Dreamlet-based interpolation using POCs method

Benfeng Wangab, Ru-Shan Wub, Yu Gengbc, Xiaohong Chena

Abstract

Due to incomplete and non-uniform coverage of the acquisition system and dead traces, real seismic data always has some missing traces which affect the performance of a multi-channel algorithm, such as Surface-Related Multiple Elimination (SRME), imaging and inversion. Therefore, it is necessary to interpolate seismic data. Dreamlet transform has been successfully used in the modeling of seismic wave propagation and imaging, and this paper explains the application of dreamlet transform to seismic data interpolation. In order to avoid spatial aliasing in transform domain thus getting arbitrary under-sampling rate, improved jittered under-sampling strategy is proposed to better control the dataset. With L0 constraint and Projection Onto Convex Sets (POCS) method, performances of dreamlet-based and curvelet-based interpolation are compared in terms of recovered signal-to-noise ratio (SNR) and convergence rate. Tests on synthetic and real cases demonstrate that dreamlet transform has superior performance to curvelet transform.

1. Introduction

Due to factors such as presence of obstacles, forbidden areas, feathering and dead traces, real seismic data is always irregularly sampled in spatial coordinates, especially for land acquisition. Since methods, such as surface-related multiple elimination (SRME), wave-equation based migration and inversion, need complete seismic data, irregular sampled seismic data can affect the performance of these algorithms, therefore, seismic data interpolation is an essential stage before multi-channel seismic data processing.

Methods for interpolation can be divided into three categories (Gao et al., 2013): First, methods based on mathematical transform and signal analysis (Herrmann and Hennenfent, 2008; Naghizadeh and Immennet, 2011; Naghizadeh and Sacchi, 2010; Trad et al., 2002; Wang et al., 2010); second, prediction filter based methods (Naghizadeh and Sacchi, 2007; Spitz, 1991); third, interpolation methods based on wave equation (Ronen, 1987). In the past several years, rank-reduction method (Gao et al., 2013; Kreimer et al., 2013; Ma, 2013) has also played an important role in seismic data interpolation. In this paper, we mainly focus on Projection Onto Convex Sets (POCS) method which belongs in the first interpolation method category. The POCS method was first proposed by Bregman (Bregman, 1965, 1967), and was used in image reconstruction (Stark and Oskouii, 1989; Youla and Webb, 1982). The POCS method was introduced into irregular seismic data interpolation by Abma and Kabir (Abma and Kabir, 2006), however his method is time consuming and merely uses linear threshold. With the rapid development of sparse transform, e.g., Fourier transform and Curvelet transform, other researchers have proposed additional strategies based on the POCS method. Gao et al. (Gao et al., 2010) used Fourier transform and Curvelet transform with exponential threshold to interpolate irregular seismic data and they also compared different threshold functions to further improve convergence rate (Gao et al., 2012). Curvelet transform (Candes et al., 2006), which works better for curved seismic events compared with Fourier-based methods, is also used in seismic interpolation (Herrmann and Hennenfent, 2008). Yang et al. (Yang et al., 2012) proves the equivalence of Iterative Shrinkage Threshold Algorithm (IST) and the POCS method with soft threshold when enough iterations are used, and the performance using IST and the POCS method based on Curvelet transform indicates that POCS method with soft threshold is superior over IST in the first few iterations. In order to use the prior information on transform-domain coefficient, Mansour et al. (Mansour et al., 2013) uses the weighted one-norm minimization which seeks the corrections between locations of significant curvelet coefficients, to interpolate the unknown seismic data. There are also some authors who do not use the specific sparse transform, instead they construct the tight frame, which can represent seismic data sparsely, according to the observed seismic data. Liang et al. (Liang et al., 2014) used the tight frame, which is constructed through learning, and the spgl1 method to interpolate unknown data, but this data-driven method is time consuming because a bank of compactly supported filters should be determined first from the observed seismic data. The data-driven method has also been used in seismic data denoising, especially random noise attenuation (Beckouche and Ma, 2014), but it is not widely used because of computational cost. Dreamlet transform (Geng et al., 2009; Wu et al., 2009, 2010).
2011, 2013) is an efficient and alternative sparse transform with respect to curvelet transform. This is because dreamlet is a physical wavelet which is defined on an observation plane and satisfies wave equation. Wu et al. (Wu et al., 2013) did a preliminary study on post-stack seismic data recovery in compressive sensing based on dreamlet transform and $L_1$ constraint, and the performance is superior to curvelet transform. An alias, generated by regular sampling or random sampling with a big gap, will make seismic data interpolation difficult to handle. Anti-aliasing seismic data interpolation is achieved by Naghizadeh and Sacchi (Naghizadeh and Sacchi, 2007) using a multistep autoregressive algorithm in Fourier transform domain. Using mask function, anti-aliasing seismic data interpolation is obtained in Fourier domain (Gao et al., 2012; Naghizadeh, 2012) and curvelet domain (Naghizadeh and Sacchi, 2010), while performance is easily affected by the parameter setting. A Jittered under-sampling algorithm is proposed (Hennenfent and Herrmann, 2008), which can keep random property as well as control the biggest gap between traces, avoiding the shortcomings of regular sampling and random sampling. Shahidi et al. (Shahidi et al., 2013) extended the Jittered under-sampling strategy from 1D to 2D, and developed Poison Disk sampling and Farthest point sampling strategy which can be used to sample seismic data.

In this paper, we propose a novel method which is based on dreamlet (drumbeat-beamlet) transform and POCS method to interpolate irregular sampled seismic data. In order to avoid spatial aliasing, an improved Jittered under-sampling method is proposed to get arbitrary sampling rate. Tests on synthetic and real data approve the validity of this proposed method.

2. Theory

2.1. Dreamlet (drumbeat-beamlet) theory

In dreamlet transform, each atom is a physical wavelet which satisfies wave equation automatically (Wu et al., 2011, 2013). In this paper, we use Gabor frame as the local decomposition atom. The basic dreamlet atom is as follows,

$$d_{\tau\Omega}(x,t) = g_{\tau\Omega}(t) b_{\Omega}(x)$$

where $g_{\tau\Omega}(t) = W(t - \tau)e^{-i\Omega t}$ is time-frequency atom: “drumbeat” with $W(t)$ as a Gaussian window function, and $b_{\Omega}(x) = B(x - \Omega)e^{i\Omega x}$ is space-wavenumber atom: “beamlet” with $B(x)$ as a Gaussian window function. Then a dreamlet atom is a localized wavepacket in time-space plane:

$$d_{\tau\Omega}(x,t) = W(t - \tau) B(x - \Omega)e^{-i(\tau x + \Omega t)}$$

where $\tau, \Omega, \Omega$ and $\Omega$ are local time, frequency, space and wavenumber, respectively. A time-space atom constructed in this way satisfies the causality automatically, which can represent seismic data efficiently, resulting in sparse coefficient in dreamlet domain. Fig. 1 shows some of the dreamlet basis, and the atom is the local basis which can be used to represent the seismic data locally.

![Fig. 1. Gabor dreamlet atoms.](image1)

![Fig. 2. (a) Shot gather; (b) recovered SNR using part of coefficients.](image2)
Seismic data decomposition based on dreamlet transform can be expressed as,

\[ u(x, t) = \sum_{\tau\in\mathbb{T}_k} \langle u(x, t), \tilde{d}_{\tau \mathbb{Z}}(x, t) \rangle \tilde{d}_{\tau \mathbb{Z}}(x, t) = \sum_{\tau\in\mathbb{T}_k} c_{\tau \mathbb{Z}} \tilde{d}_{\tau \mathbb{Z}}(x, t) \]  

where \( u(x, t) \) is seismic data, \( \tilde{d}_{\tau \mathbb{Z}}(x, t) \) is basic dreamlet atom, \( c_{\tau \mathbb{Z}} \) is dreamlet coefficient,

\[
\langle \tilde{g}_{\tau \mathbb{Z}}(t), g_{\tau \mathbb{Z}}(t) \rangle = 1 \quad \text{and} \quad \langle \tilde{b}_{\tau \mathbb{Z}}(t), b_{\tau \mathbb{Z}}(t) \rangle = 1 .
\]

Table 1

<table>
<thead>
<tr>
<th>Threshold function</th>
<th>Expression</th>
</tr>
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<tbody>
<tr>
<td>Linear threshold</td>
<td></td>
</tr>
<tr>
<td>Exponential threshold</td>
<td>( r_k = r_{\max} - (r_{\max} - r_k)(k - 1)/(N - 1), ) ( k = 1, 2, ..., N. )</td>
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2.2. POCS-based interpolation method

Irregular sampled seismic data \( d_{\text{obs}} \) can be related to the complete seismic data \( d_0 \) using sampling matrix \( R \), shown as,

\[ d_{\text{obs}} = Rd_0 \]  

where \( * \) stands for complex conjugate and \( \tilde{g}_{\tau \mathbb{Z}}(t) \) and \( \tilde{b}_{\tau \mathbb{Z}}(x) \) are the unitary dual vectors of \( g_{\tau \mathbb{Z}}(t) \) and \( b_{\tau \mathbb{Z}}(x) \), respectively, subjecting to the condition \((Chen \ et \ al., 2006)\).

Fig. 4. Under-sampling strategy demo. (a) Regular sampling; (b) the Jittered under-sampling; (c) random sampling.

Fig. 3. Flow chart of dreamlet-based interpolation using POCS method.

\[ \begin{align*}
\Phi(x) & = \left\| d_{\text{obs}} - RD^k x \right\|^2_2 + \lambda P(x) \\
\end{align*} \]  

where \( x \) is dreamlet coefficient vector, \( D^k \) is inverse dreamlet transform and \( P(x) \) is sparse constraint which can be chosen as \( l_0 \) or \( l_1 \) norm constraint. \( l_0 \) norm constraint is the sparsest constraint which can be solved through iterative hard threshold algorithm (IHT) \((Blumensath \ and \ Davies, 2008, 2009; Loris \ et \ al., 2010)\). Equivalence of IST or IHT and POCS with soft or hard threshold has been proved when using enough iterations \((Yang \ et \ al., 2012)\) and POCS method is superior in the first few iterations compared with IST or IHT algorithm. Therefore, the POCS method is used to solve Eq. (7), and an interpolation iterative algorithm is as follows,
where \( T_\lambda(x) \) is the threshold function. When \( P(x) = \|x\|_0 \) (L0 norm), it subjects to,
\[
T_\lambda(x) = \begin{cases} 
  x, & \text{if } |x| \geq \tau \\
  0, & \text{if } |x| < \tau 
\end{cases}
\]
(9)

where \( \tau = \sqrt{\lambda} \) is the threshold. When \( P(x) = \|x\|_1 \) (L1 norm), it subjects to,
\[
T_\lambda(x) = \begin{cases} 
  x - \tau, & \text{if } x > \tau \\
  0, & \text{if } |x| \leq \tau \\
  x + \tau, & \text{if } x < -\tau 
\end{cases}
\]
(10)

where \( \tau = 0.5\lambda \) is the threshold and \( \lambda \) is regularization factor. The threshold can be obtained through linear threshold function or exponential threshold, though the exponential threshold function is most frequently used and is superior to the linear one. The corresponding threshold functions are listed in Table 1. Where \( \tau_{\text{max}} = p_1 + D_{\text{max}}, \tau_{\text{min}} = p_2 D_{\text{max}} \) and \( D_{\text{max}} \) is the maximum coefficient.

A flow chart of dreamlet-based interpolation using POCS is shown in Fig. 3.

### 2.3. Under-sampling method

Random sampling can turn the aliasing effect into lower valued noise in the transform domain when using sparse transforms. Though random sampling has random property compared with regular sampling, it lacks the ability to control maximum gaps between adjacent traces which can change the original spectrum greatly. Random property and the ability to control maximum gaps between adjacent traces can...
be guaranteed by the Jittered sampling strategy (Hennenfent and Herrmann, 2008). The Jittered under-sampling demo is shown in Fig. 4 together with regular sampling and random sampling strategy.

The exact formula for Jittered under-sampling is,

\[ y(k) = f(j_k), k=1,2,...,n \]

where \( j_k = \left\lfloor \frac{\gamma}{2} + k \gamma + \sigma_k \right\rfloor \) is an integer subjecting to \( \sigma_k \in \left[ -\frac{1}{2}(\varepsilon - 1)/2l, \frac{1}{2}(\varepsilon - 1)/2l \right] \). Number of sampling traces \( n \), under-sampling factor \( \gamma \) (an odd integer) and total number of traces \( N \) subject to \( n = N/\gamma; \varepsilon \) subject to \( 0 \leq \varepsilon \leq \gamma \). The optimal Jittered under-sampling can be obtained when \( \varepsilon = \gamma \) and \( \gamma \) can also be chosen as even integer. The \( \gamma \), chosen as integer, limits range of under-sampling rate and in order to obtain arbitrary under-sampling rate, formula (11) can be modified into,

\[ y(k) = f(j_k), k=1,2,...,n \]

where, \( j_k = (k-0.5)\gamma + \sigma_k + 1; \sigma_k \in \left[ -\frac{1}{2}, \frac{1}{2} \right] \); \( \gamma = N/n; N \) is total number of traces, \( n \) is number of observed seismic data and \( \varepsilon \) subject to \( 0 \leq \varepsilon \leq \gamma \). Optimal Jittered under-sampling can be obtained when \( \varepsilon = \gamma \).

3. Numerical examples

Firstly, we design a four-layer model to approve the validity of dreamlet-based interpolation using POCS method and L0 norm constraint. In this synthetic test, traces with 40%, 50% and 60% missing are tested to show the superiority of dreamlet-based method. Then, real seismic data application is also given, in which 50% traces are missing.

3.1. Four-layer model test

Fig. 5(a) shows a synthetic seismic data based on a four-layer model including 201 traces (trace interval is 12.5 m) with 1001 samples in each trace (time sampling interval is 2 ms). 40%, 50% and 60% traces are missing based on improved Jitter under-sampling (formula 12), as shown in Fig. 5(b), (c) and (d), respectively. This indicates that random property is satisfied as well as the maximum gap between adjacent traces.
traces is controlled using the improved Jittered under-sampling strategy. The interpolated results and residuals of the 40% traces missing, based on dreamlet transform and curvelet transform with hard threshold, are plotted in Fig. 6, and Figs. 7–8 demonstrate the results and residuals of 50% and 60% traces missing, respectively.

Figs. 6–8 show that interpolated results are almost the same with the complete seismic data shown in Fig. 5(a). As the missing traces increase, the residual increases. In order to further compare these performances, the SNR curves with respect to iterations are plotted in Fig. 9.

Fig. 9 indicates that the dreamlet-based method is superior to the curvelet-based method and the recovered SNR is lower as missing traces increase. The final SNRs are 29.2, 22.3 and 18.6 dB using dreamlet transform with 40%, 50% and 60% traces missing, respectively, while the SNRs corresponding to curvelet transform are 25.4, 19.9 and 17.6 dB, respectively. Performances in this simple synthetic seismic data prove the validity of the proposed method to some extent. To further confirm the validity of the proposed method, application of a real seismic data from a marine acquisition system is given.

3.2. Real data application

Fig. 10(a) shows the complete field data with 180 traces (trace interval is 12.5 m) and 1500 samples in each trace (time sampling interval is 4 ms). Incomplete seismic data with 50% traces missing using improved Jitter under-sampling strategy, shown in Fig. 10(b).

Fig. 10(b) indicates that the maximum gap between adjacent traces is controlled and random property is preserved. For convenience of comparisons, dreamlet-based and curvelet-based methods are tested with the same parameters. Interpolated results and residuals are shown in Fig. 11, respectively.

The interpolated results shown in Fig. 11(left column) agree well with the complete marine seismic data shown in Fig. 10(a), which
demonstrates the validity of sparse transform based interpolation method and the performance is satisfactory. In order to further compare the performances, the SNR curves with iterations for this marine datasets are shown in Fig. 12.

Fig. 12 indicates that results based on dreamlet are superior to the curvelet-based ones. From the SNR curves shown in Fig. 12, we can conclude that dreamlet transform is sparser and the performance is better compared with the curvelet-based ones in this real data example, which demonstrates the validity of the dreamlet-based POCS method. From Fig. 9 and Fig. 12, we can draw a rough conclusion that the POCS method is efficient when using sparse transform with $l_0$ norm as sparse constraints. When the maximum iteration number is set as 50, the recovered SNR is stable and the curve becomes horizontal after nearly 20–30 iterations which means that the quality of the recovered seismic data improves very little as the iteration increases after 20–30 iterations in the synthetic data test. While for the real seismic data, the recovered seismic data quality improves very little after 30–40 iterations. Therefore, the iteration can be terminated reasonably if we want to improve the efficiency further. Synthetic and real seismic data applications demonstrate the validity of the proposed method.

4. Conclusion

Interpolated seismic data is obtained from the newly developed sparse transform: dreamlet transform, using POCS method. Tests on synthetic and real seismic data show that the performances based on dreamlet transform are better than those based on curvelet transform in terms of sparsity and recovered SNR. The results in this paper are based on improved Jittered under-sampling strategy, which controls the maximum gap between adjacent traces and keeps random property. If the gap between adjacent traces is big or alias occurs in transform domain when using regular sampling strategy, anti-aliasing methods should be used (Gao et al., 2012; Naghizadeh, 2012; Naghizadeh and Sacchi, 2010). Our future research topic is anti-aliasing interpolation using dreamlet transform and sparse constraints.
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References


Fig. 9. SNR curves with iterations.

Fig. 10. One shot record from a real marine datasets. (a) Complete datasets with 180 traces (spatial interval is 12.5 m) and 1500 samples in each trace (time interval is 4 ms). (b) Incomplete seismic data with 50% traces missing using the improved jitter under-sampling strategy.
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Fig. 12. SNR curves with iterations.