Propagation of a monopulse in layered and inhomogeneous media using an adaptive multiscale wavelet collocation method

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SUMMARY

In this paper, an adaptive multiscale wavelet collocation method [1] is applied to analyze the propagation of a monopulse in layered and inhomogeneous media. It is found that enough resolution can be obtained with this method even in solution’s regions where large gradient and high-frequency oscillations occur. Therefore, the new multiscale wavelet method can provide excellent algorithms for solving wave equations in layered and inhomogeneous media.

INTRODUCTION

Partial differential equations with discontinuous coefficients are usually difficult to obtain analytical solutions. Conventional numerical methods for partial differential equations basically fall into three classes: finite difference methods, finite element methods and spectral methods. If the solution of a partial differential equation is regular, any of above-mentioned three numerical techniques can be applied successfully. However, large gradient and high-frequency oscillations in solutions can be observed in many physical phenomena, such as the propagation of a monopulse in layered and inhomogeneous media. A characteristic feature of such phenomena is that the complex behavior occurs in a small region of space and possibly intermittent in time. This makes them particularly difficult to resolve numerically using the above-mentioned three methods. Accurate representation of the solution in regions where large gradient or high-frequency oscillations occur requires the implementation of adaptive finite difference or finite element methods [2-6]. In these methods an automatic error estimation step determines locally whether the current resolution of the numerical solution is sufficient or if a finer grid is necessary. The main difficulty of such adaptive methods is to find stable accurate difference operators at the interface between grids of possibly very different sizes.

Wavelet approximation is a new numerical concept which allows one to represent a function in terms of a set of basis functions, called wavelets, which have advantageous properties of localizations in both space and frequency domains [7] and vanishing moments [8]. Wavelet method seems to be an excellent candidate for solving PDEs which may have solutions varying dramatically both in space and time with large gradient and high-frequency oscillations [9].

SOLVING WAVE EQUATION USING AN ADAPTIVE MULTISCALE WAVELET COLLOCATION METHOD

SCALING FUNCTIONS AND WAVELETS

We define an interior scaling function \( \phi(x) \) and a boundary scaling function \( \phi_b(x) \) as follows

\[
\phi(x) = \frac{1}{4} \sum_{j=0}^{4} (-1)^j (x - j)^3
\]

\[
\phi_b(x) = \frac{3}{2} x^2 - \frac{11}{12} x^3 + \frac{3}{4} (x - 1)^3
\]

and \( \text{supp} \phi_i(x) = [0,3] \)

At the same time, we also define an interior wavelet...
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\( \psi(x) \) and a boundary wavelet \( \psi_b(x) \):

\[
\psi(x) = -\frac{3}{7} \phi(2x) + \frac{12}{7} \phi(2x-1) - \frac{3}{7} \phi(2x-2)
\]

\[
\psi_b(x) = \frac{24}{13} \phi_b(2x) - \frac{6}{13} \phi(2x)
\]  

A MULTISCALE WAVELET COLLOCATION SCHEME FOR WAVE EQUATION

Let \( E_{yJ}(x,t) \) represent the wavelet approximation of the solution \( E_y(x,t) \) of the following wave equation of TE incident plane wave from scale zero to scale \( J \) :

\[
\mu(x) \frac{\partial}{\partial \chi} \left( \frac{1}{\mu(x)} \frac{\partial}{\partial \chi} \right) E_y(x) - \mu(x) e(x) \frac{\partial^2 E_y}{\partial \chi^2} = 0
\]

\[
a < x < b, \quad t > 0
\]

\[
E_y(a,t) = g_0(t) \quad E_y(b,t) = g_1(t)
\]

\[
E_y(x,0) = f_0(x) \quad \frac{\partial E_y}{\partial \chi}(x,0) = f_1(x)
\]  

(5)

Adaptability

Adaptive scheme is accomplished in the following two ways:

1) Omitting collocation points

Let \( \varepsilon \geq 0 \) is a resolution threshold and all those wavelet coefficients \( \{d_{j,k}\}_{k=-1}^{2^J-2} \), which satisfy

\[
|d_{j,k}| \leq \varepsilon
\]  

(7)

will be omitted.

2) Increasing the finest resolution (or the finest scale)

If

\[
\max_{-1 \leq k \leq 2^J \text{, } n} \left| \frac{u^n}{u^{n'}-u^{-}} \right| > \varepsilon
\]  

(8)

where subscript \( n \) indicates the solution at the \( n \)-th time step, we should increase the finest resolution \( J \) of wavelet spaces to \( J' \) (\( J' > J \)).

NUMERICAL RESULTS AND DISCUSSION

We let TE pulse plane wave vertically incident to layered and inhomogeneous media (Figure 1). The first and third layer are air, and the second layer is an inhomogeneous medium. The relative permittivity, permeance of the second layer are \( \varepsilon_r = 2.56 + 10x, \mu_0 \), respectively. We see that the second layer is a linear profile inhomogeneous medium and the coefficient of wave equation (5) is discontinuous. Here, incident wave is selected as Gaussian pulse with unit amplitude and in Eq.(5):

\[
g_0 \equiv 0, \quad g_1 \equiv 0, \quad f_0 = \exp(-\alpha(x-\beta \chi)^2), \quad f_1 = 2\alpha c_0(x-\beta \chi) \exp(-\alpha(x-\beta \chi)^2)
\]
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\[ a = 0, \quad b = 2.5 \]
where \( \beta = 64, \quad \Delta x = 10^{-3} \text{m}, \quad \alpha = (4 / \beta \Delta x)^2, \]
and \( J = 6, \quad L = 20, \quad \Delta t = 10^{-12} \text{s}, \quad \varepsilon = 10^{-5}, \]
\[ c_0 = 3 \times 10^8 \text{m/s} \]

The numerical results are shown in Figure 2. From Figure 2, we see that at \( t = 5 \text{ ns} \), the incident wave just crossed the interface between air and the inhomogeneous medium (\( x = 1.5 \text{ m} \)). The reflected wave goes back and the transmitted wave enters the inhomogeneous medium. We only analyze the transmitted wave. Its amplitude is \( 0.38 \text{V/m} \), in the meantime, the amplitude of reflected wave is \(-0.62 \text{V/m}\). We can see that the behavior of the pulse is correct according to Ref. [10]. At \( t = 5 \text{ ns} \), the number of collocation points is 145. As time goes on, the transmitted pulse starts to be disturbed and to have a high-frequency oscillatory tail. The amplitude is getting smaller and smaller with the pulse width becoming wider and wider. At \( t = 6 \text{ ns}, 7 \text{ns}, 8 \text{ns} \), the number of collocation points is 206, 217, 235, respectively. From the numerical results, we can see that large gradient and high-frequency oscillations appear in the solution of wave equation of varying coefficients with discontinuities. However, enough resolution can be achieved in these regions by our adaptive multiscale (multiresolution) wavelet collocation method.

![Figure 1: Layered and inhomogenous (linear profile) media](image)

CONCLUSION

In this paper, an adaptive multiscale wavelet collocation method is applied to solve the wave equation in layered and inhomogeneous media. Enough resolution can be obtained in regions where large gradient and high-frequency oscillations appear. Hence, the new multiscale wavelet method is an efficient and competitive method for such problems.

REFERENCES


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Figure 2: Electric field in layered and inhomogeneous (linear profile) media. The evolution of the transmitted pulse in the inhomogeneous layer can be seen.