Multi-screen backpropagator for fast 3D elastic prestack migration

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ABSTRACT

Due the huge amount of computation and internal memory required, wave backpropagation becomes the bottleneck of prestack migration or other 3D imaging/inversion procedures. We propose to use the multi-screen backpropagator for 3D prestack migration in laterally inhomogeneous background (depth migration). Multi-screen (phase-screen for scalar waves, elastic complex-screen for elastic waves) backpropagator shuttles between space-domain and wavenumber-domain using FFT and therefore avoids the time-demanding matrix multiplication. The time saving is tremendous for large-size elastic wave problems. Because it needs to store the medium parameters only one grid-plane for each step, the enormous computer memory saving makes it capable of handling large 3D problem prohibitive to other methods. The method of elastic complex screen (ECS) is a one-way propagation algorithm by neglecting the backscattered waves. However, all the forward multiple-scattering effect, such as the focusing/refocusing, diffraction, interference, wave conversion between P and S, interface waves, guided waves, etc., can be correctly handled. In this paper first the Love integral and Love migration integral are introduced. The formulation of elastic complex-screen as elastic wave one-way propagator is summarized. Numerical tests and comparisons with other full-wave methods (elastic wave finite difference and eigenfunction expansion method) are presented to show the validity of the propagator. Finally, two numerical examples of single-shot prestack migration using the ECS backpropagator, one for homogeneous background and the other for inhomogeneous background, are shown to demonstrate the feasibility of the proposed scheme.

Keywords: geophysical imaging, 3D migration, prestack migration, elastic wave migration, one-way propagation, backpropagation, phase-screen, elastic complex-screen, split-step, holographic imaging.

1. INTRODUCTION

The two most popular methods of 3D prestack migration are the Kirchhoff-ray algorithm (Wapenaar and Berkhout\textsuperscript{1}, 1989; Blacquiere et al.\textsuperscript{2}, 1989; Tatham and McCormack\textsuperscript{3}, 1991) and the reverse-time finite difference algorithm (Chang and McMechan\textsuperscript{4, 5}, 1990, 1991, 1994) or reverse-time finite element algorithm (Teng and Dai\textsuperscript{6}, 1989). The former uses 3D ray-tracing, and the latter uses 3D finite difference or finite element, as the backpropagator, rendering the procedure a time-consuming one and prohibitive for large-size problem. The problem becomes much severer for elastic prestack migration. We propose to use the multi-screen backpropagator for 3D prestack migration in laterally inhomogeneous background (depth migration). Multi-screen backpropagator (phase-screen for scalar waves, elastic complex-screen for elastic waves) shuttles between space-domain and wavenumber-domain using FFT and therefore avoids the time-demanding matrix multiplication. The time saving is tremendous for large-size elastic wave problems. The method of elastic complex screen (Wu\textsuperscript{7}, 1994) is a one-way propagation algorithm by neglecting the backscattered waves. However, all the forward multiple-scattering effect, such as the focusing/refocusing, diffraction, interference, wave conversion between P and S, interface waves, guided waves, etc., can be correctly handled. For weak heterogeneous media as encountered in most cases for geophysical applications, this ECS one-way propagation algorithm produces synthetic seismograms in good agreement with full elastic wave methods. In the following we first introduce in section 2 the Love integral and Love migration integral. Then the formulation of elastic complex-screen is summarized in section 3. Numerical tests and comparisons with other full-wave methods (elastic wave finite difference and eigenfunction expansion method) are also presented in section 3 to show the validity of the propagator. Finally, in section 4 two numerical examples of single-shot prestack migration using the ECS backpropagator are shown. One is for point scatterers with different parameter perturbations (density, bulk modulus, or shear modulus) illuminated by P or S incident waves in homogeneous background. The other is for a point scatterer in inhomogeneous background composed by a low-velocity solid sphere buried in a constant medium. The results demonstrate the feasibility of the proposed scheme.

2. PRESTACK LOVE MIGRATION USING ELASTIC ONE-WAY BACKPROPOAGATOR

If the elastic wave field is known on a closed surface S, then from the representation theory (Aki and Richards\textsuperscript{1},
we can calculate the wave filed at any point, e.g. point $x_1$, inside the surface by a surface integral (propagation integral)

$$u(x_1) = \int_S \left\{ T(x) \cdot G(x,x_1) \cdot u(x) \cdot \Gamma(x,x_1) \right\}$$

(1)

where $\sigma(x)$ is the stress field on the surface, $T(x) = \hat{n} \cdot \sigma(x)$ is the corresponding traction field, $\hat{n}$ is the outward normal, $u(x)$ is the displacement field on the surface, $G(x,x_1)$, $\Sigma(x,x_1)$, and $\Gamma(x,x_1) = \hat{n} \cdot \Sigma(x,x_1)$ are the Green's displacement, stress, and traction tensors, respectively. Here we did not write explicitly the frequency or time dependence. For time domain formulation, the Green's tensor operators imply time-domain convolutions. Fig. 1 is the geometry used in many geophysical applications, where the closed surface is composed of a flat surface $S$ covered with a large hemisphere. The influence from the hemisphere can be neglected due to the limited observation time window and the long propagation distance involved. Therefore the field at $x_1$ is determined solely by the field on the flat surface $S$. This representation integral in the present form was first introduced and discussed by Love in 1904 (in a more general form first by Betti in 1872 e.g., Aki and Richards 1980), and is called the Love integral (Wu 1989a) or the elastic-wave Kirchhoff integral (Pao and Varatharajulu, 1976). Note that the Love integral (the surface integral) is from the contribution of outside sources, i.e. the sources outside the surface. The known field on the surface can be generated by actual sources or by scattering of the heterogeneities.

![Figure 1. Geometry of the formulation.](image)

It is known that by replacing the Green's tensors in (1) by their conjugates, the surface integral becomes a back-propagation integral or a migration integral, here the Love migration integral

$$I(x_1) = \int_S \left\{ T(x) \cdot G^*(x,x_1) \cdot u(x) \cdot \Gamma^*(x,x_1) \right\}$$

(2)

where $I(x_1)$ is the image field at $x_1$. $G^*$ and $\Gamma^*$ are the conjugates of $G$ and $\Gamma$. If we concern only the amplitude or
the real-part of the image field, (2) can be equivalently written as

\[ I(x_i) = \int_S dS \left\{ T^*(x) \cdot G(x, x_i) \cdot u^*(x) \cdot \Gamma(x, x_i) \right\} \]

(3)

where \( u^*(x) \) and \( T^*(x) \) are the complex-conjugates of \( u(x) \) and \( T(x) \). If we measure the backscattered field on the free surface, then the stress field vanishes and only the second term on the right hand side exists. The equation becomes the Love migration for surface reflection data. In equation (2) or (3) the Green's tensors can be of any form. If the full elastic-wave finite difference is used to calculate the Green's functions, Love migration is equivalent to the full elastic-wave reverse-time migration. Being perhaps the most precise migration, it is however a time-consuming process. A common approximation to the Green's tensors is the ray-approximation, which is valid in smoothly varying media. In this case, the Green's function can be separated into P and S wave parts and calculated separately after the neglect of the wave conversion. The Love migration becomes an elastic-wave ray-Kirchhoff migration. The operation involved in this version of migration is also time-consuming because of the space-domain convolution and 3D ray-tracing. We propose to use the newly developed elastic-wave wide-angle one-way propagation algorithm, the elastic complex-screen (ECS) method, as the backpropagator of the Love migration. The ECS method, which is a one-way propagation method, assumes that the heterogeneities of the background medium have much larger sizes than the wavelength and therefore the backscattered waves can be neglected (the forward scattering approximation). The elastic-wave one-way propagator has tremendous saving on computer CPU time and internal memory over the full wave propagator and therefore makes the large-size 3D elastic-wave prestack migration possible in today's computer environment. Because of the equivalence of (2) and (3), we discuss here only the forward propagator, i.e. the replacement of Green's tensors in (3) by the ECS propagator. In the following we first outline the basic idea of the theory and derivation, and then give a brief summary of the formulation.

3. SUMMARY OF FORMULATION OF ELASTIC COMPLEX SCREEN (ECS) PROPAGATOR

3.1. Thin-slab formulation in wavenumber domain

The geometry of the derivation is schematically shown in Fig. 2. Assume seismic waves propagate from left to right along the x-direction. The 3D heterogeneous medium is divided into thin slabs facing the propagation direction. Under the forward scattering approximation, the wave can march from one slab to another successively using the Love integral. Because of the nature of the forward scattering approximation, the thickness of the thin-slab can be always made small enough so that the local Born approximation can be applied to the heterogeneities inside the thin-slab. Taking the background of the thin-slab as constant medium, the variations of the parameters can be expressed as perturbations in the form

\[ \rho(x) = \rho_0 + \delta\rho(x) \]
\[ c(x) = c_0 + \delta c(x) \]

(4)

where \( \rho \) is the density and \( c \) is the elastic constant tensor of the medium. \( \rho_0 \) and \( c_0 \) are the parameters of the background medium. \( \delta\rho \) and \( \delta c \) are the corresponding perturbations. After this decomposition, the Love integral (1) can be replaced by another equivalent form of Love integral:

\[ u(x_i) = \int_S dS \left\{ T(x) \cdot G_0(x, x_i) \cdot u(x) \cdot \Gamma_0(x, x_i) \right\} + \int_V dV Q(x) \cdot G_0(x_i, x) \]

(5)

where \( G_0 \) and \( \Gamma_0 \) are the Green's displacement and traction tensors in homogeneous media, the volume integral is over only the thin-slab, and

\[ Q = -\left[ \delta\rho \hat{u} \cdot \nabla \cdot \left[ \frac{1}{2} \delta c : (\nabla u + u \nabla) \right] \right] \]

(6)

is the equivalent body force due to scattering (Aki and Richards\(^1\), 1980; Wu and Aki\(^5\), 1985; Wu\(^4\), 1994). Here \( \nabla u \) is the gradient of \( u \) which is a second rank tensor, \( u \nabla \) stands for the transpose of \( \nabla u \), and \( \cdot \) stands for double scalar product of tensors defined through \( (ab) : (cd) = (b \cdot c)(a \cdot d) \). In the perturbation approach, the total field is decomposed into
a) 3D model of elastic medium and "thin slabs"

b) Elastic complex-screens

Figure 2. Schematic explanation of "thin-slab" and "complex screen" formulations. A 3-D model of heterogeneous elastic medium is divided into thin-slabs perpendicular to the propagation direction $x$. After making small-angle approximation, the thin-slabs are compressed into screens. For P-P or S-S scattering the screen is a phase-screen; for converted waves, the screen becomes complex, having both phase and amplitude modulations.

$$u(x) = u^0(x) + U(x)$$

where $u^0(x)$ and $U(x)$ are the incident and scattered fields, respectively. Replacing the total field in (6) by the incident field results in the Born approximation.

The surface integral in (5) is much easier to calculate than that in (1), since the Green's tensors now are those for constant media. Combining the wavenumber domain formulations of elastic-wave Rayleigh integral (ERI) (see Wu¹², 1989a, Wapenaar and Berkhout¹¹, 1989) for the surface integral and the local elastic Born scattering (EBS) (Wu and Aki¹⁵, 1985; Wu¹³, 1989b) for the volume integral, a thin-slab formulation for elastic-wave wide-angle one-way propagation can be obtained (Wu¹⁴, 1994, Wu and Xie¹⁶, 1993).
The wave field marches slab by slab (see Fig. 2). The total field at the exit of a thin-slab is calculated as the sum of the primary field and the scattered field which is generated by the interaction between the incident field and the heterogeneities within the thin-slab. Because of the neglect of backscattered waves, the heterogeneities in front of the slab exit have no influence on the calculation. The influence of all heterogeneities behind the slab entrance has been taken into account in previous steps. The total field at the exit depends only on the field at the entrance and the heterogeneities inside the slab. Therefore the total field at the slab exit \( x_1 \) due to the incident field at the entrance \( x \) is expressed as

\[
\begin{align*}
\mathbf{u}^P(x_1, \mathbf{K}_T) &= e^{i\gamma_p x_1} \left\{ \mathbf{u}_{0,0}(x, \mathbf{K}_T) + \int d \mathbf{K}_T \left[ \mathbf{U}^{PP}(\mathbf{K}_T, \mathbf{K}_T) + \mathbf{U}^{SP}(\mathbf{K}_T, \mathbf{K}_T) \right] \right\} \\
\mathbf{u}^S(x_1, \mathbf{K}_T') &= e^{i\gamma_s x_1} \left\{ \mathbf{u}_{0,0}(x, \mathbf{K}_T') + \int d \mathbf{K}_T \left[ \mathbf{U}^{SS}(\mathbf{K}_T, \mathbf{K}_T) + \mathbf{U}^{PS}(\mathbf{K}_T, \mathbf{K}_T) \right] \right\}
\end{align*}
\]

where \( \mathbf{u}^P \) and \( \mathbf{u}^S \) are the total P and S field, \( \mathbf{K}_T \) is the input transverse wavenumber, \( \mathbf{K}_T' \) is the output transverse wavenumber, \( \mathbf{u}_{0,0} \) and \( \mathbf{u}_0^0 \) are incident fields and \( \mathbf{U}^{PP}, \mathbf{U}^{PS}, \mathbf{U}^{SS} \) and \( \mathbf{U}^{PS} \) are the P-P, S-P, S-S and P-S scattered fields:

\[
\begin{align*}
\mathbf{U}^{PP}(\mathbf{K}_T', \mathbf{K}_T) &= i \frac{k_x^2 \alpha_0 \alpha_0'}{2 \gamma} \left[ (\mathbf{k}_\alpha \cdot \mathbf{k}_\alpha') \frac{\delta \rho(\mathbf{k})}{\rho_0} - \frac{\delta \lambda(\mathbf{k})}{\lambda_0 + 2 \mu_0} \right] - (\mathbf{k}_\alpha \cdot \mathbf{k}_\alpha') \frac{2 \delta \mu(\mathbf{k})}{\lambda_0 + 2 \mu_0} \\
\mathbf{U}^{PS}(\mathbf{K}_T', \mathbf{K}_T) &= i \frac{k_x^2 \alpha_0 \beta_0 (\mathbf{k}_\alpha \cdot \mathbf{k}_\beta})(\mathbf{k}_\alpha' \cdot \mathbf{k}_\beta') \left[ \frac{\delta \rho(\mathbf{k})}{\rho_0} - \frac{2 \beta_0}{\alpha_0} (\mathbf{k}_\alpha \cdot \mathbf{k}_\alpha') \frac{\delta \mu(\mathbf{k})}{\mu_0} \right] \\
\mathbf{U}^{SP}(\mathbf{K}_T', \mathbf{K}_T) &= i \frac{k_x^2 \beta_0 \beta_0'}{2 \gamma} \left[ (\mathbf{u}_\beta \cdot \mathbf{k}_\beta) \left\{ \frac{\delta \rho(\mathbf{k})}{\rho_0} - \frac{2 \beta_0}{\alpha_0} (\mathbf{k}_\beta \cdot \mathbf{k}_\beta') \frac{\delta \mu(\mathbf{k})}{\mu_0} \right\} \right] \\
\mathbf{U}^{SS}(\mathbf{K}_T', \mathbf{K}_T) &= \frac{i \gamma}{2 \gamma^2} \left( \hat{\mathbf{m}} \cdot \mathbf{M} \right) \left[ \frac{\delta \rho(\mathbf{k})}{\rho_0} - (\mathbf{k}_\beta \cdot \mathbf{k}_\beta') [1 - \frac{(\mathbf{k}_\beta \cdot \mathbf{\hat{a}})^2}{|\mathbf{M}|^2}] \frac{\delta \mu(\mathbf{k})}{\mu_0} \right] \\
&\quad - \left( \frac{\mathbf{u}_\beta \cdot \mathbf{k}_\beta}{\mu_0} (\hat{\mathbf{m}} \cdot \mathbf{k}_\beta) \frac{\delta \mu(\mathbf{k})}{\mu_0} \right)
\end{align*}
\]

where \( u_{0,0} = \mathbf{u}_0^0(\mathbf{K}_T) \), \( \mathbf{u}_{0,0} = \mathbf{u}_0^0(\mathbf{K}_T) \), \( \mathbf{\hat{a}} = \mathbf{u}_0^0(\mathbf{K}_T) \), and \( \hat{\mathbf{m}} \) and \( \mathbf{\hat{f}} \) are the in-plane and off-plane unit vectors. Here \( \mathbf{k} = \mathbf{k}^\alpha - \mathbf{k}^\beta \) is the exchange wavenumber with \( \mathbf{k}^\alpha \), \( \mathbf{k}^\beta \) as the incident and scattering wavenumbers respectively, and

\[
\begin{align*}
\mathbf{k}_\alpha &= \gamma_\alpha \mathbf{\hat{e}}_x + \mathbf{K}_T \\
\mathbf{k}_\beta &= \gamma_\beta \mathbf{\hat{e}}_x + \mathbf{K}_T \\
\mathbf{k}'_\alpha &= \gamma'_\alpha \mathbf{\hat{e}}_x + \mathbf{K}'_T \\
\mathbf{k}'_\beta &= \gamma'_\beta \mathbf{\hat{e}}_x + \mathbf{K}'_T
\end{align*}
\]

with

\[
\begin{align*}
\gamma_\alpha &= \sqrt{k_\alpha^2 - K_T^2} \\
\gamma_\beta &= \sqrt{k_\beta^2 - K_T^2} \\
\gamma'_\alpha &= \sqrt{k'_\alpha^2 - K'_T^2} \\
\gamma'_\beta &= \sqrt{k'_\beta^2 - K'_T^2}
\end{align*}
\]

3.2. Elastic complex-screen formulation in wavenumber domain

The thin-slab formulas involve with matrix multiplication in the wavenumber domain, and therefore is computationally intensive. After making small-angle scattering approximation for the interaction between the incident wave and the thin-slab, the scattering effect of the thin-slab can be approximately equated to that of passing through a screen. As shown in the lower part of Fig. 2, the thin-slabs are squeezed into screens. It is shown that the P-P and S-S common-mode scatterings can be represented by pure phase-screens, but the P-S, S-P conversions must be represented by complex screens having both phase and amplitude modulations, which are not standard phase-screens. Therefore, the method is called elastic complex-screen (ECS) (or "generalized phase-screen") method.
After making parabolic approximation for the wavenumbers
\[ \gamma = \sqrt{k^2 - K_T^2} \approx k \left(1 - \frac{K_T^2}{2k^2}\right) \quad |K_T| \ll k. \]

the 3D thin-slab spectra are compressed into 2D screen spectra:

\[
\begin{align*}
\delta \rho (k' \alpha \cdot k_\alpha) &= \delta \rho (0, K'_T \cdot K_T) \\
\delta \rho (k' \beta \cdot k_\alpha) &= \delta \rho (0, K'_T \cdot K_T) \eta(\Delta x) \\
\delta \rho (k' \alpha \cdot k_\beta) &= \delta \rho (0, K'_T \cdot K_T) \eta^*(\Delta x) \\
\delta \rho (k' \beta \cdot k_\beta) &= \delta \rho (0, K'_T \cdot K_T)
\end{align*}
\tag{11}
\]

with
\[
\delta \rho (0, K_T) = \int_0^{\Delta x} \delta \rho (x, K_T) dx = \Delta x \delta \rho (K_T)
\tag{12}
\]

\[
\eta(\Delta x) = \text{sinc} \left(\frac{(k' \beta - k_\alpha) \Delta x}{2}\right)
\tag{13}
\]

Same can be done for \( \delta \lambda \) and \( \delta \mu \). Then the scattered fields become

\[
\begin{align*}
U^{PP}(K'_T, K_T) &= -ik_\alpha \Delta x \ u_\alpha (0)(K_T) \frac{\delta \alpha(\tilde{K}_T)}{\alpha_0} \tilde{k}'_\alpha \\
U^{PS}(K'_T, K_T) &= -ik_\beta \Delta x \ u_\alpha (0)(K_T) \tilde{k}'_\beta \tilde{k}'_\alpha \eta(\Delta x) \left[ \frac{\delta \beta(\tilde{K}_T)}{\beta_0} + \left(\frac{2\beta_0}{\alpha_0} - 1\right) \frac{\delta \mu(\tilde{K}_T)}{\mu_0} \right] \\
U^{SP}(K'_T, K_T) &= ik_\alpha \Delta x \left( u_\beta (0)(K_T) \tilde{k}'_\alpha \right) \tilde{k}'_\beta \eta^*(\Delta x) \left[ \frac{\delta \beta(\tilde{K}_T)}{\beta_0} + \left(\frac{2\beta_0}{\alpha_0} - 1\right) \frac{\delta \mu(\tilde{K}_T)}{\mu_0} \right] \\
U^{SS}(K'_T, K_T) &= -ik_\beta \Delta x \left[ u_\beta (0)(K_T) \tilde{k}'_\beta \right] \frac{\delta \beta(\tilde{K}_T)}{\beta_0}
\end{align*}
\tag{14}
\]

where \( \tilde{K}_T = K'_T - K_T \) is the transversal exchange wavenumber, \( \delta \alpha, \delta \beta \) are the P and S wave velocity perturbations respectively. For the Complex Screen method, the cross-coupling term can be neglected, because it is a higher small quantity for forward scattering.

We see that for common-mode scattering (P-P or S-S), the screens are pure phase-screens, but for converted waves the screens become Complex screens.

3.3. Dual-domain implementation of elastic complex-screen method

We see From (14) that the scattered waves are expressed in the form of convolution integrals in the wavenumber domain. Therefore in the space domain the perturbation response will be a local operator, similar to passing "screens". However, wave propagation is a convolution integral in space domain. The calculation would be inefficient in either the space domain or the wavenumber domain alone due to the convolution integral involved. The ideal way is to propagate in wavenumber domain, but interact with screens in space domain, and shuffle between these two domains using FFT. In this way the convolution operation is avoided entirely. In the following we change (7) and (14) into a dual-domain formulation.

For P-P scattering: The total field:
\[
u^{PP}(x, K'_T) = u_\alpha (x, K'_T) + U^{PP}(x, K'_T)
\]

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\[ e^{i \gamma_{\alpha} \Delta x} \hat{k}_{\beta} \int d x_T e^{i \mathbf{K}' \cdot x_T} u_0(\mathbf{x}, x_T) \exp[-ik_{\beta} \Delta x] \]

(15)

For P-S scattering: The scattered field:

\[ U^{PS}(x_1, \mathbf{K}') = -e^{i \gamma_{\beta} \Delta x} \hat{k}_{\beta} \times \hat{k}_{\alpha} u^{(1)}(x_1, \mathbf{K}') \]

(16)

where

\[ u^{(1)}(x, \mathbf{K}') = \int d x_T e^{i \mathbf{K}' \cdot x_T} u_0(x, x_T) C_B(x, x_T) \]

(17)

\[ C_B(x, x_T) = -ik_{\beta} \eta(\Delta x) \int d x' \tilde{B}(x', x_T) \approx -ik_{\beta} \eta(\Delta x) \Delta x \tilde{B}(x, x_T) \]

(18)

\[ \eta(\Delta x) = \text{sinc} \left( (k_{\beta} \cdot k_{\alpha}) \Delta x / 2 \right) e^{-i(k_{\beta} \cdot k_{\alpha}) \Delta x} \]

(19)

\[ \tilde{B}(x_T) = \eta(\Delta x) \left[ \frac{\delta \beta(x_T)}{\beta_0} + \left( \frac{2 \beta_0}{\alpha_0} - 1 \right) \frac{\delta \mu(x_T)}{\mu_0} \right] \]

(20)

For S-P scattering: The scattered field:

\[ U^{SP}(x_1, \mathbf{K}') = e^{i \gamma_{\beta} \Delta x} \hat{k}_{\beta} \cdot \hat{k}_{\alpha} \]

(21)

where

\[ u^{(1)}(x, \mathbf{K}') = \int d x_T e^{i \mathbf{K}' \cdot x_T} u_0(x, x_T) C^*_B(x, x_T) \]

(22)

\[ C^*_B(x, x_T) = -ik_{\alpha} \eta^*(\Delta x) \int d x' \tilde{B}(x', x_T) \approx -ik_{\alpha} \eta^*(\Delta x) \Delta x \tilde{B}(x, x_T) \]

(23)

For S-S scattering: The total field:

\[ u^{SS}(x_1, \mathbf{K}') = u_0(x_1, \mathbf{K}') + U^{PS}(x_1, \mathbf{K}') \]

\[ = -e^{i \gamma_{\beta} \Delta x} \hat{k}_{\beta} \times \hat{k}_{\alpha} \int d x_T e^{i \mathbf{K}' \cdot x_T} u_0(x, x_T) \exp(-ik_{\beta} \Delta x) \]

(24)

The time saving of the dual-domain implementation over the convolution operation is tremendous for large-size problems. For example, with a grid-size of 128 x 128, the time saving factor is 330; while for 512 x 512, the factor is 4148 (see Wu, 1994). Because it needs to store the medium parameters only one grid-plane for each step, the tremendous computer memory saving makes it capable of handling large 3D problem prohibitive to other methods.

3.4. Numerical tests of ECS propagator

Fig. 3 shows an example of 3D modeling. The model is an elastic sphere in a constant background (Fig. 3a). A plane P wave is incident on the sphere along the x-axis and the receiver line parallel with z-axis is located 2 radius apart from the center of the sphere. Fig. 3b shows comparisons of P-P scattering for a solid sphere with 10% P-velocity perturbation, and Fig. 3c is for P-S conversion for a solid sphere with 10% S-velocity perturbation. On the left of Fig. 3b are the x- (in the propagation direction) and z-components of scattered waves by 3D finite difference (Peng and Toksöz, 1994) calculations; On the left of Fig. 3c are those by the exact solutions of eigenfunction expansion method (Korneev and Johnson, 1993). On the right of Fig. 3b and 3c are the results from ECS. These results generally agree very well. For the finite difference calculation of the first example, \( N_x \times N_y \times N_z = 100 \times 96 \times 96 \), and \( N_z = 1500 \). The calculation is performed on a 128-node nCUBE parallel computer. The CPU time for this example is about 29 min. Note that due to the symmetry of the problem, only one quarter of the model space is taken into FD calculation. For more general problems, more CPU time will be taken. For our ECS calculation, the CPU time is less than 30 min on a SUN SPARC-2 workstation.
4. NUMERICAL EXAMPLES OF 3D PRESTACK MIGRATION USING ECS BACKPROPAGATOR

To test the complex phase screen method as a backpropagator for prestack migration, we backpropagate the synthetic scattered wave field from single-shot experiment. The implementation includes generation of the synthetic data.

Figure 3. Comparison of 3D modeling using ECS method with full elastic wave methods. a). The model is an elastic sphere in a constant background. A plane P wave is incident on the sphere along the x-axis and the receiver line parallel with z-axis is located 2 radius apart from the center of the sphere. b). Comparisons of P-P scattering by a low-velocity sphere with 10% P-velocity perturbation. On the left are the x- (in the propagation direction) and z-components of scattered waves calculated by 3D finite difference. On the right are the results from ECS. c). Comparison of P-S conversion by a low-velocity sphere with 10% S-velocity perturbation. On the left are scattered waves calculated by the exact solutions of eigenfunction expansion method. On the right are the results from ECS.
and reverse-time extrapolation. Shown on Fig. 4 is the model geometry used for the numerical examples. The left panel is a model of homogeneous background, and the right panel is that of inhomogeneous background composed by a low velocity sphere immersed in a constant medium. In the numerical examples, we change the coordinate system according to the surface reflection survey geometry. As seen from Fig. 4, the receiver array is put on the surface (X-Y plane with z = 0), and a point scatterer is put under the array (1 km in depth). The size of the model space is 3.2 x 3.2 x 2.1

Figure 4. Model geometry for forward modeling and reverse-time migration. The left panel is a model of homogeneous background, and the right panel is that of inhomogeneous background composed by a low velocity sphere immersed in a constant medium.

Figure 5. Synthetic data along a receiver line (y = 0) across the receiver array for scattered waves from different point perturbations in constant background. A: bulk-modulus perturbation, B: density perturbation, and C: shear-modulus perturbation. On the upper row are x-components and on the bottom row are z-components.
$km^3$. To avoid the possible contamination of the wrap-around effect from the artificial boundaries, only the results within a 2.0 x 2.0 x 2.1 $km^3$ volume are shown here. The elastic parameters for the constant background are $V_p = 3.2 \text{ km/s}$, $V_s = 1.85 \text{ km/s}$ and $\rho = 2.20 \text{ g/cm}^3$. For the inhomogeneous background, the sphere has a P-wave velocity 15% lower than the background velocity. The radius of the sphere is 0.3 km. Its center is located at $x = 0.7 \text{ km}$, $y = 1.0 \text{ km}$ and $z = 0.5 \text{ km}$. We use a 128 x 128 node phase-screen with a node interval of 0.025 km. The interval between the screens is 0.1 km. A total of 65 frequencies are calculated with a Nyquist frequency of 50 Hz. The calculated time length is 1.28 seconds.

For forward modeling, we assume the point scatterer is illuminated by a plane wave incident along Z-direction and the backscattered wave field can be calculated by the Rayleigh scattering approximation (e.g. Wu, 1989b). The backscattered elastic wave field generated by the point scatterer then propagates to the receiver array on the surface.

In the first example, the background is a homogeneous elastic medium and therefore the scattered waves can be calculated analytically on the surface (Wu, 1989b). Fig. 5 shows the synthetic data along a line (at Y = 0) across a 2D receiver array. Only X and Z-components are nonzero since the receiver profile is along the X-direction and located inside the symmetry plane. Fig. 5A shows the scattered field generated by a point bulk-modulus perturbation for an

A: Bulk-modulus perturbation

B: Density perturbation

C: Shear modulus perturbation

Figure 6. Snapshots of reverse-time migrations using synthetic data in Fig. 5 for three types of point perturbations. A: bulk-modulus perturbation, B: density perturbation and C: shear-modulus perturbation.
incident P-wave. In this case the equivalent point source is an explosion (or contraction). Only scattered P-wave is generated. Fig. 5B is from the scattered waves generated by a point density perturbation for an incident P-wave. In this case the equivalent point source is a single-force in Z-direction. The scattering patterns for P and S waves are orthogonal to each other. Fig. 5C is from the scattered waves generated by a point shear-modulus perturbation for an incident S-wave polarized in X-direction. The equivalent point source for this case is a double-couple forces. Due to the limited aperture restricted by the geometry of data acquisition, we can record only a quarter of the scattering patterns, which will limits our ability of recovering the equivalent sources by inverse scattering.

![Graphs showing Z-component, X-component, and Y-component](image)

Figure 7. Synthetic data along a receiver line (y = 0) across the receiver array for inhomogeneous background composed by a low-velocity spherical inclusion buried in a homogeneous medium. The equivalent point source is an explosion (bulk modulus perturbation). The focusing/defocusing and scattering effects can be seen from the synthetic profiles.

For backward extrapolation, the synthetic data are complex conjugated (time-reversed) and used as incident wave field on the X-Y plane at z = -0.1 km. This process reconstruct the wavefield in the model space in a reversed order. At the time zero, backpropagated wave field will focus on the locations where the scattering sources are. For multi-shots prestack migration, imaging time has to be calculated separately for each shot. Fig. 6 shows the snapshots of back-extrapolated wave fields for the three types of scatterers. During backpropagation, both P-wave and S-wave fronts converge gradually. At the imaging time, wave energy converges to the scattering source points. The scattered waves from three different type point scatterers carry the information about the property of the scatterer. As seen from the snapshots, when close to their source locations, the polarizations of the wave fields are quite different depending on the wave types and the properties of the scatterers. They are similar to the equivalent source moment tensors of the corresponding Rayleigh scatterers, from which the physical properties of the scatterers can be inferred. This shows the potential that multi-dimensional, multi-component full elastic wave migration with different source excitations can provide us more subsurface information than acoustic wave migration.

The second example is for an inhomogeneous background composed of a homogeneous medium and a low P-velocity sphere (see Fig. 4). The synthetic data are generated by forward propagation using our ECS propagator and are shown on Fig. 7. The time delay and focusing/defocusing effects caused by the low-velocity sphere are clearly seen in the synthetic. Fig. 8 displays the snapshots for both forward propagation and backward extrapolation. In forward propagation, after passing through the low-velocity sphere, the wave front is delayed and the amplitude within the shadow of the sphere increases due to focusing effect. The amplitudes at the surrounding places decrease due to defocusing. A weak phase can be seen well behind the primary wave front after it passing through the sphere. This phase is the interface wave (creeping waves) along the boundary between the sphere and the surrounding medium. During backward extrapolation, the distorted wavefront gradually recovered after re-passing through the inhomogeneous body (the sphere). Even the creeping wave is migrated back to the wavefront. At the imaging time, all the energy converges to the scattering point.

These three-dimensional elastic wave examples are calculated on a SUN SPARC-10 workstation with 2.5 hours of CPU time for each model. Whenever more information are needed to generate fine image, additional extrapolation between two successive screens are needed.

The above numerical tests, though not a full prestack migration, demonstrate that the elastic complex screen algorithm can be successfully used as a backpropagator in both prestack and poststack migrations.
5. CONCLUSION

Due to the enormous computer CPU-time and memory saving the multi-screen (phase-screen for scalar waves, elastic complex-screen for elastic waves) method is very attractive and promising as the backpropagator for 3D prestack migration. It can handle large 3D problems prohibitive to other methods. The numerical tests and examples shown in the paper demonstrate that the elastic complex screen (ECS) algorithm can be successfully used as both propagator in forward modeling and backpropagator in prestack or poststack migrations.

A. Forward modeling

Figure 8. Snapshots on the X-Z plane (y = 0) for the forward modeling and reverse-time migration for the inhomogeneous background, A: forward modeling and B: reverse-time migration. Focusing and defocusing effect can be seen from both forward propagation and reverse extrapolation. The distorted wave front is gradually recovered during the backpropagation.

B. Backward extrapolation

6. ACKNOWLEDGEMENT

The work was supported by a grant from the Los Alamos National Laboratory Institutional Supported Program, under the auspices of the United States Department of Energy, and by the Airforce Office of Scientific Research through contract F49620-92-J-0461 administered by the Phillips Laboratory of the Air Force. The support from the W.M. Keck Foundation is also acknowledged. Institute of Tectonics, University of California, contribution 253.

7. REFERENCES