**Lg-wave simulation in heterogeneous crusts with surface topography using screen propagators**

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**SUMMARY**

We develop a numerical simulation method that can efficiently model the combined effects of large-scale structural variations and small-scale heterogeneities (e.g. random media) on Lg-wave propagation at far regional distances. The approach is based on the generalized screen propagator (GSP) method, which has previously been used to simulate SH Lg waves in complex crustal waveguides. In this paper, we extend the GSP method to treat complex crustal models with irregular or rough topography by incorporating surface flattening transformation into the method. The transformation converts surface perturbations into modified volume perturbations. In this way the range-dependent boundary condition becomes a stress release boundary condition on a flat surface in the new coordinate system where the half-space GSP can be applied.

To demonstrate the accuracy and efficiency of the extended GSP method, synthetic seismograms are generated for various crustal waveguides, including uniform crusts, a Gaussian hill half-space, and crustal models with mild and moderately rough surfaces. The results are compared with those generated by the exact boundary element method. It is shown that the screen method is efficient for modelling the effect of surface topography on Lg waves. The comparison of synthetic seismograms generated by the screen method and the traditional parabolic equation method shows that the screen method can handle wider-angle waves as well as rougher topography than the parabolic equation method. Finally, we apply the method to complex crustal waveguides with both small-scale heterogeneities (random media) and random rough surfaces for Lg propagation to far regional distances. The influence of random heterogeneities and rough surfaces on Lg attenuation is significant.

Key words: boundary element, Lg waves, synthetic seismograms, topography.

**1 INTRODUCTION**

Regional phases, generated by shallow events, have drawn wide interest because of their potential uses in seismic source characterization, magnitude estimation and crustal structure determination. The study of regional waves is also important in Comprehensive Nuclear Test Ban Treaty (CTBT) research. However, due to the complexity of Earth structures and the limited modelling capability of the existing numerical simulation techniques, the mechanisms that influence Lg attenuation and blockage have not been fully understood. The numerical simulations of Lg blockage by large-scale crustal structures have not succeeded in matching the observations (Campillo et al. 1993; Gibson & Bouchon 1994). It has been noted that the influence of small-scale heterogeneities including rough topography could be more efficient than large-scale structures for Lg attenuation in many cases (Wu et al. 1996, 1997, 1998, 1999, 2000a,b).

Substantial efforts have been made in developing various numerical simulation methods to model regional wave (Lg) propagation for complex crustal models. For the purpose of monitoring the CTBT, the method should have the capability of fast computation, which is required for Lg modelling with high frequencies—up to 25 Hz—and far regional distances—greater than 1000 km. The finite difference method (Xie & Lay 1994; Goldstein et al. 1996; Jih 1996) is a general numerical method to simulate wave propagation in complex media. However, it has some severe limitations for regional wave simulation. First, it is susceptible to progressive dispersion errors when simulating wave propagation over distances of hundreds of wavelengths (Fornberg 1987; Sei 1993). The accuracy and stability of solutions might therefore be questionable for high frequencies and long distances. Second, it is relatively complicated to apply the finite difference method to the free-surface topography of complex geometric shapes (Jih et al. 1988;
Robertsson 1996). Also, the finite difference method is time-consuming, even formidable, for long distances and high frequencies.

The boundary integral equation methods or boundary element (BE) methods are widely adopted numerical methods in regional wave simulations (Bouchon 1985; Campillo & Bouchon 1985; Bouchon et al. 1989). The traction-free condition for a rugged free surface is easy and natural to treat in an accurate and stable manner using the BE method. Therefore, the BE method can be applied to any severe topography. However, it can only be applied to simple models that consist of finite locally homogeneous subdomains. In other words, the BE method cannot deal with volume heterogeneities. Another serious limitation to the BE method is that the computation time increases dramatically with increasing model size and frequency because a large full-rank matrix equation must be inverted for each frequency. In fact, only very simple geological structures can be treated.

The parabolic equation (PE) method is based on a Fourier split-step technique and has been extensively applied to the study of the propagation of radio waves in the troposphere over irregular terrain (Kuttler & Dockery 1991; Barrios 1994) as well as to the study of the propagation of underwater sounds in ocean environments including irregular ocean bottoms (Collins 1990; Evans 1998). The ability of the parabolic equation method to predict propagation losses over ranges extending up to a few hundred kilometres, as well as to accommodate range-dependent refractive index variations and irregular free surfaces or ocean bottoms has been demonstrated (Barrios 1994; Evans 1998). However, direct application of the parabolic equation method in modelling Lg-wave propagation in the crusts with irregular free surfaces is problematic due to its inherent small-angle limitation.

Wu et al. (1996, 1997, 1998, 1999; 2000a,b) developed a half-space GSP for modelling the main characteristics of Lg (2-D SH case) in smoothly varying, heterogeneous crustal waveguides. The method is based on the one-way wave equation and small-angle approximation. The one-way propagator GSP neglects backscattered waves but handles all the forward scattering effects, for example, focusing/defocusing, diffraction and interference. The successful applications of the one-way approximation to Lg simulation is based on the fact that the majority of Lg energy in the crustal waveguide environment is carried by forward-propagating waves bouncing up and down between the free surface and major geophysical discontinuities such as the Moho and Conrad discontinuities. Beyond the critical angle, these waves are dominated by small-angle waves and are trapped in the crustal waveguides. Therefore, the neglect of backscattered waves and the small-angle approximation in the simulation will not change the main features of regional phases in most cases (Wu et al. 2000a,b). The main advantages of the GSP method are the great savings in computation time and memory. However, the method in its previous version cannot handle surface topography in crustal waveguides.

In this study, we extend the GSP method to treat complex crustal models with irregular topography by incorporating a surface-flattening transformation into the method. Synthetic seismograms are generated by the extended screen method for various crustal waveguides, including uniform crusts, a Gaussian hill half-space, and crustal models with mild and moderately rough surfaces. The results are compared with those generated by the exact BE method. Numerical comparisons show that the extended screen method is effective for modelling the effect of surface topography on Lg phases. Finally, we apply the method to complex crustal waveguides with small-scale heterogeneities (random media) and random rough surfaces for Lg modelling to far regional distances; in this case the BE method is not applicable because of the presence of heterogeneities in the crust. The influence of random heterogeneities and rough surfaces on Lg amplitude attenuation is significant.

\section{2 Theory}

The partial differential equation used to describe SH-wave propagation in arbitrarily heterogeneous media can be written as

\[
\nabla \cdot \mu \nabla u + \rho \omega^2 u = 0
\]

(Aki & Richards 1980), where \( u \) is the displacement of the SH wave (\( y \)-component), \( \mu \) and \( \rho \) are the shear modulus and density, respectively. They are functions of the position in space (\( x, z \)). The time-dependent complex displacement \( u \) is assumed to be periodic with frequency \( \omega \). For 2-D SH problems, the gradient operator is \( \nabla = (\partial \hat{e}_x) \hat{e}_x + (\partial \hat{e}_z) \hat{e}_z \), where \( \hat{e}_x \) and \( \hat{e}_z \) are the unit vectors in the range, \( x \)-, and depth, \( z \)-directions, respectively. To solve eq. (1) for a complex half-space problem with an irregular surface with height \( h(x) \), a stress-free boundary condition on the free surface must be satisfied, i.e.

\[
\hat{\epsilon}_u \hat{\epsilon}_n (x, z = h(x)) = 0,
\]

which is a Neumann boundary condition, where \( \hat{\epsilon}_u \hat{\epsilon}_n \) represents the gradient of \( u \) at an outgoing direction normal to the surface. The fact that the boundary condition is range-dependent makes a straightforward solution very difficult.

\subsection{2.1 Wide-angle thin-slab approximation}

Wu et al. (2000a,b) successfully applied the GSP method to the study of Lg-wave modelling for complex waveguides with flat surfaces. The image method (Morse & Feshbach 1953) for constructing a half-space Green function was employed in their derivation. For irregular surfaces, the image method is no longer applicable. To use the GSP method for solving a range-dependent boundary condition problem, we employ a simple flattening transformation given by Beillis & Tappert (1979). The transformation is defined as

\[
\begin{cases}
 Z = x \\
 \zeta = z - h(x)
\end{cases}
\]

where \( h(x) \) is the height function of the free surface. Eq. (3) shows that the transformation shifts only the depth variable \( z \), that is, depth measurement starts from the free surface. Under the above transformation, the medium parameters in the new coordinate \( \{ Z, \zeta \} \) can be expressed by \( \hat{\mu}(Z, \zeta) = \hat{\mu}(x, z - h(x)) = \mu(x, z) \) and \( \hat{\rho}(Z, \zeta) = \hat{\rho}(x, z - h(x)) = \rho(x, z) \).

Strictly speaking, there is no such one-to-one relationship for the displacement fields between the two different coordinate systems \( \{ x, z \} \) and \( \{ Z, \zeta \} \). However, if \( h(x) \) is smooth enough and the height fluctuations are small, the displacement field

\begin{center}
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\end{center}
Thus, the imaging method can be applied.

Comparing eqs (1) and (5), the simple surface flattening transformation introduces three extra terms proportional to the slope of the free surface. At first glance, it seems that the transformation makes the problem more complicated. However, it turns a complex waveguide with an irregular surface into a simple one with a flat surface. Under the new coordinate system \((\tilde{x}, \tilde{z})\), the boundary condition eq. (2) becomes

\[
\tilde{\mu} \frac{\partial \tilde{u}(\tilde{x}, \tilde{z})}{\partial \tilde{z}} (\tilde{x}, \tilde{z} = 0) = 0 .
\]  

Thus, the imaging method can be applied.

For volume heterogeneities of \(\tilde{\rho}(\tilde{x}, \tilde{z})\) and \(\tilde{\mu}(\tilde{x}, \tilde{z})\), we follow the procedure given by Wu et al. (2000a) to decompose \(\tilde{\rho}(\tilde{x}, \tilde{z})\) and \(\tilde{\mu}(\tilde{x}, \tilde{z})\) into

\[
\tilde{\rho}(\tilde{x}, \tilde{z}) = \rho_0 + \delta \tilde{\rho}(\tilde{x}, \tilde{z}) ,
\]

\[
\tilde{\mu}(\tilde{x}, \tilde{z}) = \mu_0 + \delta \tilde{\mu}(\tilde{x}, \tilde{z}) ,
\]

where \(\rho_0\) and \(\mu_0\) are the parameters of the background medium and \(\delta \tilde{\rho}(\tilde{x}, \tilde{z})\) and \(\delta \tilde{\mu}(\tilde{x}, \tilde{z})\) are the corresponding perturbations. Eq. (5) can then be rewritten as

\[
(\nabla^2 + k_0^2) \tilde{a}(\tilde{x}, \tilde{z}) = -k_0^2 \left\{ \nabla \tilde{F}_V(\tilde{x}, \tilde{z}) + \nabla \tilde{F}_S(\tilde{x}, \tilde{z}) \right\} \tilde{u}(\tilde{x}, \tilde{z}) ,
\]

where \(k_0 = \omega \sqrt{\mu_0/\rho_0}\) and the operators

\[
\tilde{\nabla}^2 = \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{z}^2} ,
\]

\[
\tilde{F}_V(\tilde{x}, \tilde{z}) = \tilde{\epsilon}_p(\tilde{x}, \tilde{z}) + \frac{1}{k_0^2} \tilde{\nabla} \cdot \tilde{\nabla} ,
\]

\[
\tilde{F}_S(\tilde{x}, \tilde{z}) = -\frac{1}{k_0^2 \mu_0} \left[ \frac{\partial \tilde{h}}{\partial \tilde{x}} + \frac{2 \partial \tilde{h}}{\partial \tilde{x}} \frac{\partial^2}{\partial \tilde{z}^2} + \left( \frac{\partial \tilde{h}}{\partial \tilde{x}} \right)^2 - \frac{\partial^2}{\partial \tilde{z}^2} \right] ,
\]

with

\[
\tilde{\epsilon}_p(\tilde{x}, \tilde{z}) = \frac{\tilde{\rho}(\tilde{x}, \tilde{z}) - \rho_0}{\rho_0} ,
\]

\[
\tilde{\epsilon}_\mu(\tilde{x}, \tilde{z}) = \frac{\tilde{\mu}(\tilde{x}, \tilde{z}) - \mu_0}{\mu_0} .
\]

Eq. (1) describes \(SH\)-wave propagation in arbitrarily heterogeneous media, while eq. (9) describes \(SH\)-wave propagation in a homogeneous medium (background medium), but with non-zero equivalent volume forces. The equivalent volume forces \(\tilde{F}_V(\tilde{x}, \tilde{z})\) and \(\tilde{F}_S(\tilde{x}, \tilde{z})\) in eq. (9) are from the volume heterogeneities and the surface flattening transformation, respectively. We assume that the change in the fields caused by such volume forces is small when compared with the primary background field. The perturbation theory can thus be applied to solve eq. (9). Using Green’s theorem, the scattered field by the equivalent forces \(\tilde{F}_V(\tilde{x}, \tilde{z})\) and \(\tilde{F}_S(\tilde{x}, \tilde{z})\) can be written as

\[
U(\tilde{x}_1, k_1) = k_0^2 \int_{\mathcal{V}} d^2 \tilde{V} \tilde{V}^b(\tilde{x}_1, k_0; \tilde{x}, \tilde{z}) \times \left[ \tilde{F}_V(\tilde{x}, \tilde{z}) + \tilde{F}_S(\tilde{x}, \tilde{z}) \right] \tilde{u}(\tilde{x}, \tilde{z}) ,
\]

where the 2-D volume integration is over the volume \(\mathcal{V}\) including all the heterogeneities in the modelling space. \(g^b\) is a half-space scalar Green’s function (Wu et al. 2000a). Comparing eq. (15) with eq. (10) in Wu et al. (2000a), we see that the only difference between the two equations is the additional term \(\tilde{F}_S(\tilde{x}, \tilde{z})\). Therefore, we can apply the half-space GSP developed by Wu et al. (2000a) to the calculation of eq. (15). Under the forward-scattering approximation, for each step of the marching algorithm, the total field at \(\tilde{x}_1\), the exit of a particular thin slab, is calculated as the sum of the primary field, which is the field freely propagated in the half-space from \(\tilde{x}_0\) to \(\tilde{x}_1\) (\(\tilde{x}_0\) is the entrance of the thin slab), and the scattered fields caused by the heterogeneities including topography in the thin slab between \(\tilde{x}_0\) and \(\tilde{x}_1\). The thickness of the thin slab should be made thin enough to ensure the validity of the local Born approximation. Under this condition, the Green’s function can be approximated by the homogeneous half-space Green’s function. Using the image method, Wu et al. (2000a) derived a half-space Green’s function as follows:

\[
g_0^b(\tilde{x}_1, k_1; \tilde{x}, \tilde{z}) = \frac{i}{2\pi} e^{i(\tilde{x}_1 - \tilde{x})} \cos(k_1 \tilde{z}) .
\]

The scattered field at the exit, \(\tilde{x}_1\), of the thin slab by the volume heterogeneities \(\tilde{F}_V\) is

\[
U_P(\tilde{x}, k_1) = \frac{i k_0^2}{2\pi} \int_{\tilde{x}_0}^{\tilde{x}_1} d\tilde{x} e^{-i(\tilde{x}_1 - \tilde{x})} \left\{ \frac{\partial}{\partial \tilde{z}} \tilde{u}_0(\tilde{x}, \tilde{z}) - \tilde{u}(\tilde{x}, \tilde{z}) \right\} \tilde{\nabla} \tilde{u}_0(\tilde{x}, \tilde{z}) \right\} ,
\]

where \(\tilde{u}_0(\tilde{x}, \tilde{z})\) is the incident field at the entrance \(\tilde{x}_0\) of the thin slab and can be calculated by

\[
\tilde{u}_0(\tilde{x}, \tilde{z}) = g^{0-1} \left[ e^{i(\tilde{x}_0 - \tilde{x})} \tilde{u}_0(\tilde{x}_0, k_0^2) \right] ,
\]

and

\[
\tilde{\nabla} \tilde{u}_0(\tilde{x}, \tilde{z}) = g^{0-1} \left[ e^{i(\tilde{x}_0 - \tilde{x})} \frac{\partial}{\partial \tilde{z}} \tilde{u}_0(\tilde{x}_0, k_0^2) \right] ,
\]

\[
\tilde{\nabla} \tilde{u}_0(\tilde{x}, \tilde{z}) = g^{0-1} \left[ e^{i(\tilde{x}_0 - \tilde{x})} \frac{\partial}{\partial \tilde{z}} \tilde{u}_0(\tilde{x}_0, k_0^2) \right] ,
\]

with \(g^{0-1}\) and \(\tilde{g}^{0-1}\) are the cosine and sine transforms and \(\tilde{\nabla}^{0-1}\) and \(\tilde{\nabla}^{0-1}\) are the corresponding inverse transforms (Wu et al. 2000a). In a similar way, we can obtain the scattered field at the exit, \(\tilde{x}_1\), by the equivalent force \(\tilde{F}_S\),

\[
U_S(\tilde{x}, k_1) = \frac{i}{2\pi} \int_{\tilde{x}_0}^{\tilde{x}_1} d\tilde{x} e^{-i(\tilde{x}_1 - \tilde{x})} \tilde{u}(\tilde{x}, \tilde{z}) \tilde{\nabla} \tilde{u}(\tilde{x}, \tilde{z}) \right\} ,
\]
where
\[ A = \frac{\partial^2 u_0(z_\xi)}{\partial z^2} = -i\mathcal{G}^{-1}\left[k_z^2 e^{i(z-x_0)}\hat{u}_0(x_0, k_z^2)\right], \]
(22)
\[ B = \frac{\partial^2 u_0(z_\xi)}{\partial z^2} = -\mathcal{G}^{-1}\left[k_z^2 e^{i(z-x_0)}\hat{u}_0(x_0, k_z^2)\right], \]
(23)
\[ D = \frac{\partial^2 u_0(z_\xi)}{\partial z^2} = -\mathcal{G}^{-1}\left[k_z^2 e^{i(z-x_0)}\hat{u}_0(x_0, k_z^2)\right]. \]
(24)
Eqs (17)–(24) are the dual-domain expressions of the wide-angle thin-slab propagator for SH problems in an arbitrary half-space model with arbitrary heterogeneities and an irregular surface. However, the wide-angle version is not efficient in fast computation, it is also unstable due to the large number of multiplications required between the background field and heterogeneities and the singularity of the factor \( V_n \) in eqs (17) and (21). In the following section we derive the corresponding small-angle version.

### 2.2 Small-angle approximation

When the energy of crustal guided waves is carried mainly by small-angle waves (with respect to the horizontal direction), the small-angle approximation can be invoked to simplify the theory and calculations. For small-angle waves, \( k_z \ll \gamma \approx k_0 \), \( U_{F}(\xi, k_z) \) can be approximated by (Wu et al. 2000a)
\[ U_{F}(\xi, k_z) \approx ik_0 e^{ik_0 z_0} \{ \mathcal{F}_s(\xi) \hat{u}_0(x_0, \xi) \}, \]
(25)
where \( \Delta z = z_1 - z_0 \) is the thickness of the thin slab and
\[ \mathcal{F}_s(\xi) = \frac{1}{2} \int_0^{\infty} d\xi \left[ \hat{e}_\xi (\xi, \xi) - \hat{e}_\mu (\xi, \xi) \right]. \]
(26)
For \( U_{F}(\xi, k_z) \), substituting eqs (22)–(24) into eq. (21), we have
\[ U_{F}(\xi, k_z) = \frac{i}{2\gamma} \int_0^{\infty} d\xi \hat{e}_{\xi}^{(z-x_0)} \int_0^{\infty} d\xi \hat{2} \cos(k_z^2) \frac{\hat{\mu} (\xi)}{\mu_0} \times \int_{-\infty}^{\infty} dk_z \hat{e}_{\xi}^{(z-m_0)} \hat{e}_{\xi}^{(z-n_0)} \times \left[ \frac{2 \hat{m}_g}{c_x} k_z^2 - \frac{2 \hat{m}_x}{c_x^2} k_z^2 - \left( \frac{c_y}{c_x} \right)^2 k_z^2 \right] \hat{u}_0(x_0, k_z^2) \]
(27)
When \( h(x) \) is sufficiently smooth with respect to \( x \), under the small-angle approximation the last two terms in the above equation are high-order small quantities. Neglecting the last two terms, we obtain
\[ U_{F}(\xi, k_z) = \frac{i}{2} e^{ik_0 z_0} \int_0^{\infty} d\xi \hat{2} \cos(k_z^2) \frac{\hat{\mu} (\xi)}{\mu_0} \int_0^{\infty} d\xi \frac{2 \hat{m}_g}{c_x} k_z^2 \]
\[ \times \int_{-\infty}^{\infty} dk_z \hat{e}_{\xi}^{(z-m_0)} \hat{e}_{\xi}^{(z-n_0)} \hat{u}_0(x_0, k_z^2) \]
\[ = -\mathcal{F}(\xi) e^{ik_0 z_0} \left\{ \frac{\hat{\mu} (\xi)}{\mu_0} \mathcal{G}^{-1}\left[k_z^2 \hat{u}_0(x_0, k_z^2)\right]\right\}, \]
(28)
with
\[ \mathcal{F}(\xi) = h(x_1) - h(x_0). \]
(29)
It is clear that under the small-angle approximation the scattered field \( U_{F} \) is proportional to the height difference of the two adjacent screens for each forward step. The total field can be obtained through summing the primary field, which freely propagates in the background medium, and the scattered fields (25) and (28),
\[ \hat{u}(z_1, k_z) = e^{ik_0 z_0} \left\{ \hat{u}_0(x_0, \xi) + ik_0 \mathcal{F}(\xi) \hat{u}_0(x_0, \xi) \right\} \]
\[ -\mathcal{F}(\xi) \left\{ \frac{\hat{\mu} (\xi)}{\mu_0} \mathcal{G}^{-1}\left[k_z^2 \hat{u}_0(x_0, k_z^2)\right]\right\} \]
\[ \approx e^{ik_0 z_0} \left\{ e^{ik_0 z_0} \mathcal{G}^{-1}\left[k_z^2 \hat{u}_0(x_0, k_z^2)\right]\right\}. \]
(30)
Eq. (30) is the dual-domain expression of the small-angle approximation for SH-wave simulation in complex waveguides with arbitrary heterogeneities and irregular surfaces. It cannot be written in a pure phase-screen propagator since it is implemented by a mixed transform consisting of cosine and sine transforms. The stability of eq. (30) depends strongly on the second term in the brackets, especially for a large number of steps (e.g. several thousands). The normalization of energy in the wavenumber domain for each forward step is required. In addition, a single-sided cosine taper function is applied in the space domain to eliminate the artificial boundary reflections.

### 3 NUMERICAL TESTS

To test the validity of the extended GSP method, i.e. eq. (30), for modelling \( L_g \)-wave propagation in complex crustal waveguides with irregular surfaces, we have conducted extensive numerical tests and compared the results with the more accurate BE method. Because of the computational intensity of the BE method, we performed the comparisons at short propagation distances (250 km) for relatively low frequencies (0 to 8 Hz). In our calculation of the BE method, each wavelength contains five boundary elements at least. All synthetic seismograms are plotted with reduced time (\( t - X V_0 \)) and \( X \) denotes horizontal distance and \( V_0 \) denotes the shear velocity of background medium.

### 3.1 A uniform crustal model

First, we use a simple crustal model, i.e. a uniform crust, to show the accuracy of the screen method by comparing synthetic seismograms with those calculated by the BE method. We also compare the screen method with the traditional PE method (Kutlter & Dockery 1991) to demonstrate the advantage of the GSP approach over the PE method. The parameters of the crust and the mantle are \( V_{\text{crust}} = 3.5 \text{ km s}^{-1}, \rho_{\text{crust}} = 2.8 \text{ g cm}^{-3}, \) \( V_{\text{mantle}} = 4.5 \text{ km s}^{-1} \) and \( \rho_{\text{mantle}} = 3.1 \text{ g cm}^{-3} \). The thickness of the crust is 32 km. The source is located at a depth of 8 km. The source time function is a Ricker wavelet with a dominant
3.2 A Gaussian hill

Second, we apply the screen and PE methods to a Gaussian hill (Fig. 2) for calculating synthetic seismograms. The height versus range dependence of the Gaussian hill is given by

\[ h(x) = -h_0 \exp\left[-\frac{(x-x_0)^2}{2\sigma^2}\right], \]  

where \( x_0 = 62.25 \) km, \( h_0 = 4 \) km, \( \sigma = 9.129 \) km. The maximum slope is 0.268 (15° relative to the horizontal direction). The parameters for the half-space are \( V = 3.5 \) km s\(^{-1}\), \( \rho = 2 \) g cm\(^{-3}\). The dominant frequency of the source time function is 3 Hz. The marching step length is \( \Delta x = 125 \) m and the spatial sampling interval in depth \( \Delta z = 250 \) m. Fig. 3 shows the accuracy of the screen method by comparing the synthetic seismograms calculated by the screen method (solid lines) with those calculated by the BE (dotted lines) method. Figs 3(a) and (b) correspond to different source depths of 8 and 32 km, respectively. Fig. 3(a) corresponds to the small incident angle case (less than 15° to the normal of the screen for all receivers), while Fig. 3(b) corresponds to the case of a medium incident angle (less than 35° to the normal of the screen). The excellent agreement between the screen and BE methods is clearly seen except in the vicinity of the hilltop in the case of a medium incident angle, where a small discrepancy exists in both wave shapes and amplitudes. The error will decrease if the step length \( \Delta x \) reduces. For forward marching algorithms, the step length \( \Delta x \) may be adjusted according to the roughness of the topography. In a practical calculation the step length \( \Delta x \) can be determined by letting \( \partial^2 x \) be a constant. The more severe the topography is, the finer the \( \Delta x \) should be. Fig. 3(a) reveals the effect of the hill on the ground motion. At and near the top of the hill the motion is amplified when compared with the motion at the base of the hill (Sills 1978; Geli \textit{et al.} 1988; Bouchon & Barker 1996). Fig. 4 shows the accuracy of the PE method (Barrios 1994) applied to the above Gaussian hill model for synthesizing seismograms. Comparing Figs 4 and 3, even for small-angle incidence (Fig. 4a) significant errors in traveltimes are accumulated when waves propagate through the hill. For a much smoother hill, for instance, \( h_0 \) in eq. (31) is less than 1 km, the results from the traditional PE method agree very well with those from the BE method. This means that the traditional PE method can only handle cases with much smoother topography.

3.3 Crustal waveguides with irregular surfaces

Fig. 5 is a crustal model with a mildly irregular surface. The maximum, minimum and mean heights are 1.14, −1.32 and −0.1 km, respectively. The maximum slope of the irregular surface is 0.3 (16.5° relative to the horizontal direction). The source is located at a depth of 8 km and the dominant frequency of the source time function is 1 Hz. The formation
parameters and configuration are also shown in Fig. 5. For this crustal model the synthetic seismograms calculated by the screen (solid lines) and BE (dotted lines) methods are given in Fig. 6(a). Fig. 6(b) shows the corresponding energy attenuation versus horizontal distance. The results of the two methods agree in general. The relatively large differences in energy level between the two methods are found at horizontal distances of 50–100 and 140–190 km, where large-angle, pre-critical Moho reflections occur. In this case, the screen method has low accuracy. The Moho reflections are usually overestimated. However, the discrepancy appears only locally near the critical reflections, and the errors do not propagate with the guided waves ($Lg$) as can be seen in Fig. 6(b) (and Fig. 8b). The energy attenuation for a crust with a flat free surface is also given in Fig. 6(b) to show the

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**Figure 3.** Comparison of synthetic seismograms calculated by the screen method (solid lines) and the BE method (dotted lines) for the Gaussian hill shown in Fig. 2. The source is located at depths of 8 km (a) and 32 km (b). The dominant frequency of the source time function is 3 Hz.

**Figure 4.** Comparison of synthetic seismograms calculated by the PE method (solid lines) and BE method (dotted lines) for the model shown in Fig. 2.

**Figure 5.** A crustal model with an irregular surface. The maximum, minimum and mean heights of the irregular surface are 1.14, −1.32 and −0.1 km, respectively. The corresponding maximum slope is 0.3 (a slope angle of 16.5°).

**Figure 6.** Comparison between the screen method and the BE method for a crustal waveguide with a smoothly irregular surface. (a) Synthetic seismograms; (b) energy distribution with horizontal distance. The thick smoothly varying curve in (b) is calculated with the finite difference method for a waveguide similar to that shown in Fig. 5 but with a flat free surface.
effect of an irregular surface. Fig. 6(b) demonstrates that even for a smoothly irregular surface, the energy level (amplitude) can vary dramatically. Fig. 7 shows a similar crustal model to that shown in Fig. 4, but with a random rough surface. The correlation length is 2.5 km, the rms fluctuation is 0.6 km (1.75~−1.5 km). Fig. 8 shows the synthetic seismograms calculated by the screen and BE methods and the corresponding energy attenuation curves. We see that the presence of a random rough surface makes the waveforms and attenuation curves more complicated. Except for large-angle Moho reflections, the results of the screen method agree well with those of the BE method. Comparing Figs. 8(a) and 6(a), the amplitude of multiple reflections in Fig. 8(a) decreases rapidly due to the scattering by the rough surface. The scattering by the rough surface also causes more rapid energy attenuation (Fig. 8b).

Figure 7. A crustal model with a random rough surface. The correlation length is 2.5 km, the rms perturbation is 0.6 km (1.75~−1.5 km).

Figure 8. Comparison between the screen method and the BE method for a crustal waveguide with a random rough surface. (a) Synthetic seismograms; (b) energy distribution with horizontal distance. The thick smoothly varying curve in (b) is calculated with the finite difference method for a waveguide similar to that shown in Fig. 7 but with a flat free surface.

4 AN APPLICATION EXAMPLE: COMBINING THE EFFECTS OF ROUGH TOPOGRAPHY AND VOLUME HETEROGENEITY ON LG PROPAGATION

Numerical examples in the previous sections have demonstrated the accuracy and efficiency of the screen method in modelling $Lg$-wave propagation in a Gaussian hill half-space and the crustal waveguides with mildly and moderately rough surfaces. Because of the computational intensity of the BE method, comparisons have been made only for short propagation distances (<250 km) and low frequencies. In this section the screen method is applied to simulate $Lg$-wave propagation in a crustal waveguide with both rough surface topography and volume heterogeneities. This example is intended only to demonstrate the capability of the half-space screen propagator with topographic transformation in studying long-range, high-frequency $Lg$ propagation. Systematic investigation of the combined effects of topography and volume heterogeneities on $Lg$ attenuation and blockage will be conducted and presented in future publications. Fig. 9 shows the spatial spectrum of the surface topography (random rough surface: its correlation length is 2.5 km, the rms perturbation is 0.6 km, the spatial sampling interval is 125 m and the total number of samples is 4000) used in this example. The horizontal axis is spatial frequency ($\text{km}^{-1}$). The broad spectrum shows the richness in small-scale variation. The crustal waveguide is 32 km thick and is filled with a random medium with an exponential correlation function. The volume heterogeneities include only variation in velocity. The correlation lengths are 6 km in range and 4 km in depth and the rms values are 5 and 10 per cent, respectively. The background medium parameters for crust and mantle are the same as those used in Fig. 5. The source is located at a depth of 8 km. The dominant frequency of the source time function is 2 Hz. The corresponding wavelength is 1.75 km in the crust. The forward step length and spatial sampling interval for the screen method are 125 and 250 km. The total number of screens is 4000. Fig. 10 shows the energy attenuation. Fig. 10(a) corresponds to a uniform crust, Fig. 10(b) to a heterogeneous crust with an rms of 5 per cent and Fig. 10(c) to a heterogeneous crust with an rms of 10 per cent. The dashed line, calculated using the finite difference method for a uniform crustal waveguide, is used as a reference. We see that random heterogeneities combined with rough topography drastically increase the attenuation of the high-frequency $Lg$ waves. This further demonstrates that small-scale volume heterogeneities and rough surface topography could be important factors in $Lg$ attenuation and blockage (Wu et al. 2000b).

So far, the GSP method has been developed to handle large-sized 2-D crustal waveguides with moderately rough surfaces and random heterogeneities for $Lg$ modelling. It is two to three orders of magnitude faster than the BE and FD methods even for the small-sized crustal models given in this paper. It is anticipated to be an efficient simulation tool for the study of $Lg$ attenuation and blockage in some realistic Earth structures. Future efforts will be concentrated on the systematic investigation of $Lg$ attenuation and blockage using the screen method and the development of a $P-SV$ elastic screen method for $Lg$ simulation.

5 CONCLUSIONS

In this paper, the GSP method has been developed to model the combined effects of large-scale structure variations, small-scale
the dominant frequency of the source time function is 2 Hz.

The comparisons of the results between the screen and boundary element methods for various crustal waveguides, including uniform crusts and a Gaussian hill half-space, and crustal models with mildly and moderately rough surfaces show that the screen method works well for mildly to moderately irregular surfaces. The screen method is numerically very efficient because of the use of the one-way wave approximation and fast Fourier transforms in the implementation. For a model with propagation distances of 250 km and a dominant frequency of 1 Hz for the source time function, the screen method takes about 35 min, while the BE method takes about 72 hr with the same accuracy. It is also shown that the traditional parabolic equation method has a very limited capability for handling large-angle waves and moderate topography compared with the screen method. An application example of the screen method shows that the influence of random heterogeneities and rough surfaces on $L_g$ amplitude attenuation is significant. Systematic investigation of the combined effects of topography and volume heterogeneities on $L_g$ attenuation and blockage will be conducted and presented in a future study.

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