Modeling porous layers in elastic media with a hybrid method

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SUMMARY

A hybrid method which combines one-way approximation method and the propagator matrix method (the full-wave solution) is introduced and extended to including two-phase media based on the equivalent propagator matrix of two-phase media derived in this paper. It can handle more complicated structure than one-way approximation method and the propagator matrix method used alone. It can handle strong heterogeneous and/or two-phase media layers embedded in arbitrarily weak heterogeneous media. Numerical examples show good agreement between one-way approximation method and the matrix method for weak heterogeneous media. By means of the hybrid method, the reflection seismograms in elastic solid and the transmission fields through two-phase media with different porosi ve and permeability, are modeled, respectively.

INTRODUCTION

The recent series of studies show that one-way and one return approximation methods can be well applied to seismic modeling (Wu and Huang, 1992, 1995; Wu and Xie, 1993, 1995; Wu, 1994, 1996 Xie and Wu, 1995, 1996). Compared with the traditional till-wave methods such as finite element and fmite difference, one-way approximation method has the fast speed of computation and huge saving of internal memory, especially for large-sized 3-D modeling, migration/inversion problems. However, one-way approximation method is good only when the heterogeneity of the media is weak. Furthermore one-return approximation can only model the primary reflections and is incapable to model multiples.

In this paper we present a hybrid method which combines one-way approximation method and the propagator matrix method. The layer, which have strong contrast with the background medium but are homogeneous inside, will be treated with the matrix method. The weak, inhomogeneous area will be handled with one-way approximation method. In addition, an equivalent propagator matrix for two-phas emedia is also constructed in this paper. Using the equivalent matrix, we extend the hybrid. method to two-phase media. It aims at modeling the reflection and cross-well seismograms in complex elastic, media including porous layers.

PHYSICAL MODEL

Figure 1 shows a simple 2-D physical model. Source is located at the upper boundary of the model, and receivers is located at the upper boundary for reflection field and at the lower boundary for transmission field. The shaded layer between Zi and zj is a homogenous and isotropic elastic solid or a two-phas emedium. In order to use the propagator matrix method for our target layer (the shaded slab), we have to calculate both displacement and stress fields at the entrance of the target layer (the upper boundary)/ The relevant formulas can be found in Wu(1994,1996).Note that, in order to connect well with the propagator matrix method, the wide-angle version of one-way propagator in Wu(1996) must be used.

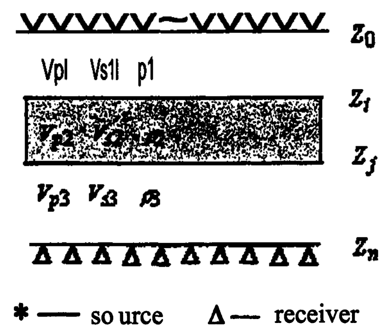

Fig. 1 The schematic illustration for a 2-D model.

PROPAGATOR MATRIX FOR AN ELASTIC S OLID

Propagator matrix for an elastic solid is well known. For a 2-D homogeneous and isotropic elastic solid, we take the displacement-stress vector on zi plane as

$$\vec{S}_{z_i}(\omega,k)=(u_x/k,\tau_{zz}/\omega^2,u_z/k,\tau_{xz}/\omega^2)^T_{z_i} \quad (1)$$

where u_x and u_z are the horizontal and vertical components of displacement, respectively. τ_{zz} and τ_{xz} are the normal and shear components of stress, respectively. k is the horizontal wavenumber, ω is the circle Frequency. T denotes transpose. The displacement-stress vector on different interfaces in the same media can be calculated as

$$\vec{S}_{z_j}(\omega,k)=M\vec{S}_{z_i}(\omega,k) \quad (2)$$

where $S_{z_j}(\omega,k)$ is the displacement-stress vector on z_j plane, and M is a 4 x 4 matrix related to formation constants (velocities ofP- and' S-waves and density) as well as thickness and frequency. M is the propagator matrix of the elastic solid layer. Unlike one-way approximation method, the field solved by the matrix method includes all multiple reflections within the layer.

PROPAGATOR MATRIX FOR TWO-PHASE MEDIA

Assume that the target layer is a two-phase medium described by Biot's simultaneous equations (**Biot,** 1956a). Biot's theory predicts that there are three types of waves : fast and slow P-waves and shear wave in two-phase media.. So we take the displacement-stress vector on z_i plane as

$$\vec{S}^d_{z_i}(\omega,k) = (u^d_x / k, -P^d_f / \omega^2, \tau^d_{zz} / \omega^2, \tau^d_{xz} / \omega^2, w^d_z / k, u^d_z / k)^T_{z_i} \quad (3)$$

where superscript d stands for two-phase media, u^d_x and u^d_z are the horizontal and vertical components of the displacement of the matrix (skeleton), respectively. w^d_x is the horizontal component of the permeable displacement defined as $\phi(\vec{U}^d - \vec{u}^d)$, ϕ is porosity, $\vec{U}^d$ is the displacement of fluid in pore. $-P^d_f$ is the pressure of the fluid in pore. τ^d_{zz} and τ^d_{xz} are the normal and shear components of the total stress. Using equation (3) and the relations of stress/pressure with displacements (in detail see Wu etc., 1993) , we can get a similar recursive formula as

$$\vec{S}^d_{z_j}(\omega,k) = H\vec{S}^d_{z_i}(\omega,k) \quad (4)$$

where $\vec{S}^d_{z_j}(\omega,k)$ is the displacement-stress vector of two-phase media on z_j plane. H is a 6 x 6 matrix related to elastic constants and reservoir parameters (such as porosity, permeability and viscosity) of two-phase media as well as thickness and frequency. H is a propagator matrix of two-phase media. Note that application of equation (4) is convenient only to the wave field propagation inside two-phase media. Difficulty will be met when wave propagates through the interface between the elastic solid and two-phase medium, because $-P^d_f$ in equation (4) on the interface is not independent. In order to solve this problem, we introduce an equivalent propagator matrix of two-phase media.

EQUIVALENT PROPAGATOR MATRIX FOR TWO-PHASE MEDIA

The displacement-stress continuous conditions on both z_i and z_j planes can be written as

$$\begin{pmatrix} u_x / k \\ \tau_{zz} / \omega^2 \\ \tau_{xz} / \omega^2 \\ 0 \\ u_z / k \end{pmatrix}_{z_i} = \begin{pmatrix} u^d_x / k \\ \tau^d_{zz} / \omega^2 \\ \tau^d_{xz} / \omega^2 \\ w^d_z / k \\ u^d_z / k \end{pmatrix}_{z_i} \quad (5)$$

and

$$\begin{pmatrix} u^d_x / k \\ \tau^d_{zz} / \omega^2 \\ \tau^d_{xz} / \omega^2 \\ w^d_z / k \\ u^d_z / k \end{pmatrix}_{z_j} = \begin{pmatrix} u_x / k \\ \tau_{zz} / \omega^2 \\ \tau_{xz} / \omega^2 \\ 0 \\ u_z / k \end{pmatrix}_{z_j} \quad (6)$$

The left of equation(5) and the right of equation(6) are the displacement-stress vectors for the elastic solid. Using equations (5) and (6), we can eliminate $-P^d_f$ **in** equation (4) and obtain

(7)

with D *as*

$$\begin{pmatrix} H_{11} - \frac{H_{12}H_{51}}{H_{52}} & H_{13} - \frac{H_{12}H_{53}}{H_{52}} & H_{14} - \frac{H_{12}H_{54}}{H_{52}} & H_{16} - \frac{H_{12}H_{56}}{H_{52}} \\ H_{31} - \frac{H_{32}H_{51}}{H_{52}} & H_{33} - \frac{H_{32}H_{53}}{H_{52}} & H_{34} - \frac{H_{32}H_{54}}{H_{52}} & H_{36} - \frac{H_{32}H_{56}}{H_{52}} \\ H_{41} - \frac{H_{42}H_{51}}{H_{52}} & H_{43} - \frac{H_{42}H_{53}}{H_{52}} & H_{44} - \frac{H_{42}H_{54}}{H_{52}} & H_{46} - \frac{H_{42}H_{56}}{H_{52}} \\ H_{61} - \frac{H_{62}H_{51}}{H_{52}} & H_{63} - \frac{H_{62}H_{53}}{H_{52}} & H_{64} - \frac{H_{62}H_{54}}{H_{52}} & H_{66} - \frac{H_{62}H_{56}}{H_{52}} \end{pmatrix}$$

where $H_{ij}(i, j = 1,6)$ are the components of matrix H . Equation (7) shows that when the target layer is a two-phase layer, we first calculate the propagator matrix H of the two-phase layer, and then construct a 4 x 4 matrix D. D is called the equivalent matrix of two-phase media because it is the same in form as M in equation (2). The displacement fields computed from one-way approximation method at the entrance of the target layer may be connected directly by D .

NUMERICAL EXAMPLES

In order to test the connection algorithm of our hybrid method, we first compare the results form our hybrid method to those by the one-return approximation method alone for a weakly perturbed solid layer. The model shown in Fig. 1 is defined on a 1024 x 300 rectangular grid with $z_0 = 0$, $z_i = 0.75$km and $z_j = 125$km. The grid spacing is 10m in horizontal direction and is 5m in vertical direction. A point source is located at the center of z_0 plane. The frequency range is from 5Hz to 35Hz. Formation parameters are:

$V_{p1} = V_{p3} = V_p^0 = 5km/s$; $V_{s1} = V_{s3} = V_s^0 = 3km/s$;

$\rho_1 = \rho_3 = \rho^0 = 2g/cm^3$; V_{p2} and V_{s2} are both of 10% perturbations. Figure 2 shows the reflection seismograms at z_0 plane. The solid curves are calculated by one-return approximation alone, the dashed lines are from the hybrid method. They have good agreement. Since the wade-angle version of the one-way approximation method is used, the records with wide incident angle also have good agreement. For small perturbations (less than 15% of the background parameters), the one-return approximation method can be reliably applied for the modeling.

Figure 3 shows the transmission field at z_n plane through a two -phase medium with different porosity. Formation parameters are:

$V_{p1} = V_{p3} = V_p^0 = 4km/s$; $V_{s1} = V_{s3} = V_s^0 = 2.2km/s$;

$\rho_1 = \rho_3 = \rho^0 = 2g/cm^3$; $V_{p2} = 4.175km/s$; $V_{s2} = 2.154km/s$ (V_{p2} and V_{s2} are the characteristic velocities of the compressional and the shear waves in two-phase media); and the densities of the grain and the fluid in pore are $2.5g/cm^3$ and $1.5g/cm^3$, respectively; the mass-coupling coefficient is 2; viscosity is $10^{-8} kg/s \cdot m$. The geometry parameters are $z_0 = 0$; $z_i = 0.1km$, z_j =1.1km and $z_n = 1.2$km . The arrivals after the fast P-wave are amplified by a factor of 100. From figure 3 we see that porosity has only a slight influence on velocities of fast P- and S-waves, but has strong effect on velocity of slow P-wave. Effect of permeability on the transmission filed is shown in Figure 4. Both fast P- and S-waves are hardly affected by permeability, but the amplitude of slow P-wave increases with increasing permeability. The detailed investigation will be reported in the future study.

CONCLUSIONS

A hybrid method which combines the one-return approximation method and the propagator matrix method (the full-wave solution) is presented and extended to including two-phase media. The method can be used in modeling elastic waves in complex media with strong reflecting layers and porous layers.

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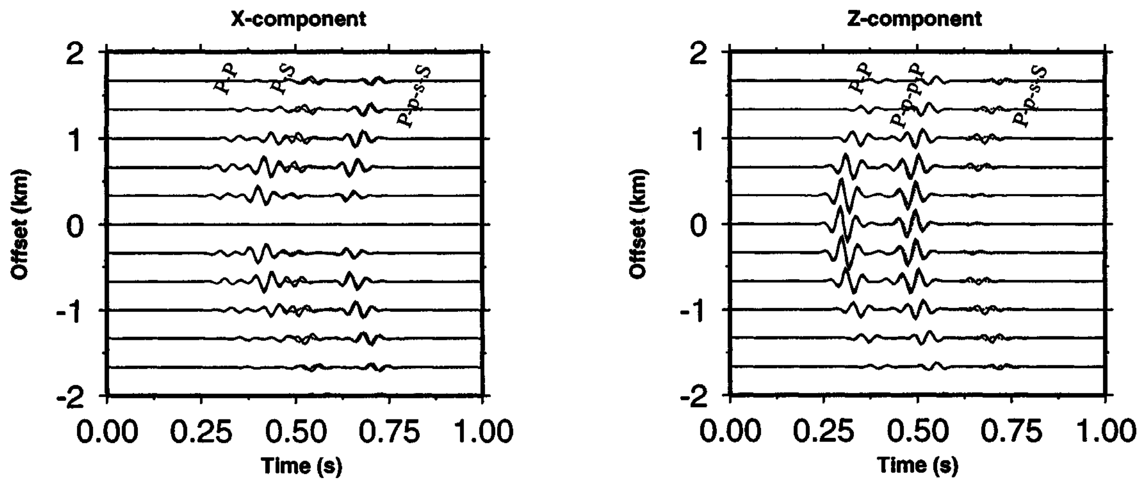

Fig.2 Reflection seismograms: The solid curves are calculated from one-way approximation alone and the dashed lines are from the hybrid method. P and S stands for P- and S-waves propagating in background medium, and p and s for P- and S-waves propagating in the target layer.

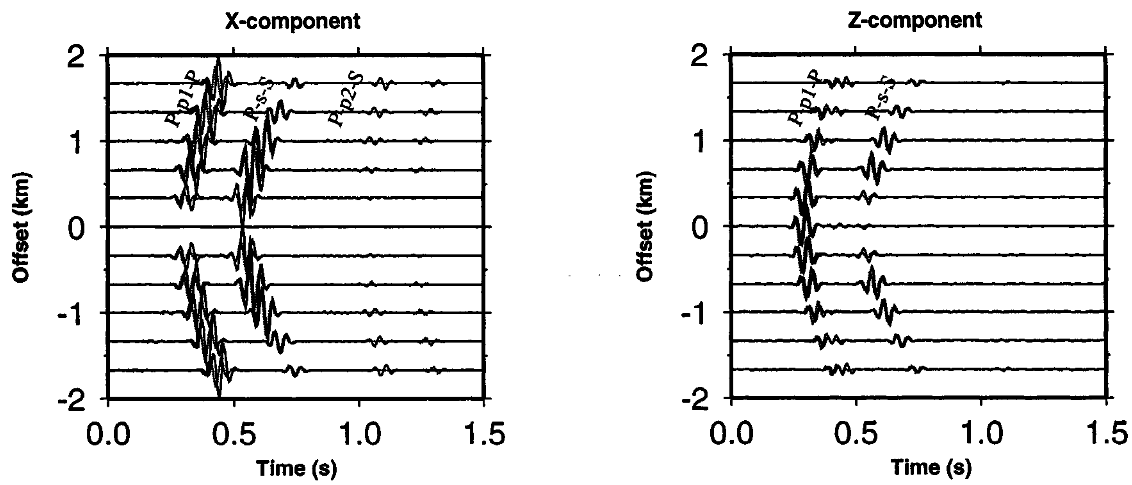

Fig.3 Transmission field through two-phase media with different porosity: Porosity is taken as 0.2 (solid) and 0.1 (dashed), respectively, permeability is 100mD; pl,p2 and s stands for fast and slow P-wavs and S-wave propagating in two-phase medium, P and S for P- and S-waves propagating in elastic solid.)

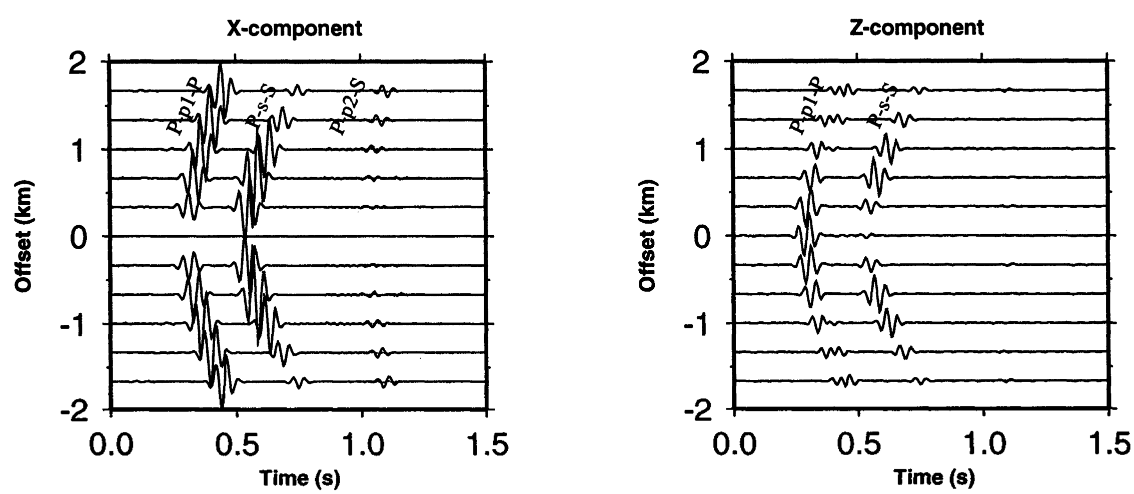

Fig.4 Transmission field through two-phase media with different permeability: Permeability is 100mD (dashed) and lD(solid), respectively, porosity is 0.2.