Imaging diffraction points using the local image matrices generated in prestack migration

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ABSTRACT

Energy-angle distributions in the local image matrix, which is a function of local incident and receiving angles at a subsurface point, are different for a planar reflector and a diffraction point. The former exhibits a linear distribution along a certain dip direction, whereas the latter shows a scattered distribution in the entire matrix. Therefore, the singular values of the local image matrix in the local angle domain indicate the energy distribution along different dip directions. The difference between the auto- and crosscorrelation coefficients of the sets of singular values between adjacent image points can be used to distinguish these two types of targets and obtain prestack images which contain only diffraction points. Using three synthetic examples, we found that the method can effectively image diffraction points. Seismic images of diffraction points may provide important information about geological discontinuities and diffraction points can be taken as secondary sources for potential applications in imaging subsalt structures which are poorly illuminated by primary waves.

INTRODUCTION

Diffraction points contain valuable information about subsurface structures such as faults, pinchouts, rough edges, fractures, channels, salt bodies, small-sized scatterers, and any sudden changes in structures. The significance of diffracted waves in seismic imaging was recognized more than fifty years ago (Krey, 1952). Diffraction can be used to detect local heterogeneities. Upon illumination by a plane wave, a plane specular reflector responds with a plane wave, while a point diffractor acts as a point source. These different responses to an incident plane wave can be used to separate specular reflections and diffraction events (Taner et al., 2006). Plane-wave destruction filters have been used to suppress specular events to get plane wave sections of diffractions (Fomel, 2002; Taner et al., 2006; Fomel et al., 2006, 2007; Liu et al., 2007).

Several workflows have been tested to enhance diffraction-like signals and remove ordinary reflections (Bansal and Imhof, 2005) by working directly on the data. The results show that the eigenvector filter is the most efficient one for both 2D and 3D data sets. The coherence of seismic data measured by the crosscorrelation between each seismic trace and its neighboring traces has proven to be an effective method for imaging geological discontinuities. The seismic coherence highlights subtle features which may not be readily apparent in the seismic data. The coherence algorithms have been developed from using only three traces (Bahorich and Farmer, 1995) to multitraces. The multitrace coherence measurement is based on the eigenstructure of the covariance matrix formed from the traces in the analysis cube (Gersztenkorn and Marfurt, 1999; Marfurt and Kirlin, 2000).

In this paper, we propose a diffraction point imaging method in the local angle domain based on the difference between the scattering patterns of local planar reflectors and those of diffraction/scattering points. The beamlet or the local plane wave decomposition provides localizations in both space and wavenumber domains for the wavefield (Wu et al., 2000, 2008; Wu and Chen, 2001, 2006; Chen et al., 2006). Using an imaging condition in the local angle domain, we can obtain the local image matrix (LIM) for each image point during the migration process. The scattering patterns of local heterogeneities are imprinted in the local image matrices (LIMs). For a planar reflector, the energy of the LIM concentrates linearly along a certain dip direction; while for a diffraction point, the energy of the LIM scatters around the entire matrix. In this study, we apply the singular value decomposition (SVD) technique to the LIMs. We then use the auto- and crosscorrelation methods on the sets of singular values of those LIMs to identify and image diffraction points in the model. The choice of thresholds in separating planar reflectors and diffraction points is an important issue, and is discussed in detail in this paper.
In this paper, we describe the theory of an imaging condition in the beamlet domain and the LIM. We then discuss the energy distributions of different objects in the LIMs and show the data processing flow in separating diffraction points from planar reflectors. Finally, we use two simple models and the 2D SEG-EAGE salt model as examples to demonstrate the feasibility of our approach.

METHOD

Imaging condition in the beamlet domain and the local image matrix

We use a wave-equation-based wavefield extrapolation formulation based on local perturbation theory to extrapolate wavefield from the surface to depth $z$ (Wu and de Hoop, 1996; Wu et al., 2000; 2008; Chen et al., 2006; Wu and Chen, 2006). By performing beamlet decomposition to the wavefields (Figure 1) and applying an imaging condition (Wu and Chen, 2001, 2002a, 2002b) to the decomposed wavefields, we obtain the image in the beamlet domain and in the local angle domain. For each point source, the forward-propagated wavefield can be decomposed locally at the image point $(x, z)$ (Figure 1):

$$u^S(x, z, \omega) = \sum_{l} \sum_{j} u^S(\vec{x}_l, \vec{\xi}_j, z, \omega) g_{jl}(x)$$

$$= \sum_{j} e^{i\vec{\xi}_j} \sum_{l} g(x - \vec{x}_l) \hat{u}^S(\vec{x}_l, \vec{\xi}_j, z, \omega),$$

(1)

with

$$\hat{u}^S(\vec{x}_l, \vec{\xi}_j, z, \omega) = \langle u^S(x, z, \omega), g_{jl}(x) \rangle$$

$$= \int dx u^S(x, z, \omega) \hat{g}(x - \vec{x}_l) e^{-i\vec{\xi}_j},$$

where $u^S(x, z, \omega)$ is the incident wavefield at $(x, z)$ propagated from the source on the surface (Figure 1); $\hat{g}(\vec{x}_l, \vec{\xi}_j, z, \omega)$ is the corresponding beamlet coefficient of the incident wavefield at depth $z$, the parameters $\vec{x}_l = l \Delta x$ and $\vec{\xi}_j = j \Delta \xi$ $(l, j = 1, 2, 3, \ldots)$ are the $l$th window location and the $j$th local wavenumber position, respectively; here $\Delta x$ and $\Delta \xi$ are the space and wavenumber sampling intervals of the frame vectors, respectively; we use oversampling $(\Delta x, \Delta \xi < 2\pi)$ for stable reconstruction; a bar over a variable means the localization in either the space domain ($\vec{x}_l$) or the wavenumber domain ($\vec{\xi}_j$); $\omega$ is the circular frequency; the operator $\langle \rangle$ denotes the inner product; $g_{jl}(x)$ is the Gabor-Daubechies (G-D) frame atom; the Gabor-Daubechies frame is a windowed Fourier frame with Gaussian window (Daubechies, 1990, 1992; Feichtinger and Strohmer, 1998; see more detail in Appendix A in Chen et al., 2006 and Appendix A in Wu and Chen, 2006); $\hat{g}(\vec{x}_l) = e^{i\vec{\xi}_j}g(x - \vec{x}_l)$ is the dual frame atom; $g(x - \vec{x}_l) = (2\pi)^{-1/2}e^{-i(x - \vec{x}_l)^2}$ is the Gaussian window function for spatial localization and $\hat{g}(x - \vec{x}_l)$ is the dual window function which can be calculated by a generalized inverse method (Daubechies, 1992; Mallat, 1998; Chen et al., 2006); the asterisk denotes complex conjugate. The recorded scattered wavefield at each receiver on the surface can be back-propagated to the image space and decomposed locally at the same image point $(x, z)$ (Figure 1):

$$u^{RS}(x, z, \omega) = \sum_{q} \sum_{p} u^{RS}(\vec{x}_q, \vec{\xi}_p, z, \omega) g_{pq}(x)$$

$$= \sum_{p} e^{i\vec{\xi}_p} \sum_{q} g(x - \vec{x}_q) \hat{u}^{RS}(\vec{x}_q, \vec{\xi}_p, z, \omega),$$

(2)

with

$$\hat{u}^{RS}(\vec{x}_q, \vec{\xi}_p, z, \omega) = \langle u^{RS}(x, z, \omega), g_{pq}(x) \rangle$$

$$= \int dx u^{RS}(x, z, \omega) \hat{g}(x - \vec{x}_q) e^{-i\vec{\xi}_p},$$

where $u^{RS}(x, z, \omega)$ is the scattered wavefield at $(x, z)$ back-propagated from the receiver at $x$, on the surface due to the source at $x$, in the frequency domain (Figure 1); $\hat{g}(\vec{x}_q, \vec{\xi}_p, z, \omega)$ is the corresponding beamlet coefficient of the scattered wavefield at $(x, z)$. By crosscorrelating of $u^S(x, z, \omega)$ and $u^{RS}(x, z, \omega)$ and summing up the contributions from all the source-receiver pairs, we obtain the image for each frequency:

$$I_{\omega}(x, z, \omega) = \sum_{S} u^S(x, z, \omega) \sum_{Rs} u^{RS}(x, z, \omega)$$

$$= \sum_{S} \sum_{p} e^{i(\vec{\xi}_j - \vec{\xi}_p)} \sum_{l} \sum_{q} M_{lqps}(\vec{x}_l, \vec{\xi}_j, \vec{x}_q, \vec{\xi}_p, z, \omega)$$

$$\times g(x - \vec{x}_l) g(x - \vec{x}_q),$$

(3)

with

$$M_{lqps}(\vec{x}_l, \vec{\xi}_j, \vec{x}_q, \vec{\xi}_p, z, \omega) = \sum_{S} \hat{u}^{S}(\vec{x}_l, \vec{\xi}_j, z, \omega) \sum_{Rs} \hat{u}^{RS}(\vec{x}_q, \vec{\xi}_p, z, \omega),$$

where $I_{\omega}(x, z, \omega)$ is the image at point $(x, z)$ for a specific frequency $\omega$; the summation indices $S$ and $Rs$ denote the sums of the wavefields or the beamlet coefficients of the wavefields from all the sources and receivers, respectively; $M_{lqps}(\vec{x}_l, \vec{\xi}_j, \vec{x}_q, \vec{\xi}_p, z, \omega)$ is the image produced by the windowed incident plane wave in the $l$th window $\vec{x}_l$ with the $j$th wavenumber $\vec{\xi}_j$ and the windowed scattered plane wave in the $q$th window $\vec{x}_q$ with the $p$th wavenumber $\vec{\xi}_p$. Summing up the contributions from all the beamlets with the same local wavenumber in the neighboring windows, we obtain the local image matrix (LIM) in the wavenumber domain for a specific frequency $\omega$:  

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**Figure 1.** Schematic diagram showing the imaging process. Here $u^S(x, z)$ is the incident wavefields at the image point $(x, z)$ propagated from the source on the surface and $u^{RS}(x, z)$ is the scattered wavefield at the image point $(x, z)$ backpropagated from the receiver on the surface.
\[ L_{ip}(\vec{x}_p, z, \omega) = \sum_{\Omega} \left( W^S_{ip}(x, \vec{x}_p, z, \omega) \sum_{Rs} W^{Rs}_{i}(x, \vec{x}_p, z, \omega) \right), \]

with
\[ W^S_{ip}(x, \vec{x}_p, z, \omega) = e^{i\vec{\xi}_p \cdot \vec{x}} g(x - \vec{x}_p), \]
\[ W^{Rs}_{i}(x, \vec{x}_p, z, \omega) = e^{i\vec{\xi}_p \cdot \vec{x}} g(x - \vec{x}_p), \]

where \( L_{ip}(\vec{x}_p, z, \omega) \) is the LIM at point \((x, z)\) for a specific frequency \( \omega \) in terms of the local incident plane wave with the wavenumber \( \vec{\xi}_p \) and the local scattered plane wave with the wavenumber \( \vec{\xi}_p \). \( W^S_{ip}(x, \vec{x}_p, z, \omega) \) and \( W^{Rs}_{i}(x, \vec{x}_p, z, \omega) \) are the local incident plane wave with the wavenumber \( \vec{\xi} \) and the local scattered plane wave with the wavenumber \( \vec{\xi}_p \), respectively; \( i \) is the window length.

From now on, we denote the local wavenumber of an incident plane wave as \( \vec{\xi}_i \) and the local wavenumber of a scattered plane wave as \( \vec{\xi}_s \). The local wavenumbers \( \vec{\xi}_s \) and \( \vec{\xi}_s \) can be related to \( \theta_i \) and \( \theta_s \), which are the local incident and receiving angles, respectively (Figure 2a). Figure 2a shows the definitions of the reflection and dip angles: \( \theta_i, \theta_s, \) and \( \theta_s \) are the source angle, incident angle, and receiving angle, respectively; here \( \theta_r = \theta_s \_s \) and \( \theta_s \) are the normal angle of the dipping reflector and the reflection angle with respect to the normal, respectively. In Figure 2b and c, \( \theta_i, \theta_s, \) and \( \theta_s \) are the local incident and receiving angles, respectively; \( \theta_i \) and \( \theta_s \) are the local normal and reflection angles, respectively; here, \( \theta_s = (\theta_x + \theta_s)/2 \) and \( \theta_s = (\theta_s + \theta_s)/2 \). The local wavenumbers and the local incident and receiving angles are related by

\[ \vec{\xi}_i = k_0 \sin \theta_i, \]
\[ \vec{\xi}_s = k_0 \sin \theta_s, \]
\[ |d\vec{\xi}_i| = k_0 \cos \theta_i |d\theta_i|, \quad \text{and} \]
\[ |d\vec{\xi}_s| = k_0 \cos \theta_s |d\theta_s|. \]

where \( I(\theta_i, \theta_s, x, z) \) is the common dip-angle image at point \((x, z)\) with normal angle \( \theta_i \). \( I(\theta_i, \theta_s, x, z) \) is the LIM in the local angle domain at point \((x, z)\) by transforming coordinates from the incident-receiving angle pair \((\theta_i, \theta_s)\) to the normal-reflection angle pair \((\theta_i, \theta_s)\).

The energy distributions of different objects in the local image matrices

The local scattering matrix is defined as the matrix of scattering amplitude represented in terms of the incident-receiving angle pair.
Hence, it is the intrinsic property of the scattering medium. It is independent of the acquisition system and free from propagation effects. It also contains the information of the local structure and the elastic properties revealed by the imaging experiment at a local heterogeneity point. The LIM is the local scattering matrix distorted by acquisition and propagation effects. However, the basic difference between a local planar reflector and a diffraction point remains unchanged in the LIM. Also, the acquisition aperture correction (Wu et al., 2004; Cao and Wu, 2008) can be applied to the LIM. For a planar reflector, most of the energy in LIM is distributed linearly along a certain dip direction \( \theta_s \) (Figures 2c and 3a); while for a diffraction point, the energy in the LIM scatters widely in the entire matrix (Figures 2c and 3b) because it does not have a well-defined normal direction.

**Separation of diffraction points from planar reflectors**

Recognizing that the energy of a planar reflector in the LIM distributes along a certain dip direction, we transform the LIM from the form represented in terms of the incident and receiving angles pair \((\theta_i, \theta_r)\) (Figure 2b) to that represented in terms of the normal and reflection angles pair \((\bar{\theta}_n, \bar{\theta}_r)\) (Figure 2c). In the latter representation of the LIM, the energy distribution of a common dip angle is along a horizontal line and the distributions belonging to different common dip angles are along different horizontal lines. We perform singular value decomposition to the matrix

\[
I_\text{nr}(\bar{\theta}_n, \bar{\theta}_r) = U \Sigma V^T,
\]

with

\[
\Sigma = \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix},
\]

\[
\Sigma_r = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r), \quad (\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0),
\]

where \(I_\text{nr}(\bar{\theta}_n, \bar{\theta}_r)\) is a LIM in terms of the normal-reflection angle pair \((\bar{\theta}_n, \bar{\theta}_r)\). For the sake of simplicity we drop the position variables \((x, z)\) in \(I_\text{nr}(\bar{\theta}_n, \bar{\theta}_r, x, z)\). \(\Sigma\) is an \(m \times n\) matrix, \(U\) an \(m \times m\) matrix and \(V\) an \(n \times n\) matrix. Both \(U\) and \(V\) have orthogonal columns. \(V^T\) denotes the transpose of the matrix \(V\). The singular values \((\sigma_1, \sigma_2, \ldots, \sigma_r)\) can be viewed as the projections of the energy distribution along the set of dip directions \(\bar{\theta}_r\) (Figure 2c).

We use Equation 7 to reconstruct the LIM; matrices \(U\) and \(V\) are obtained using singular value decomposition to the original LIM. In order to see the representations of the different singular values, we show the matrix which has only the \(i\)th singular value \(\Sigma_i\), that is,

\[
\Sigma_i^l = \text{diag}(\ldots, 0, \sigma_i, 0, \ldots), \quad (1 \leq i \leq r).
\]

Figure 4a and b shows the singular values of the original LIMs represented in terms of the normal and reflection angles pair in Figure 3a and b, respectively; Figure 5 and 6 show the relationships of LIMs with their singular values for a planar reflector and a diffraction point, respectively; Figure 5a and Figure 6a are the original LIMs; Figure 5b and Figure 6b are the recovered LIMs using only the four biggest singular values in Figure 4a and b, respectively; Figure 5c-f shows the LIMs of a planar reflector recovered using only the first, second, third, and fourth singular values (see Figure 4a), respectively; Figure 6c-f shows the LIMs of a diffraction point recovered using only the first, second, third, and fourth singular values (see Figure 4b), respectively. We can see that the original LIM can be recovered by its first few singular values which correspond to the different energy distribution patterns. The singular value distribution of a diffraction point (Figure 4b) scatters more than that of a planar reflector (Figure 4a). Hence, the LIMs of some neighboring planar reflectors will have similar singular values distributions and the auto- and crosscorrelation coefficients of these sets of singular values should be more concentrated than those of diffraction points. The difference between these two kinds of correlation coefficients corresponds to the difference between the local structure at a diffraction point and that at a planar reflector. Therefore, we can use this as a criterion to separate diffraction points from planar reflectors and finally obtain the images of diffraction points. The image amplitude of the diffraction point results from summing up all the values in the corresponding LIM.

In summary, the workflow for separating diffraction points from planar reflectors is

1) Calculate the LIMs for all the points in the image space using the beamlet imaging technique in the local angle domain.
2) Transform the LIM at each point \((x, z)\) from the incident-receiving angle domain \((I_\text{nr}(\bar{\theta}_n, \bar{\theta}_r))\) to the normal-reflection angle domain \((I_\text{nr}(\theta_i, \theta_r))\).
3) Compute the set of singular values of the LIM at each point \((x, z)\) using singular value decomposition technique.
4) Calculate crosscorrelation coefficients between the reference point and its neighboring points: Select a point as a reference point \(O\) (Figure 7a and b) and compute the crosscorrelation co-

![Figure 3](image-url)  
**Figure 3.** Amplitude distributions in the two local image matrices (LIMs) of different objects: (a) a planar reflector and (b) a diffraction point.

![Figure 4](image-url)  
**Figure 4.** Singular values of the original local image matrices (LIMs) represented in terms of the normal and reflection angles in Figure 3a and b, respectively. (a) the singular values of the original local image matrix in Figure 3a (b) the singular values of the original local image matrix in Figure 3b.
efficient \( C_{OQ} \) and \( C_{OQ'} \) of sets of singular values between the reference point \( O \) and its neighboring points \( Q \) (and \( Q' \) in the opposite direction of \( Q \)) in all possible directions. There are two different cases:

a) For the directions which pass through the nodes (i.e., grid points in the space domain), such as the directions of type 1 and 2 in Figure 7b, we choose point E (or F) as the neighboring point for the direction of type 1 (or type 2).

b) For the directions which do not pass through the nodes, such as the direction of type 3 in Figure 7b, we choose point G as the neighboring point and there is no value for point G during the computation. The crosscorrelation coefficient \( C_{OG} \) of sets of singular values between point O and G are obtained by weighted average value of those crosscorrelation coefficients \( C_{OF} \) and \( C_{OG} \) and the weighting factor is inversely proportional to the distance between point F (or point H) and point G. These crosscorrelation coefficients are to be compared with the autocorrelation coefficient \( C_{OO} \) of the reference point O in step (6).

5) Take each point \((x, z)\) in the original model as a reference point and repeat the same processes as described in step (4). Then sum up all the elements of the LIM at each image point to get

6) Select the diffraction points by the following criterion (Equation 11). By setting a proper threshold \( \alpha \), the selection can be done for any given point as

\[
\text{diffraction point imaging for } a, b, c, f, \text{ and } g
\]

Figure 6. Local image matrices corresponding to a diffraction point and its recovered local image matrices (LIMs) using different singular values (see Figure 4b): (a) the original local image matrix; (c-f), the recovered local image matrices correspond to the first, second, third, and fourth singular values (see Figure 4b), respectively; (b) the sum of all the local image matrices (c-f).

Figure 7. Diagram of a reference point and its neighboring points in all possible directions. (a) all possible directions corresponding to the reference point (the reference point is \( O \) and \( QQ' \) can be \( AA' \), \( BB' \), \( CC' \) or \( DD' \) which are different directions); (b) strategy for choosing neighbor points for two kinds of directions: one passes through the nodes (the directions of type 1 and 2); the other does not pass through the nodes (the direction of type 3).
\[ |C_{00} - C_{00'}| \leq \alpha \times C_{00'} \quad \text{planar reflector} \]
\[ |C_{00} - C_{00'}| > \alpha \times C_{00'} \quad \text{diffraction point} \]

where \( Q \) can be points \( A, B, C, D, \ldots \) (Figure 7a) at one side and \( Q' \) can be points \( A', B', C', D', \ldots \) at the opposite side. Through extensive numerical testing the threshold \( \alpha \) is chosen between 0 and 0.3.

**NUMERICAL EXAMPLES**

**Model 1**

We construct a test model composed of two high velocity (4700 m/s) horizontal reflectors in a background velocity model, which has a constant vertical gradient (Figure 8). We generate the data using a finite difference method with a Ricker wavelet of dominant frequency 15 Hz. For local angle domain decomposition, we use two types of window systems. In type A, the dominant frequency is 10 Hz, the frequency bandwidth is from 1 Hz to 20 Hz, and the window length is 810 m. In type B, the dominant frequency is 15 Hz, the frequency bandwidth is from 1 Hz to 30 Hz, and the window length is 510 m.

![Figure 8. Velocity model of two horizontal reflectors: Two high velocity (4700 m/s) horizontal reflectors in a background velocity model with a constant vertical gradient.](image)

![Figure 9. Images of regular migration and diffraction points for model 1 with type A window (dominant frequency \( f_0 = 10 \) Hz). (a) Regular image after prestack migration and (b) the images of diffraction points (\( \alpha = 0.10 \) for the directions of type 1 and 2 which pass through the nodes).](image)

![Figure 10. Images of regular migration and diffraction points for model 1 with type B window (dominant frequency \( f_0 = 15 \) Hz). (a) Regular image after prestack migration and (b) the images of diffraction points (\( \alpha = 0.12 \) for the directions of type 1 and 2).](image)

![Figure 11. Velocity model of two dipping reflectors: Two high velocity (4700 m/s) dipping reflectors in a background velocity model with a constant vertical gradient.](image)

**Model 2**

Model 2 is the same as model 1 except that the two high velocity (4700 m/s) reflectors are dipping (Figure 11). We generate the data using the same method and same Ricker wavelet as those used in model 1. The dominant frequency, frequency bandwidths and window lengths for type A and B are also the same as those used in model 1. The images of regular migration of the two types (type A, dominant frequency \( f_0 = 10 \) Hz and type B, dominant frequency \( f_0 = 15 \) Hz) are shown in Figures 12a and 13a, respectively. The images of diffraction points of the two types are shown in Figures 12b and 13b, respectively. The corresponding thresholds are \( \alpha = 0.10 \) for the horizontal and vertical directions and \( \alpha = 0.15 \) for the other directions.

Figure 9a and Figure 10a are the two regular images of prestack beamlet migration using the background velocity model 1. Figure 9b and Figure 10b depict the images of diffraction points of the two types (dominant frequency \( f_0 = 10 \) Hz for type A and \( f_0 = 15 \) Hz for type B) with different thresholds \( \alpha = 0.10 \) (Figure 9b) and \( \alpha = 0.12 \) (Figure 10b) for the directions of type 1 and 2 which pass through the nodes and \( \alpha = 0.0 \) for the directions of type 3 which do not pass through the nodes. We set the default value of \( \alpha = 0.0 \) for all the directions which are not explicitly specified in the rest of the paper. The results show that most of the energy of planar reflectors has been removed and the images of diffraction points coincide with the exact geological discontinuities.
tions of type 1 and 2 in Figure 12b, and $\alpha = 0.15$ for the directions of type 1 and 2 in Figure 13b. The results demonstrate the validity of the method even for finite dipping reflectors.

2D SEG-EAGE salt model

The acquisition for this experiment consists of 325 shots and each shot has 176 receivers on the left side. The LIM at each point is computed using the shot migration and a smoothed velocity model (Figure 14). We still use two types of window systems. Figure 15 shows the results with the type A window (dominant frequency $f_0 = 10$ Hz). Figure 15a shows the image after regular prestack beamlet migration and Figure 15b-d shows the images of diffraction points. In Figure 15b, $\alpha = 0.15$ for the horizontal and vertical directions and $\alpha = 0.20$ for the other directions of type 1 and 2; in Figure 15c, $\alpha = 0.10$ for the directions of type 1 and 2 and $\alpha = 0.004$ for the directions of type 3; in Figure 15d, $\alpha = 0.12$ for the directions of type 1 and 2 and $\alpha = 0.004$ for the directions of type 3. In Figure 16, the dominant frequency is $f_0 = 15$ Hz. Figure 16a is the regular image of the prestack beamlet migration and Figure 16b-d shows the images of diffraction points. In Figure 16b, $\alpha = 0.20$ for the directions of type 1 and 2; in Figure 16c, $\alpha = 0.12$ for the directions of type 1 and 2 and $\alpha = 0.005$ for the directions of type 3; in Figure 16d, $\alpha = 0.15$ for the directions of type 1 and 2 and $\alpha = 0.005$ for the directions of type 3.
By comparing the images of diffraction points in Figure 15 (Figure 15b-d from type A) and Figure 16 (Figure 16b-d from type B) with those images of regular migration (Figure 15a and Figure 16a), it is clear that most of the energy of reflected waves has been removed in the diffraction point imaging. By comparing the images of diffraction points in Figure 15 and 16, we find that the images of diffraction points become sharper for broader frequency bandwidth and higher frequency components. However, due to the roughness of some reflection planes in the original model, it is difficult to draw a hard line between a reflection and a diffraction for broadband signals. The images of diffraction points are frequency dependent.

By comparing our results with those of diffraction images obtained based on a plane-wave destruction filter method (Yilmaz and Taner, 1994; Fomel, 2002; Taner et al., 2006), such as Figure 5 (right), 6 (right) and 7b in the paper by Taner et al., 2006, we find that there are some separated events in deeper part of their results which are the triplications of the propagating plane wave caused by lateral velocity variations; however, there are no such artifacts in the diffraction points images (Figures 15b-d and 16b-d) obtained using our method.

**DISCUSSION**

In our method, the choice of the threshold \( \alpha \) (see the workflow of separating diffraction points from planar reflectors in the method section) depends on the structural dips of the subsurface, the acquisition aperture and frequency bandwidth of the seismic data. We can take advantage of the knowledge of structural dips to search for the possible directions in choosing the neighboring points of the reference points; we can also use the knowledge of acquisition aperture and frequency bandwidth of the seismic data to choose a proper threshold \( \alpha \). Local image matrices (LIMs) generated using seismic data with wider acquisition aperture and broader frequency bandwidth will contain more information about the local structures and the threshold \( \alpha \) will be more sensitive to the final results. Through extensive numerical tests we find that for those directions which do not pass through the nodes (directions of type 3), the thresholds should be chosen more carefully than those of other directions (directions of type 1 and 2) because there are no real nodes on those directions during computation and the weighted average crosscorrelation coefficients are approximate values.

We use local cosine beamlets decomposition (Wu et al., 2000; Wu and Chen, 2006) to transform the wavefield and image into the local angle domain in this paper. The local cosine beamlets decomposition is orthogonal and very efficient. However, the decomposition always has two symmetric lobes with respect to the vertical axis due to the inherent property of the local cosine basis function. Hence, the local cosine beamlets contain incomplete direction information. We expect that this shortcoming of the local cosine transform does not greatly affect the separation of diffraction points from the specular reflector. Nevertheless, we may test other schemes of local angle decomposition to further improve the performance of the diffraction point imaging.

The time consuming part of the procedure is computing the local image matrix at each point in the image space (see the work flow described in the method section). However, if the local image matrix at each point is available when performing the prestack depth migration, such as in the case of beamlet migration, or through local slant stack for acquisition aperture correlation (Wu et al., 2004; Cao and Wu, 2008), the extra computational time we need is only about 10% more than that of the prestack migration. Hence, the image of diffraction point imaging offers a valuable byproduct of prestack depth migration with little extra work.

Another potential application of diffraction point imaging is imaging with multiple scattered waves. Diffraction points with strong scattered waves, such as sharp tips and rough edges of irregular salt bodies, can be taken as secondary sources for imaging some subsalt structures which are hardly illuminated by primary waves. Diffraction point imaging can help in identifying the potential secondary sources. Combining with the techniques in interferometry in retrieving the Green’s functions of these secondary sources, it may provide additional illumination in the shadow zones of primary sources for subsalt imaging.

**CONCLUSIONS**

This paper proposes a method to separate diffraction points from planar reflectors based on the different energy distributions between...
the planar reflectors and the diffraction points in local image matrices. Through three numerical examples, we have demonstrated that the method is effective in obtaining the images of diffraction points. The results show that the diffraction points are well preserved and the planar reflectors are removed almost completely. Separating and imaging diffraction points from planar reflectors can provide valuable information about geological discontinuities, such as faults, pinchouts, rough edges, fractures, salt bodies, and any sudden changes in structures. The process of diffraction point imaging can be performed during the regular procedure of prestack migration, provided that the wavefield information in the local angle domain is available, such as in the process of beamlet migration, directional illumination analysis, or acquisition aperture correction.

For further study, we can use the local exponential beamlets decomposition which can provide complete direction information of local image matrices and therefore better resolution for images of diffraction points. The choice of thresholds is crucial for a successful application of this method and it needs to be further explored.

ACKNOWLEDGMENTS

This work is supported by WTOP (Wavelet Transform On Propagation and Imaging for seismic exploration) Research Consortium at University of California, Santa Cruz. Xiaosan Zhu acknowledges the support from the China Scholarship Council. The authors thank Xiao-Bi Xie, Jun Cao, YaoFeng He, Yingcai Zheng, Hui Yang, Jian Mao and Xiaofeng Jia for valuable comments and suggestions. The authors thank Xiaofei Chen for support. This manuscript will be published as contribution No. 501 of the Center for Study of Imaging and Dynamics of the Earth, Institute of Geophysics and Planetary Physics.

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