spacing on the surface is 20m instead of 40m in example one.
3D beamlet propagator

Fig. 3(a) is the velocity model of a depth slice at $Z=40$, and (b) is the reconstructed image by 3D poststack beamlet migration. The high image quality can be seen clearly.

Conclusions

The local cosine beamlet propagator has been extended from 2D to 3D, and applied as a 3D poststack migration on two synthetic datasets. The good image quality is obtained. We also processed real datasets and the results are encouraging.

Local cosine beamlet has good localization characteristics in both space and wavenumber (direction). The use of local reference (background) velocity and local perturbations allows optimization of the local free beamlet propagators and easily handles the strong lateral velocity variations. Further improvements in computational efficiency include more effective sparsifying the propagator matrices and faster sparse-matrix-sparse-vector manipulation. These features demonstrate the great potential of this method for practical use in seismic exploration.

References


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