Numerical tests on generalized diffraction tomography

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Abstract

In this paper we formulate the generalized diffraction tomography based on the volume scattering model in heterogeneous media. The frequency-dependent effect due to the finite spatial aperture of the acquisition system is corrected in the local angle domain, resulting in less artifacts and more balanced amplitude for recovering velocity perturbations. Using multiple frequencies, the blind area in the spectral domain in the traditional diffraction tomography can be partially filled and the qualities of the local spectra can be improved in both coverage and uniformity in the local wavenumber domain. Through the preliminary tests using a simple box model in a smoothly varying v(z) medium, the generalized diffraction tomography can recover the long-wavelength component of velocity perturbations up to 23% with respect to the background velocity. It can also reconstruct the low-wavenumber-component Marmousi velocity model very well when low-frequency waves are used in the process.

1. Introduction

In most cases, inversion theories and methods depend on forward modeling methods. Different modeling methods emphasize different parameters of the models and may obtain quite different inversion results. The traditional diffraction tomography applies a filtered propagation to the scattered field to reconstruct the parameter perturbation based on a homogeneous reference model (Beydoun and Mendes, 1989; Beylkin et al., 1986; Clayton and Stolt, 1981; Devaney, 1982, 1984; Harris, 1987; Ikelle et al., 1986; Lambaré et al., 1992, 2003; Miller et al., 1987; Slaney et al., 1984; Wu and Toksöz, 1987). Through the analysis of travel times of the seismic waves, the traditional tomography focuses on the reconstruction of smooth variations in the velocity model. Devaney (1984) presented the foundation of diffraction tomography for both vertical seismic profiling (VSP) and cross-well tomography. Wu and Toksöz (1987) derived the reconstruction formula of diffraction tomography for the case of surface reflection profiling, the VSP and cross-hole measurements based on the Born or Rytov approximation, which were restricted to weak inhomogeneities.

In order to overcome the limitation of the classic diffraction tomography and obtain more accurate inversion results, formulation of the diffraction tomography in the spatial domain (Woodward et al., 1998, 2008) and nonlinear full waveform inversion (FWI) method (Crase et al., 1990; Forgues and Lambaré, 1997; Gauthier et al., 1986; Jin et al., 1992; Joncour et al., 2011; Liu et al., 2005; Min and Shin, 2006; Operto et al., 2003; Pica et al., 1990; Pratt, 1999; Pratt and Worthington, 1988, 1990; Pratt et al., 1998; Sheng et al., 2006; Tarantola, 1984a, 1984b, 2005; Xu and Lambaré, 2006, 2009) have been developed for heterogeneous background media. Pratt and Worthington (1990) used a nonlinear waveform inversion in the frequency domain to incorporate the rigorous finite-difference modeling technique into the inverse procedure. In order to use the full information content of the recorded waveform and to reduce the waveform misfit between the observed and the modeled data, Pratt (1999) applied and evaluated a frequency-space domain approach to waveform inversion, which was a local descent algorithm that proceeds from a starting model to refine the model iteratively. However, in these formulation and waveform inversion approaches in spatial domain, the intuitive spectral inversion and its efficiency are lost (Wu, 2007). Hence, some good linear inversion methods, including diffraction tomography, are still desirable, since it can serve as the basis for an efficient nonlinear inversion.

In the new development of generalization of diffraction tomography (Gelius et al., 1991; Wu, 2007) the scattered wave field can be calculated by the distorted-wave Born approximation in heterogeneous media (Gelius et al., 1991; Schultz and Jaggard, 1987; Taylor, 1972). Schultz and Jaggard (1987) used a distorted-wave Born and geometric-optics approximation for the reconstruction algorithm, which is based on a weighted generalized backprojection operator, to establish a connection between the coherent swept-frequency microwave scattering data and the projections of refractive objects. Gelius et al. (1991) also extended...
diffraction tomography to nonuniform background models using an asymptotic ray theory for the calculation of the background Green’s function, which can handle irregular acquisition geometries. However, the ray-approximated Green’s function cannot completely handle all the wave phenomena.

Recently, a unified theory was developed for the true-reflection imaging based on the resolution operator of Backus and Gilbert (1967, 1968, 1970), combined with different kinds of forward modeling operators, such as the Born scattering for volume heterogeneities (Wu, 2007; Zhu and Wu, 2009) and boundary scattering for sharp discontinuities. For implementation, we proposed to decompose the operators in the local wavenumber domain and performed the correlation therein (Wu, 2007; Zhu and Wu, 2009).

Wu (2007) derived the formulation of scattering tomography in heterogeneous media for the volume scattering. We use a forward scattering renormalized Green’s function based on the De Wolf approximation (De Wolf, 1971, 1985; Wu, 1996, 2003; Wu et al., 2007), which is a multiple forward scattering and single back scattering approximation and can be implemented by an iterative marching algorithm (Liu et al., 2007; Wu, 1996, 2003; Wu et al., 2007; Xie et al., 2005).

In this paper, we summarize the theory of generalized diffraction tomography based on the distorted-wave Born model, and present the deconvolution filtering using the local image matrices (LIMs) and their corresponding resolution matrices in the local wavenumber domain, and apply the amplitude correction factors to the amplitude of the wavefield. The results of several numerical tests show excellent resolving capability of the method.

The outline of this paper is the following. Firstly, we briefly review the theory and method of the generalized diffraction tomography, and we describe the distorted-wave Born modeling for the volume scattering model, the imaging condition in the local wavenumber domain, and the local image matrix. We then present the resolving kernel and deconvolution filtering in the local wavenumber domain. Following that, we provide an algorithm for amplitude correction at the dominant frequency for the Born model. We then use several numerical examples to test the spectral recovery and the velocity-anomaly reconstruction. Finally, we give some discussions and conclusions.

2. Theory and method of generalized diffraction tomography

The generalized diffraction tomography consists of backpropagation plus filtering in the local wavenumber domain. The backpropagation is a double-focusing operation, which focuses both the wavefield from the source array and the receiver array to the image point. The filtering is a deconvolution in the local wavenumber domain. In this section we describe the theory and formulation of the generalized diffraction tomography (Cao, 2008; Wu, 2007; Zhu and Wu, 2009).

2.1. Distorted-wave Born modeling for volume scattering model

The modeling operator in general is nonlinear to the model parameters, which means the response to the model perturbation at one location will depend on the responses at other locations (multiple scattering). In the true-reflection imaging approach, we try to linearize the problem in an optimal way, which can not only give an optimal reflectivity recovery, but also form a basis for developing efficient iterative inversion methods. In this section we introduce a linearized modeling procedure using the Born model.

In the Born model, the scattering of each volume element is independent from each other and no interactions between elements are taken into consideration. This approximation is valid for weak perturbations and short propagation distances. The parameters to be inverted in the Born model are perturbations of unknown. In the case of scalar media (e.g., acoustic media with a constant density) and apply the amplitude correction factors to the amplitude of the wavefield. The backpropagation is a double-focusing (imaging) process we used includes the backpropagation from the receiver array to the image point. In our method, the double-focusing and receiver array focusing and compare with the classic diffraction tomography, we slightly modify the imaging condition from the De Wolf approximation (De Wolf, 1971, 1985; Wu, 1996, 2003; Wu et al., 2007), which is a multiple forward scattering and single back scattering approximation and can be implemented by an iterative marching algorithm (Liu et al., 2007; Wu, 1996, 2003; Wu et al., 2007; Xie et al., 2005).

Following the theory and method of the generalized diffraction tomography, we describe the distorted-wave Born modeling for the volume scattering model, the imaging condition in the local wavenumber domain, and the local image matrix. We then present the resolving kernel and deconvolution filtering in the local wavenumber domain. Following that, we provide an algorithm for amplitude correction at the dominant frequency for the Born model. We then use several numerical examples to test the spectral recovery and the velocity-anomaly reconstruction. Finally, we give some discussions and conclusions.

where \( p^{\text{sc}}(x_0, x_s) \) is the scattered field; \( G_f(x_0, x_s) \) and \( G_f(x_0, x_s) \) are the scalar wave Green’s functions for the modeling process in the homogeneous background medium. The Green’s function \( G_f(x_0, x_s) \) is the wavefield at \( x \) due to a point source on the surface located at \( x_0 \). Likewise, \( G_f(x_0, x_s) \) is the Green’s function for a receiver at \( x_s \) due to a source at \( x \). \( O(x) \) is the velocity perturbation function of the medium, which is the object function in classic diffraction tomography (Devaney, 1982, 1984; Slaney et al., 1984; Wu and Toksöz, 1987). \( k = \omega/c_0(x) \) is the background wavenumber where \( \omega \) is the circular frequency and \( c_0(x) \) is the background velocity at point \( x \). \( V \) is the integral volume of the heterogeneous medium. Hence the modeling operator for the Born model is

\[
\mathbf{F}(\omega, x_0, x_s|\mathbf{x}_0) = -k^2 \int V G_f(x_0, x_s) G_f(x_0, x)|\mathbf{v}(\mathbf{x}_0).
\]

where \( F(\omega, x_0, x_s|\mathbf{x}_0) \) is the modeling operator which maps the model into the data set. We use a forward scattering renormalized Green’s function in the model, and the forward modeling is based on the De Wolf approximation (De Wolf, 1971, 1985; Wu, 1996, 2003; Wu and Xie, 2009; Wu et al., 2006, 2007; Xie et al., 2005, 2006), which updates the Green’s function by the multiple forward scattering and keeps a single backscattering approximation in the modeling process. The De Wolf approximation is a kind of local Born approximation since the Born approximation applies only locally to the individual thin-slabs (Wu and Xie, 2009; Wu et al., 2008). Hence the model defined by Eq. (1) with the De Wolf approximated Green’s function can be called the De Wolf-Born model (or the local Born model) in comparison with the Born model for the classic diffraction tomography in homogeneous background media. It can extend the application to large volume weak perturbations and can overcome some drawbacks and limitations of the Born modeling.

2.2. Imaging condition and local image matrix

Imaging process can be defined mathematically as the back-propagation (or reverse-time propagation) plus focusing (imaging principle) (Claerbout, 1971, 1985). During the process of seismic data, prestack migration is an imaging process with double-focusing operator, which focuses both the waves from the source array and those from the receiver array to the image point. In our method, the double-focusing (imaging) process we used includes the backpropagation of the wavefield and applying an imaging condition at the image point (Claerbout, 1971, 1985). In order to be symmetric for the source array focusing and receiver array focusing and compare with the classic diffraction tomography, we slightly modify the imaging condition from the standard imaging condition in the form of cross-correlation and obtain the formula for the tomographic imaging condition (Cao, 2008; Wu, 2007; Wu and Toksöz, 1987; Zhu and Wu, 2009):

\[
I(x) = \text{Re} \int_{B(o)} \text{do} \omega (\omega) I(\omega, x).
\]
with

$$L(\omega, \mathbf{x}) = 4 \int_{A(\omega)} d\omega \int_{A(\omega)} d\xi \left[ s(\mathbf{x}) \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_s)}{\partial z} \right] d\omega s(\mathbf{x}) \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_g)}{\partial z} \right\}$$

(5)

where $L(\omega, \mathbf{x})$ is the result of the double focusing process for both the source field and receiver field (scattered waves) for a single frequency $\omega$, and the final image $I(\mathbf{x})$ at point $\mathbf{x}$ is obtained by summing up the images of all the frequencies at that point. The incident wave (downgoing wave) is from the source on the surface located at $\mathbf{x}$, and the scattering wave (upgoing wave) at the imaging point $\mathbf{x}$ is obtained by a backpropagation Rayleigh integral which refocuses the scattered field at the image plane from the source on the surface to the scattering point. The "Re" denotes the real part of the complex function; the asterisk denotes complex conjugate; $G(\omega, \mathbf{x} : \mathbf{x}_s)$ and $G(\omega, \mathbf{x} : \mathbf{x}_g)$ are the Green's functions used in the imaging process, which could be different from the Green's function of forward modeling. $\omega(\mathbf{x})$ is the source distribution in space and used as unity for regularized data; $A(\omega)$ is the source spectra with $B(\omega)$ as its frequency bandwidth. $A(\mathbf{x})$ and $A(\mathbf{x})$ are the source aperture and spatial receiver aperture for a given source, respectively. According to the formula of tomographic imaging condition (Eq. (5)), the formula of the imaging operator is

$$B(\mathbf{x}) = 4 \int_{B(\omega)} d\omega s(\mathbf{x}) \left[ \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_s)}{\partial z} \right] \left[ \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_g)}{\partial z} \right]$$

(6)

where $B(\mathbf{x})$ is the imaging operator which inverts seismic data into an image. In order to obtain the local angle-spectra of an image field, the conventional space-domain imaging condition can be extended to the local wavenumber domain (or the beamlet domain) (Cao, 2008; Wu, 2007; Wu and Chen, 2002; Xie and Wu, 2002; Zhu and Wu, 2009), then the image function is no longer a scalar value but a matrix: local image matrix (LIM). Note that the source direction is defined as the direction from the image point to the source on the surface, and is opposite to the incident direction.

We decompose the Green's functions in Eq. (5) into the local plane waves surrounding the scattering point. For the case of one-way propagation, we decompose the incident wave along a surface at level $z$ near the image point and represent the incident wave at $\mathbf{x}' = (x', z')$ as the superposition of local plane waves (Cao, 2008; Chen et al., 2006; Wu, 2007; Wu and Chen, 2001, 2006; Zhu and Wu, 2008, 2009, 2010),

$$G_i(\omega, \mathbf{x} : \mathbf{x}_s) = \sum_i e^{ik_x x'} G_i(\omega, x, z, (\xi_s) ; \mathbf{x}_s),$$

(7)

where $G_i(\omega, x, z, \xi) ; \mathbf{x}_s)$ is the local incident plane-wave coefficient of a local plane wave component of the Green's function, and $\mathbf{k}' = (x', x, z')$ is the source wavenumber vector; $\xi_s$ and $\xi$ are the corresponding local horizontal and vertical wavenumbers, respectively; here $c_s = (\omega/c(\xi))^2 - \xi^2$. A bar over a variable means the localization in the wavenumber domain ($\xi_s$ and $\xi_s$). The summation index $i$ denotes the sum of the local incident plane waves. We can assume the decomposition coefficients at the close vicinity of $\mathbf{x}$ are representative of the value at $\mathbf{x}$. When $\mathbf{x}$ approaches $\mathbf{x}'$, Eq. (7) can be written as

$$G_i(\omega, \mathbf{x} : \mathbf{x}_s) = \sum_i G_i(\omega, \mathbf{x}' : (\xi_s) ; \mathbf{x}_s),$$

(8)

where $G_i(\omega, \mathbf{x}' : (\xi_s) ; \mathbf{x}_s)$ is the incident beamlet at the image point $\mathbf{x}'$ generated by a point source on the surface at $\mathbf{x}_s$. It is seen that the original space-domain Green's function is transformed into a Green's function in the local wavenumber domain. The same decomposition and assumption can be applied to the Green's function of receiver

$$G_r(\omega, \mathbf{x}' : \mathbf{x}_s) = \sum_i e^{ik_x x'} G_r(\omega, x, z, (\xi) ; \mathbf{x}_s),$$

(9)

$$G_r(\omega, \mathbf{x}' : \mathbf{x}_s) = \sum_i G_r(\omega, \mathbf{x}' : (\xi) ; \mathbf{x}_s).$$

(10)

where $G_r(\omega, x, z, (\xi) ; \mathbf{x}_s)$ is the local outgoing (scattered) plane-wave coefficient of a local plane wave component of the Green's function; and $G_r(\omega, \mathbf{x}' : (\xi) ; \mathbf{x}_s)$ is the outgoing beamlet at the image point $\mathbf{x}'$ received by a receiver at $\mathbf{x}_s$ on the surface for a specific frequency. $\mathbf{k}_s = (\xi_s, \xi_s)$ is the receiver wavenumber vector; $\xi_s$ and $\xi_s$ are the corresponding local horizontal and vertical wavenumbers, respectively.

Substituting the decomposed Green's functions into Eq. (5), we obtain the tomographic imaging condition for a single frequency in the local wavenumber domain

$$L(\omega, \mathbf{x} : \mathbf{k}, \mathbf{k}_s) = 4 \int_{B(\omega)} d\omega s(\mathbf{x}) \left[ \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_s)}{\partial z} \right] \left[ \frac{\partial G(\omega, \mathbf{x} : \mathbf{x}_g)}{\partial z} \right]$$

(11)

where the image $L(\omega, \mathbf{x} : (\xi_s) ; \mathbf{x}_s)$ is the LIM at the image point $\mathbf{x}'$ in the local wavenumber domain for a specific frequency $\omega$. Here $\mathbf{k}_s = (\xi_s, \xi_s)$ is the source wavenumber vector; $\mathbf{k}_s$ and $\mathbf{k}_s$ are the unit vectors in the source direction and the scattering direction, respectively. Fig. 1 shows the definitions of local wavenumbers and the domain of spectral coverage for the Born model when the source and receiver arrays are both infinite (Wu and Toksöz, 1987). $\mathbf{k}_s$, $\mathbf{k}_s$, and $\mathbf{k}_s$ are the local wavenumbers of source, incident and receiving directions, respectively; due to the source direction is defined from the image point to the source on the surface and is opposite to the incident direction in this paper, hence $\mathbf{k}_s = -\mathbf{k}_s$.

We set $z' = z$ and chose the coordinate original in the $x$ axis at the decomposition level as 0, then $\mathbf{r}' = (x', 0)$. According to Eq. (11), the

![Fig. 1. Definitions of local wavenumbers and domain of spectral coverage for the Born model when the source and receiver arrays are both infinite (Wu and Toksöz, 1987; Zhu and Wu, 2009).](attachment:image.png)
image matrix at \( \mathbf{x} \) for a single frequency can be simplified as (Cao, 2008; Chen et al., 2006; Wu, 2007; Wu and Chen, 2001, 2006; Zhu and Wu, 2008, 2009, 2010)

\[
L\left(\omega, \mathbf{x} \right) = \mathbf{R} = \mathbf{L} + \mathbf{k}
\]

\[
= -4\pi r \cdot \exp\left(-i\left(\mathbf{r} + \mathbf{r}_0\right) \cdot \mathbf{R}\right) \int_{A(s)} \frac{dG_F(\omega, x, x_0; x)}{dz} \cdot \mathbf{R}(\omega, x_0; x_0, x) \cdot \mathbf{r}^2 \cdot \mathbf{R}(\omega, x_0; x, x) \cdot \mathbf{r}^2.
\]

(12)

From the formula above we can see that for a pair of local incident-scattering angle we can only detect the local spectral component at \( \mathbf{k} = \mathbf{L} + \mathbf{k}_0 \) (Fig. 1).

With multi-frequency imaging, the spectral coverage can be expanded and the local image matrix (LIM) becomes

\[
L(\mathbf{x}, \mathbf{R}) = \int_{B(s)} \text{det}(\omega)L(\omega, \mathbf{x} \cdot \mathbf{R} = \mathbf{L} + \mathbf{k})
\]

(13)

Due to the approximations in propagators and inaccuracy of velocity model, the LIM in general is a complex matrix, that is

\[
L(\omega, \mathbf{x}, \mathbf{L}, \mathbf{k}) = \left[A(\omega, \mathbf{x}, \mathbf{L}, \mathbf{k}) \exp(i\varphi(\omega, \mathbf{x}, \mathbf{L}, \mathbf{k})\right]
\]

(14)

where \( A(\omega, \mathbf{x}, \mathbf{L}, \mathbf{k}) \) and \( \varphi(\omega, \mathbf{x}, \mathbf{L}, \mathbf{k}) \) are the amplitude and phase of matrix element, respectively. For an inaccurate background velocity model, the phases are different for different pairs of \( (\mathbf{L}, \mathbf{k}) \), and those phase residuals can cause the blurriness of the final image and produce artifacts and coherent noises.

2.3. Resolving kernel and deconvolution filtering in local wavenumber domain

2.3.1. Resolving kernel for volume scattering based on the Born model

The kernel for the resolution operator (resolving kernel) (Backus and Gilbert, 1967, 1968, 1970; Tarantola, 1984a, 2005) maps one scattering point in the model space back into the model space and is also called the point spreading function (PSF) (Wu, 2007) for the imaging system, which includes the effects of both acquisition (modeling) and inversion (imaging) process. From the imaging operator (Eq. (6)) and the Born model operator (Eq. (3)) we obtain the resolving kernel of tomographic imaging condition for the Born model

\[
\mathcal{G}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) = -4\pi \int d\omega \int dx_0 \int dx_0 \int dx_0 \frac{dG_F(\omega, x, x_0; x)}{dz} \cdot G_F(\omega, x_0; x_0, x) \cdot \mathbf{r}^2 \cdot \mathbf{R}(\omega, x_0; x, x) \cdot \mathbf{r}^2.
\]

Due to the calculations of the resolving kernel and the volume deconvolution in space domain are intractable and very time-consuming, while the deconvolution process in the local wavenumber domain is easy and efficient. Therefore, we decompose the resolving kernel into the local wavenumber domain and formulate the process in that domain. The resolving kernel of tomographic imaging condition in local wavenumber domain can be obtained by performing local 3D Fourier transform on \( \mathcal{G}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) \) (Eq. (15)) with the coordinate at \( \mathbf{x}_0 \) (Wu et al., 2004)

\[
\mathcal{G}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) = 2\pi \int d\omega \int dx_0 \int dx_0 \left[ \frac{dG_F(\omega, x, x_0; x_0)}{dz} \cdot G_F(\omega, x_0; x_0, x) \cdot \mathbf{r}^2 \cdot \mathbf{R}(\omega, x_0; x, x) \right].
\]

with

\[
B_l(\omega, \mathbf{x}_0, \mathbf{k}_l, \mathbf{x}) = -2\int_{A(x_0)} \frac{dG_F(\omega, x_0, \mathbf{k}_l, \mathbf{x})}{dz} G_F(\omega, x_0; x_0, x),
\]

(17)

where \( B_l(\omega, \mathbf{x}_0, \mathbf{k}_l, \mathbf{x}) \) is the backprojection integral with a limited receiver-aperture applying to the modeling Green’s function.

2.3.2. Deconvolution filtering in local wavenumber domain

In the local wavenumber domain, the deconvolution is a division of the image matrix \( \mathbf{I}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) \) by resolving kernel \( \mathcal{G}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) \), hence the 2D space domain object function can be reconstructed by inverse beamlet transform

\[
O(\mathbf{x}) = \frac{1}{2\pi} \int \mathbf{I}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) \mathcal{G}(\mathbf{x}, \mathbf{L}, \mathbf{k}, \mathbf{L}) d\omega \int dx g(\mathbf{x}, \mathbf{L}, \mathbf{k}) d\mathbf{x}
\]

(18)

with

\[
\mathcal{J}(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0) = \frac{f(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0)}{\mathcal{G}(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0)}
\]

(19)

\[
\mathbf{J}(\mathbf{k}_l, \mathbf{x}, \mathbf{k}_l, \mathbf{x}_0) = \frac{f(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0)}{\mathbf{G}(\mathbf{k}_l, \mathbf{x}_0)}
\]

(20)

where \( f(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0) \) is the Jacobian of the coordinate transform from \( (\mathbf{k}_l, \mathbf{x}_0) \) to \( (\mathbf{k}_l, \mathbf{x}_0) \) and \( \mathcal{J}(\mathbf{x}, \mathbf{k}_l, \mathbf{x}_0) \) is the tomographic filter, which can be directly applied to the image matrix in the local wavenumber domain to obtain the object function. The above formulation is to perform the deconv-filtering to a single frequency local image matrix.

2.4. Amplitude correction factor at the dominant frequency for the Born model

Due to the limitation in acquisition aperture and frequency bandwidth, in general, the image matrix is a complex matrix with phase residual as a function of local wavenumbers, so is the resolving kernel. The recovery of the local spectra using multi-frequency data and the deconvolution-filtering is a task of diffraction tomography of general inversion. For the amplitude compensation in true-reflection imaging, the task can be reduced to a simpler one. Assuming that the background velocity we used is the exact one and the propagators do not have phase

Fig. 2. Velocity model of a square box with an average velocity perturbation 23% higher than the average background velocity.
errors, we only need to correct the amplitude of the local angular spectra of the image field.

For most of the seismic source wavelets (e.g., the Ricker wavelet), the major energy is carried by the wavefield near the dominant frequency. If we use the amplitude correction factor at the dominant frequency instead of those factors for all the frequencies to the entire wavelet, it is a proper approximation to the full resolving kernel. Therefore, the fast calculation of resolving kernel can be of orders of magnitudes. For the resolving kernel of a single frequency and for a given \( k_s \), \( K \) and \( k_g \) become one-to-one correspondence. Hence, considering only the amplitude correction, we have

\[
\gamma_i(\omega_0, K = \mathbf{k}_s; x_s) = 2\text{Re}\left\{ k_0^2 \int_{A(x_s)} dx_x \frac{\partial G \left( \omega_0, x_0, \mathbf{k}_s; x_s \right)}{\partial x} \times G_F (\omega_0, x_0, x_s) B_i (\omega_0, x_0, \mathbf{k}_s; x_s) \right\},
\]

where \( B_i (\omega_0, x_0, \mathbf{k}_s; x_s) \) is the receiver-aperture effect of a given source on the surface located at \( x_s \) and a given \( \mathbf{k}_s \) for a dominant frequency \( \omega_0 \).

In the ideal situation with an exact velocity model, the Green’s function in the focusing operator (backpropagation integral) is complex-conjugated with the modeling Green’s function. For most of cases, we

Fig. 3. Spectra of Ricker wavelets with different dominant frequencies (i.e., 0.3, 0.5, 0.75, 1.13, 1.70, 2.56, 3.86 and 5.81 Hz) and the final stacked spectra.

Fig. 4. Domain of multi-frequency spectral coverage of local image matrices for the Born model when the velocity perturbation is a real function in the model.

Fig. 5. Sampled three points in the square box velocity model for showing the spectral recovery.
take $\frac{\partial G_f(\omega_0, x_0; x_g)}{\partial z} G_f(\omega_0, x_0; x_g)$ and remove the phase residual. For the same reason, we take the absolute value of the receiver-aperture effect $B_f(\omega_0, x_0; x_g)$, that is

$$ |B_f(\omega_0, x_0; x_g) - \bar{B}_f(\omega_0, x_0; x_g)| $$

and remove the phase residual. For the same reason, we take the absolute value of the receiver-aperture effect $B_f(\omega_0, x_0; x_g)$, that is

$$ |B_f(\omega_0, x_0; x_g)| $$

for $G_f(\omega_0, x_0; x_g)$ for $G_f(\omega_0, x_0; x_g)$, then energy conservation exists

$$ |B_f(\omega_0, x_0; x_g)| = 2 \int dF(\omega_0, x_0; x_g, x_g) |G_f(\omega_0, x_0; x_g)|^2. \quad (22) $$

The receiver-aperture integral involves a point source spreading to the surface receiver array and a backpropagation to a beamlet receiver at the point. For backpropagation we prefer to use an energy-conserved Green's function. In this way, all the energy loss during refocusing will be neglected, hence we can conserve all the energy collected by the receiver array to the maximum. That is the practice of one-way wave focusing operator.

Assuming we use $G_f(\omega_0, x_0; x_g)$ for $G_f(\omega_0, x_0; x_g)$, then energy conservation exists

$$ |B_f(\omega_0, x_0; x_g)| = 2 \int dF(\omega_0, x_0; x_g, x_g) |G_f(\omega_0, x_0; x_g)|^2. \quad (23) $$

**Fig. 6.** Original spectra (without corrected by the amplitude correction factor) and their corresponding recovered spectra (corrected by the amplitude correction factor) of local image matrices of the three selected points in the same sequence as in Fig. 5. Those local image matrices are generated using the scalar wave synthetic data. (a) original spectra of LIM generated using the velocity model with the average velocity perturbation 23%; (b) recovered spectra using their corresponding local resolution matrices of (a); (c) original spectra of LIM generated using the velocity model with the average velocity perturbation 30%; (d) recovered spectra using their corresponding local resolution matrices of (c).
except for some edge beamlets. By reciprocity, the beamlet Green's function can be calculated as radiated from a point source on surface at \(x_0\) and received by a beamlet antenna at \(x_0\) with the same angle and beamwidth. Therefore, we need to calculate the resolving kernel for the modeling using the Green's function. We make the following approximation in the above equations

\[
G_F(\omega, \mathbf{x}; x_s) \approx G_I(\omega, \mathbf{x}; x_s),
\]

then the amplitude correction factor reduces to a correction factor

\[
F_a(\omega_0, \mathbf{k}_x | x_0) = |g(\omega_0, \mathbf{k}_x) = \mathbf{k}_s + \mathbf{k}_g, x_0)\|
= 4k_0^2 \int_{|k_x|} \frac{\partial G_I(\omega_0, x_0, \mathbf{k}_x, x_s)}{\partial z} G_I(\omega_0, x_0; x_s) | \int_{|k_x|} \frac{\partial G_I(\omega_0, x_0, \mathbf{k}_x, x_s)}{\partial z} G_I(\omega_0, x_0; x_s) \|^2
\]

where \(F_a(\omega_0, \mathbf{k}_x | x_0)\) is a correction factor and \(a\) is the window width. Relating the local wavenumbers to their corresponding local angles, we obtain the correction factor in the local angle domain

\[
F_a(\omega_0, \theta_0 | x_0) = 4k_0^2 \int_{|k_x|} \frac{\partial G_I(\omega_0, x_0, \theta_s, x_s)}{\partial z} G_I(\omega_0, x_0; x_s) | \int_{|k_x|} \frac{\partial G_I(\omega_0, x_0, \theta_s, x_s)}{\partial z} G_I(\omega_0, x_0; x_s) \|^2,
\]

where \(\theta\) is the local angle defined by \(\mathbf{k}_s = \mathbf{k}_g + \mathbf{k}_s\) and \(\bar{\theta} = (\bar{\theta}_s + \bar{\theta}_g)/2\) (Wu and Chen, 2002, 2006; Zhu and Wu, 2008, 2010); \(\bar{\theta}_s\) and \(\bar{\theta}_g\) are defined by \(\mathbf{k}_s\) and \(\mathbf{k}_g\), respectively; here \(\bar{\theta}_s = \sin^{-1}(\mathbf{k}_s \cdot c(x)/c(x))\) and \(\bar{\theta}_g = \sin^{-1}(\mathbf{k}_g \cdot c(x)/c(x)).\) Hence, the approximation makes the calculation of amplitude correction into an efficient procedure.

---

**Fig. 7.** Results of the velocity reconstruction based on the WKBJ approximated Green's functions using the Born data and cross-sections through these results. (a) Velocity reconstruction of the model with the average velocity perturbation 23% with respect to the average background velocity; (b) and (c) are the vertical and horizontal slices of the result (a), respectively; (d) Velocity reconstruction for the model with the average velocity perturbation 30% with respect to the average background velocity; (e) and (f) are the vertical and horizontal slices of the result (d), respectively.
3. Numerical examples

3.1. Synthetic data generated in shot domain

Our first model is a velocity perturbation in a gradient medium (Fig. 2). We generate the synthetic data (450 shots and 501 receivers for each shot profile) using both Born approximation and finite difference method, and the recording times for all those data are 4 seconds. We generate the Born data based on Eq. (1), the model (or object function) $O(x)$ is the velocity perturbation function with inhomogeneous background velocity model and the background velocity is defined according to the formula $c_0(z) = 3.5 + 0.12 \times z$ (km/s). We use two kinds of average velocity perturbations (the average value of $c(z) - c_0(z)/c_0(z)$) of the square box with respect to the average background velocity and the average velocity perturbations are +23% (Fig. 2) and +30%, respectively. Fig. 2 shows a velocity model of a square box with an average velocity perturbation 23% higher than the average background velocity. The size of the square box is 1 km × 1 km. We generate the synthetic data of the two different velocity models using a finite-difference method based on the exact 2D scalar wave equation, which will be called the scalar wave synthetic data in the rest of the paper.

$$\nabla^2 u(x, t) - \frac{1}{c^2(x)} \ddot{u}(x, t) = -f(t)\delta(x-x_s).$$  

(27)

Where $u(x,t)$ is the wavefield; $f(t)$ is the source time function located at $x_s$; $\delta(x - x_s)$ is the Dirac delta function; $\nabla^2$ is the Laplace operator; $\ddot{u}(x, t)$ is the second derivative of the wavefield $u(x,t)$ with respect to time $t$.

We use 8 Ricker wavelets with different dominant frequencies (i.e., 0.3, 0.5, 0.75, 1.13, 1.70, 2.56, 3.86 and 5.81 Hz) (Fig. 3) as the source wavelets to generate synthetic data and the dominant frequencies are selected according to the formula (Sirgue and Pratt, 2004)

$$f_{n+1} = f_n \sqrt{1 + \left(\frac{h_{\text{max}}}{z}\right)^2}, \quad (n = 1, 2, 3, ...)$$  

(28)

where $f_n$ is the $n$th selected dominant frequency of the Ricker wavelet; $h_{\text{max}}/z$ is the ratio of the maximum half offset $h_{\text{max}}$ versus the depth $z$. 

![Fig. 8. Results for the velocity reconstruction based on a LCB one-way propagator using the Born data and cross-sections through these results. (a) Velocity reconstruction for the model with the average velocity perturbation 23% with respect to the average background velocity; (b) and (c) are the vertical and horizontal slices of the result (a), respectively; (d) Velocity reconstruction for the model with the average velocity perturbation 30% with respect to the average background velocity; (e) and (f) are the vertical and horizontal slices of the result (d), respectively.](image-url)
3.2. Spectrum recovery

We decompose the wavefields into the local wavenumber domain (or local angle domain) based on the local plane wave decomposition technique (Eq. (7)) (Chen et al., 2006; Wu, 2007; Wu and Chen, 2001, 2006) and apply the imaging condition in the local wavenumber domain (Eq. (11)) to the local plane waves from both sources and receivers, we then obtain the local image matrix $L(\omega, x, K)$ and the local

![Fig. 9. Results for the velocity reconstruction based on a LCB one-way propagator using the scalar wave synthetic data and cross-sections through these results. (a) Velocity reconstruction for the model with the average velocity perturbation 23% with respect to the average background velocity; (b) and (c) are the vertical and horizontal slices of the result (a), respectively; (d) Velocity reconstruction for the model with the average velocity perturbation 30% with respect to the average background velocity; (e) and (f) are the vertical and horizontal slices of the result (d), respectively.](image)

![Fig. 10. 2D Marmousi velocity model (Bourgeois et al., 1991).](image)
Fig. 11. The 2D Marmousi velocity model, its spectra and their multi-scale representations (from 1 to 6 scales) generated by multi-resolution analysis using dyadic scaling. On the left column, from (a) to (f) are the corresponding multi-scale representations from 6 to 1 of the Marmousi velocity model, respectively; on the right column, from (h) to (n) are the corresponding spectra of those from (a) to (g) on the left column, respectively; (g) and (n) are the exact 2D Marmousi velocity model and its spectra, respectively.
resolving kernel matrix $\mathcal{R}(\omega, \mathbf{r}, \mathbf{R})$ at $\mathbf{x}$ for a specific frequency $\omega$. We compute the local image matrices of all the points in the model and their corresponding local resolving kernels matrices only for the dominant frequencies of the synthetic data. From one pair of local incident-receiving angle $(\theta_i, \theta_r)$ of a single frequency data, we can obtain only the spectrum of the object at one wavenumber $\mathbf{K} = \mathbf{K}_i$, hence the spectra of all the scattering angles from one incident angle $\theta_i$ can cover only the half circle in the spectral domain. The information of the object in spectral domain is conjugated symmetric if the velocity perturbation of the half circle in the spectral domain. The information of the object in the other half circle in spectral domain for both the local image matrix and local resolving kernel matrix can be assumed known as

$$L(\omega, \mathbf{x}, -\mathbf{K}) = L'(\omega, \mathbf{x}, \mathbf{K}).$$

However, there are still two circular blind areas in the domain of spectral coverage of a single frequency. In order to fill up the blind areas in the spectral domain, we average the spectral information of local image matrices generated using the synthetic data with different dominant frequencies. Fig. 4 shows the domain of multi-frequency spectral coverage of local image matrices, which are generated using the data with eight different dominant frequencies. The information of blind areas in the spectral domain has been recovered partially, and the more frequency spectra are used for averaging, the better coverage in the spectral domain we can obtain.

We choose the same three points, which are pointed out by the arrowheads in the models in Fig. 5, to show the original spectra of the local image matrices of them and their corresponding recovered spectra for the scalar wave synthetic data (Fig. 6), which have been corrected by the amplitude correction factors. We show the spectra of the local image matrices according to the same sequence as that shown in Fig. 5. From Fig. 6, we see that the information of the blind area in the spectral domain has been partially recovered using those of multi-frequency spectra and the qualities of the recovered spectra have been greatly improved for both the coverage and uniformity in the local wavenumber domain, especially for the lower wavenumber component. Though the original spectra of those LIMs are different from each other for the two average velocity perturbations (23% and 30%) in the velocity models, their recovered spectra are very similar.

3.3. Velocity reconstruction

3.3.1. Square box model

In order to compare the results of generalized diffraction tomography based on the different kinds of Green’s functions during the inversion process, we use two kinds of Green’s functions: the WKBJ approximated 2D scalar wave Green’s functions (Cayton and Stolt, 1981; Ursin, 1984; Wu and Cao, 2005) and the exact 2D scalar wave Green’s functions, which are implemented using a local cosine basis (LCB) one-way propagator (Wu and Mao, 2007; Wu et al., 2008). The Green’s functions based on the WKBJ approximation (Cayton and Stolt, 1981; Ursin, 1984; Wu and Cao, 2005) and the exact 2D scalar wave Green’s functions, which are implemented using a local cosine basis (LCB) one-way propagator (Wu and Mao, 2007; Wu et al., 2008), are

$$G_L(x, z; x_s, z_s) = -\frac{1}{2\pi} \int_{B(a)} d\omega \int_{-\infty}^{\infty} dk' \frac{1}{\pm 2i\sqrt{k_z(z_s)(z)}} \exp \left\{ ik_x x' \pm \int_{z}^{z_s} k_z(z')dz' \right\},$$

where ‘±’ means the forward and backward propagations of the Green’s functions. The local cosine basis (LCB) one-way propagator is a beamlet propagator, which decomposes (or reconstructs) the wavefield with a local cosine basis (LCB) and constructs the propagator matrix in the LCB beamlet domain based on local perturbation theory (Wu and Mao, 2007; Wu et al., 2008).

Due to the dominant frequencies are around 0.3–5.81 Hz, we can recover the low-frequency component of the velocity model. We obtain the results of the generalized diffraction tomography using the Born data based on the WKBJ approximated Green’s functions (Fig. 7) and a local cosine basis (LCB) one-way propagator (Fig. 8). We also obtain

![Image](Fig. 12). Smoothed layered initial velocity model and the result of the velocity reconstruction based on LCB one-way propagator using the scalar wave synthetic data. (a) Smoothed layered initial velocity model; (b) Velocity reconstruction for the Marmousi model.
the results using the scalar wave synthetic data based on a local cosine basis (LCB) one-way propagator (Fig. 9). From Figs. 7, 8 and 9, we see that all the results have very good recoveries in the low-wavenumber component of the velocity models. For the results processed using the Born data (Figs. 7 and 8), we see that the results are very similar for most of the data, expect there are slight differences in those results for both the vertical and horizontal slices (Figs. 7 and 8) from the data with average velocity perturbation 30%. From the results of the generalized diffraction tomography inversed using the scalar wave synthetic data (Fig. 9), we see that the results have good recoveries for the velocity anomalies in both vertical and horizontal slices in the model, expect there is a singularity in the vertical slice in Fig. 9e. The oscillation inside the high velocity box may be caused by the lack of some low-wavenumber component and internal multiples.

Fig. 13. Compare the different scale representations (from 1 to 6 scales) between the exact Marmousi velocity model (Fig. 10) and the result generated using the smoothed layered background velocity model (Fig. 12b). On the left column, from (a) to (f) are the corresponding scale representations from 6 to 1 of the exact Marmousi velocity model, respectively; on the right column, from (g) to (l) are the corresponding scale representations from 6 to 1 of the result generated using the smoothed layered initial velocity model, respectively.
As we know that the Born approximation is a weak scattering approximation and does not obey the energy conservation, and it is only valid when the scattered field is much smaller than the incident field, that is, the heterogeneities are weak and the propagation distance is short (Wu et al., 2007). The valid regions of the Born approximation are very different between the forward scattering and backscattering, and the forward scattering divergence is the weakest point of Born approximation (Wu et al., 2007). The total scattering field is the sum of scattered fields from all parts of the scattering volume. In the forward direction, the scattered fields from each part propagate with the same speed as the incident field, so they will be coherently superposed, leading to the linear increase of the total field, and the energy increase will result in a catastrophic divergence for long distance propagation (Wu et al., 2006, 2007). However, there is no incident wave in the backscattered fields from all scatters and all the contributions will be canceled out except the coherence region for backscattered waves whose size is about $\lambda/4$ because of the two-way travel time difference. Hence backscattering does not have the catastrophic divergence for Born approximation. Another problem is that the forward scattering is sensitive to the velocities of heterogeneities and the velocity perturbation will produce the changes of travel-time and phase of the wavefield, which can accumulate to considerable large values and cause the breakdown of the Born approximation. Therefore, the region of validity of the Born approximation for backscattering is much larger than that for forward scattering (Wu et al., 2007). However, the De Wolf approximation splits the scattering potential into the forward scattering and backscattering parts, and renormalizes the incident field and Green’s function into the forward propagated field and forward propagated Green’s function, respectively (De Wolf, 1971, 1985). The forward propagated field is the sum of an infinite sub-series including all the multiple forescattered fields and the forward propagated Green’s function is the sum of a similar sub-series including multiple forward scattering corrections to the Green’s function. With the renormalized incident field and Green’s function, the local Born approximation has proved to work very well for practical applications according to the reconstruction results of velocity anomalies in this paper (Figs. 7, 8 and 9).

In summary, the generalized diffraction tomography in the current implementation is valid for the media with moderate velocity perturbations. It can recover the long wavelength components of average velocity perturbations up to 23% with respect to the background velocity in this box model.

### 3.3.2. 2D Marmousi model

We generate the scalar wave synthetic data based on the 2D Marmousi model (the survey is an off end survey with 96 receivers to the left of the source being pulled towards the right and the total source number is 240, receiver as well as shot spacing is every 25 m, near offset is 425 m from the source, 726 time gates are recorded with 4 ms spacing) (Bourgeois et al., 1991) (Fig. 10) using the eight Ricker wavelets with different dominant frequencies (i.e., 0.3, 0.4, 0.61, 1.23, 2.49, 5.05, 8.10 and 10.27 Hz). We obtain the final results of velocity reconstruction for 2D Marmousi model (Figs. 12, 14 and 16) based on the LCB one-way propagator. We use three different velocity models as the initial models: the smoothed layered velocity model (Fig. 12a), the vertical gradient velocity model (Fig. 14a) and the heavily smoothed exact Marmousi velocity model (Fig. 16a). Their corresponding velocity reconstruction results are shown in Figs. 12b, 14b and 16b, respectively.

In order to compare the reconstruction results of the velocity model with the exact one in detail, we use multi-scale analysis to those velocity models and decompose them into multi-scale representations by multi-resolution analysis using dyadic scaling. In this paper we use curvelet transform method (Candès and Demanet, 2005; Candès and Donoho, 2000) to decompose the velocity models into multi-scale. The Curvelet transform is a higher dimensional generalization of the wavelet transform designed to represent images at different scales and different angles. Multi-scale analysis method has been used in comparing the edge detection (Dessing et al., 1996) with the migration process and AVA analysis (Wapenar, 1997).

In this paper, in order to find the validity of the generalized diffraction tomography, we use multi-scale nature of the velocity models to compare the reconstruction velocity results with the exact one. We decompose the exact Marmousi velocity model (Fig. 11a–f) and all the results of velocity reconstruction (Figs. 13g–l, 15g–l and 17g–l) into 6 different scales, then compare those results of the velocity reconstruction with the exact one in terms of the 6 different scales (Figs. 13, 15 and 17). Fig. 12a–f are the multi-scale presentations of the Marmousi velocity model (Fig. 10 or Fig. 11g) and Fig. 11h–n are their corresponding spectra of the Figs. 11a–g, respectively. Figs. 13a–f, 15a–f and 17a–f are the same as those shown in Fig. 11a–f; Figs. 13g–l, 15g–l and 17g–l are the corresponding multi-scale presentations of those velocity reconstruction results, which are obtained using the smoothed layered velocity model (Fig. 12b), vertical gradient velocity model (Fig. 14b) and heavily smoothed Marmousi velocity model (Fig. 16b) as the initial models, respectively.

From Fig. 11 we see that different scale representations of the velocity model are related to different bandwidths of the velocity spectra. The bigger the scale is, the more concentrative the spectra are. From Figs. 12 and 13 we see that the components of scales 6 and 5 of the Marmousi velocity model have been partially recovered, that is some low wavenumber component of the velocity model has been recovered. The horizontal lines in Figs. 12b and 13g–l are the tomographic artifacts caused by the high wavenumber components of the layered velocity model. From Figs. 14 and 15 we see that the components of scales 6, 5, 4 and 3 have been recovered very well and those results include broader bandwidth in the wavenumber domain and less artifacts than those obtained using the smoothed layered velocity model. From Figs. 16 and 17 we see that most of the component of the Marmousi velocity model has been recovered very well except those of scales 1 and 2, the only artifacts, which close to the boundary, are caused by Gibbs effect due to the cut off in the spectral domain and the boundary effect of the discrete curvelet transform. The low wavenumber component of the Marmousi velocity model has been reconstructed nearly completely. From Figs. 13, 15 and 17, we see that the high wavenumber component of the velocity model has not been recovered at all for all the velocity reconstruction results, because that the Born approximation is valid to the low frequency component of seismic data and it’s difficult to recover the information of high frequency component of them.

3.3.2.1. 2D Marmousi model

![Fig. 14](image-url) Vertical gradient initial velocity model (i.e., a $v(z)$ velocity model) and the result for the velocity reconstruction based on a LCB one-way propagator using the scalar wave synthetic data. (a) Vertical gradient initial velocity model; (b) velocity reconstruction for the Marmousi model.
so is the generalized diffraction tomography based on the Born model. To solve this problem we can use the generalized diffraction tomography based on the boundary scattering model (Wu, 2007) to obtain the information of high wavenumber component of velocity model in the future research, therefore we can reconstruct completely the velocity model by combining the two kinds of tomographic methods in the future.

Hence, the generalized diffraction tomography in the current implementation can recover the low wavenumber component of the Marmousi velocity model (Figs. 12b, 14b and 16b) very well with different initial velocity models (Figs. 12a, 14a and 16a), especially the heavily smoothed Marmousi velocity model (Fig. 16a).

All the results of the velocity reconstruction (Figs. 12b, 14b, and 16b) are obtained by stacking the inversion results with only 8 different

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Fig. 15. Compare the different scale representations (from 1 to 6 scales) between the exact Marmousi velocity model and the result generated using the vertical gradient initial velocity model (Fig. 14b). On the left column, from (a) to (f) are the corresponding scale representations from 6 to 1 of the exact Marmousi velocity model, respectively; on the right column, from (g) to (l) are the corresponding scale representations from 6 to 1 of the result generated using the vertical gradient initial velocity model, respectively.

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these factors will be canceled when applying the inverse Hessian to conjugate in the phase term to the modeling operator. In principle, necessary for the focusing operator which should be the complex-joint operator must have the same attenuation term. However, it is not this paper, the generalized diffraction tomography based on volume scattering model would be used to obtain the information of high-frequency component of the velocity model.

4. Discussion

Here we would like to discuss the relationship between the resolving kernel derived in this paper and the Hessian matrix in least-square inversion because they are closely related to each other. In the least-square inversion (Beylkin, 1985; Chavent and Plessix, 1999; Clapp et al., 2005; Hu et al., 2001; Kuhl and Sacchi, 2003; Lailly, 1983; Plessix and Mulder, 2004; Pratt et al., 1998; Rickett; 2003; Tarantola, 1984a; ten Kroode et al., 1994; Valenciano et al., 2005, 2006), which is based on the minimizing the misfit function using adjoint operator in the Hessian matrix, the backpropagation operator is defined as the gradient of the least-square error function, and the Hessian matrix is the second derivative of the error function (Plessix and Mulder, 2004). There are several differences between the formula of the resolving kernel and that of Hessian matrix. Firstly, they have different imaging operators although their forward modeling operators are the same. The imaging operator of the resolving kernel used in this paper is a focusing operator (backpropagation integral), while the Hessian matrix is an adjoint operator of the modeling operator. If there is an attenuation term in the modeling Green’s function, such as $e^{-i\omega\tau}$, then the adjoint operator must have the same attenuation term. However, it is not necessary for the focusing operator which should be the complex-conjugate in the phase term to the modeling operator. In principle, these factors will be canceled when applying the inverse Hessian to the image field. However, in strong attenuation media, the attenuation in the adjoint operator could kill the already weak scattered signal in the data and make the inversion unstable during the computation. Finally, for the gradient component of the Hessian matrix, the adjoint operator has the same Green’s function as that in the modeling operator, but phase conjugated; while that of the focusing operator is the normal gradient of the Green’s function, which corresponds to a propagator for boundary elements instead of volume elements. It has some advantages in calculating the resolving kernel when energy conservation is invoked for a certain kind of approximation.

Due to the intrinsic disadvantage of the Born approximation, in this paper, the generalized diffraction tomography based on volume scattering model can provide a smooth background velocity model, that is, we can obtain the information of low frequency component of the velocity model. Though the frequencies of those synthetic data used in this paper are very low (i.e., 0.3, 0.5, 0.75, 1.13, 1.70, 2.56 and 5.81 Hz), such low frequency seismic data can be received using broadband seismometers which can record the seismic signals with low frequencies close to 0.01 Hz. To recover the information of high frequency component of the velocity model, that is, the information about the sharp contrasts of the velocity model, we need the scattering tomography based on the boundary scattering model (Wu, 2007). Boundary scattering can be formulated in several different methods and Kirchhoff model is one of some convenient approximations. The Kirchhoff approximation is a kind of tangent plane approximation (Ogilvy, 1991; Wu, 2007) or optical approximation. Therefore, the Kirchhoff theory is a high-frequency asymptotic solution of a smooth interface approximation, that is, it is accurate for the smoothly varying interface but has limited use for rough surfaces (Cao, 2008; Wu, 2007). In a word, the diffraction tomography based on boundary scattering model will be used to obtain the information of high-frequency component of the velocity model.

5. Conclusions

The generalized diffraction tomography based on volume scattering model can recover the information of the low wavenumber component of the velocity model very well using multi-frequency data sets. Through the tests of the three models with different velocity perturbations (23% and 30%) with respect to the background velocity and using both the Born data and the scalar wave synthetic data, we see that the information of the blind area in the spectral domain has been partially recovered using those of multi-frequency spectra; The qualities of the recovered spectra have significant improvement in both coverage and uniformity in the local wavenumber domain; the generalized diffraction tomography in the current implementation is valid for the media with moderate velocity perturbations. It can recover the long wavelength components of average velocity perturbations up to 23% with respect to the background velocity in the square box model. Comparing the multi-scale representations of the exact Marmousi velocity model with those of the reconstruction results of the generalized diffraction tomography, which are generated using different initial velocity models, it can also reconstruct the low wavenumber components of the Marmousi velocity model with the low-frequency waves being used in the implementation.

In future research, we will try high contrast media and different background velocities models, and define the factors, which influence the recovery of the long wavelength components of the velocity model, to improve the validity of generalized diffraction tomography based on the Born model. Combining the generalized diffraction tomography based on volume scattering model with that based on the boundary scattering model to reconstruct the complete information of the velocity model.

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Fig. 17. Compare the different scale representations (from 1 to 6 scales) between the exact Marmousi velocity model and the result generated using the heavily smoothed Marmousi velocity model (Fig. 16b). On the left column, from (a) to (f) are the corresponding scale representations from 6 to 1 of the exact Marmousi velocity model, respectively; on the right column, from (g) to (l) are the corresponding scale representations from 6 to 1 of the result generated using the heavily smoothed Marmousi velocity model, respectively.

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