DISCUSSION


In the paper by Lerche (hereafter referred to as Lerche 86), the author compared his formula of effective attenuation (42) based on the mean field attenuation with formula (41), attributed to Wu (1982a), and claimed that Wu’s formula “explicitly assumed that the Born approximation is valid (i.e., that the scattering by the randomly sited centers is from the unperturbed incident wave with no modification to the wave from previous scattering),” while in his result “the modification to the incident wave is taken into account through the statistically sharp mean field” and therefore is “more correct.” However, neither the statement about the validity condition of Wu’s formula nor the author’s formula (42) from the mean field approach is correct. In fact, the mean field formalism, when improperly applied to amplitude attenuation, can generate physically meaningless results. The impotence and fallacy of the mean field formalism in dealing with amplitude attenuation in 3-D random media have been recognized since the early 1980s (Wu, 1980, 1982a, b; Sato, 1982a, b). This recognition and the developments thereafter can be considered as one of the major advances in the field of scattering attenuation of seismic waves (Herraiz and Espinosa, 1987; Wu, 1987). Retrace from this advance seems injudicious and illogical. Therefore, I would like to take this opportunity to clarify some basic problems of scattering attenuation.

Mean field attenuation (MFA) — decoherence

In calculation of the mean field, the ensemble average is used for the complex field (including the phase and amplitude). The displacement of a random seismic wave $u$ can be expressed as

$$u = Ae^{i\phi} = Ae^{i(\phi_0 + \delta\phi)},$$

(1)

where $A$ is the amplitude, $\phi$ is the phase, $\phi_0$ is the average phase delay after traversing the random medium, and $\delta\phi$ is the random phase fluctuation caused by the random heterogeneities. Suppose the medium is pure elastic (no intrinsic attenuation) and the scattering is mainly forward scattering (neglecting the energy loss due to large-angle scattering). Then, $A$ can be considered constant. The only random part of the wave is $\delta\phi$. An ensemble average of $u$ is

$$<u> = A <e^{i(\phi_0 + \delta\phi)}> = Ae^{i\phi_0} \exp(-<\delta\phi^2>/2),$$

(2)

if we assume a Gaussian probability distribution of $\delta\phi$. We see from equation (2) that even if we assume there is no amplitude attenuation, the mean field $<u>$ will attenuate due to phase randomization of the field. Knowing the propagation distance $R$, we can define the attenuation coefficient of the mean field as

$$\nu = \frac{<\delta\phi^2>}{2R}.$$  

(3)

The formula for $\nu$ can be derived from mean field theory (e.g., Karal and Keller, 1964; Uscinski, 1977). We can derive $<\delta\phi^2>$ by simple and intuitive arguments. Suppose the random medium is uniform and has a correlation length $a$. For the wave traveling through distance $a$, the phase deviation from the homogeneous case is $\delta\phi_1 = (\delta\phi)a = -(\omega/v_0)(\delta\nu/v_0)a = -k(\delta\nu/v_0)a$, where $v_0$ is the average velocity and $k$ the average wavenumber. Assume $R = na$; then, the total phase variance is

$$<\delta\phi^2> = n <\delta\phi_1^2> = k^2 <(\delta\nu/v_0)^2>aR = k^2\nu^2aR,$$

(4)

where $\nu = <(\delta\nu/v_0)^2>^{1/2}$ is the relative rms velocity perturbation. Therefore, we obtain

$$\nu = \frac{\nu^2k^2a}{2},$$

(5)

which is correct for high frequencies ($ka \gg 1$).

Because the mean field attenuation is mainly due to interference between different realizations of the random wave caused by the phase randomization, and therefore cannot be measured for a specific realization, $\nu$ is called by Wu (1982b) the “randomization coefficient” rather than “attenuation coefficient” to avoid confusion. Since the decrease of $<u>$ means the loss of coherency of the wave field, $\nu$ can be also called the decoherence coefficient or simply decoherence.

Average amplitude attenuation (AAA)

In order to derive the average amplitude attenuation (AAA), we need to remove the phase influence before taking the ensemble average. This can be done by multiplying the wave field by its complex conjugate. This corresponds to performing a phase correction for every realization of the ensemble before averaging. By this process we get

$$<uu^*> = <Ae^{i\phi} Ae^{-i\phi}*> = <A^2>.$$

(6)

Therefore, in order to derive the formulas for AAA, we have to solve the equation for the second moment $<uu*>$, not the first moment $<u>$.

To formulate the AAA, Wu (1982a) and Sato (1982a) applied different approaches (for a review, see Herraiz and Espinosa, 1987). Wu (1982a) noticed that the scattered waves in the forward direction will be rescattered back into the propagation direction. Depending on the width of the observation time window, the forward scattered energy should be partly excluded from the scattering attenuation. When the whole seismogram is considered, Wu (1982a) uses the multiple-forward-scattering approximation (or the single-backscattering approximation), which counts all the waves scattered to the backward half-space as scattering loss. This approximation underestimates the scattering attenuation, since not all the forward scattered waves can be rescattered back to the propagation direction and reach the receiver in the observation time window. The approximation can be modified to include only the forescattered waves within certain critical angle $\theta_e$ as the recycled energy. The formula becomes

$$\eta_e = \frac{1}{2} \bar{\nu}^2 k^2 [P(2k \sin \theta_e) - P(2k)],$$

(7)
where \( \tilde{v} = (\delta v / v_0)_{	ext{rms}} \), \( P(k) \) is the 1-D power spectrum of the random heterogeneities, and \( k \) is the wavenumber in the background medium. In the case of exponential correlation, taking \( \theta_e = 90^\circ \) (as in Wu, 1982a) results in (when \( ka \gg 1 \))

\[
\eta_e = \frac{q^2}{4a}.
\]  

(8)

Compared with equation (5), we see that \( \eta_e \) is inversely proportional to the scale of heterogeneities \( a \), and \( v \) is proportional to \( a \). These results are consistent with the physics of scattering. When the scale \( a \) is smaller, there will be more scatterers along the propagation path. Since the number density is proportional to \( 1/a^3 \), and the cross-section of each scatter is \( a^2 \), the scattering loss will be proportional to \( 1/a \). On the other hand, larger \( a \) will cause greater phase fluctuations; therefore, the coherency loss should be proportional to \( a \) and also increase with frequency as in equation (5).

Sato's approach is to modify the mean field formalism such that the phase (or traveltime) fluctuation is excluded from the scattering attenuation (Sato, 1982a,b). He noticed that the low wavenumber components of the heterogeneity spectrum are the dominant contributors to the traveltime fluctuations. Therefore, he cut off the part of the spectrum below a certain critical wavenumber components of the dominant contributors to the traveltime fluctuations. Thereafter, the scattering attenuation using the mean field approach (Sato called this the traveltime-corrected mean wave formalism). He also proved that, with \( K_c = \pi/a \), the corresponding \( \eta_e \) is about \( 29^\circ \).

In Lerche 86, the apparent attenuation due to scattering, from equation (39) of that paper, is

\[
\eta_L = \frac{k}{2} Q_\tau^{-1} = 4\tilde{v}^2 k^4 (4k^2 + a^2)^{-1} = 4\tilde{v}^2 k^4 (4k^2 a^2 + 1)^{-1},
\]  

(9)

where \( c \) is a constant and \( a = 1/k_i, k_i \) is the corner frequency in Lerche's notation. For high frequencies, expression (9) becomes

\[
\eta_L = \tilde{v}^2 k^2 a,
\]  

(10)

which is the typical result of mean field attenuation (compared with equation (5)). The increase of \( \eta_L \) with the scale length of the heterogeneities indicates clearly that \( \eta_L \) is a measure of coherence loss and bears no relation to the amplitude attenuation.

Note also that the frequency dependence of decoherence \( v \), i.e., MFA [such as equations (5) and (10)] is also drastically different from the amplitude attenuation. The decoherence \( v \) increases with frequency for two powers higher than the amplitude attenuation. Field observations (Wu, 1982b; Sato, 1982b; for a summary, see Herrera and Espinosa, 1987), laboratory measurements (Menke et al., 1985; Dubendorff and Menke, 1986; Matsunami, 1983, 1987), and numerical simulation (Frankel and Clayton, 1986) confirmed the AAA formulation of Wu and Sato rather than the MFA results.

Lerche 86 also claimed (p. 1536) that Wu's formula as in equation (7) explicitly assumed the Born approximation, while his formula had taken into account the modification to the incident wave "through the statistically sharp mean field." His result (9) can be expressed as [formula (42) of Lerche 86]

\[
\eta_L = \tilde{v}^2 k^2 [P(0) - P(2k)].
\]  

(11)

We immediately recognize that this result is nothing but the familiar result of Born approximation. In fact, Wu (1982b) has shown that the single scattering approximation (Born approximation) for the average energy loss (second moment) leads to the same result as the bilocal approximation for the mean field (first moment). The bilocal approximation also known as the first-order smoothing approximation, binary interaction approximation, etc. (for review, see Frisch, 1968; Wu, 1982b), is used in most mean field derivations, including Lerche 86. Physically, it can be understood as follows. The incident wave is totally coherent; by assuming the total loss of the scattered energy, the propagating wave remains coherent. Putting the coherence loss into an exponential form implies the multiple scattering process of the coherent field (mean field). However, this multiple scattering of the mean field is equivalent to the single scattering approximation of the energy loss. The scattered waves can be rescattered back to the forward direction, but they become incoherent due to the random nature of the velocity perturbations. Therefore, they are dropped totally in the mean field formalism. Wu (1982a) considered regaining scattered energy in the forward direction in his derivation; therefore, Wu's formula is a multiple-forward scattering, single-backscattering approximation and is not a Born approximation.

In conclusion, replacing \( P(\sqrt{2}k) \) in Wu's formula with \( P(0) \), as in equation (11) [equation (42) in Lerche 86], is not "more correct" but incorrect and can generate physically meaningless results in the high-frequency range. Interestingly, the phase correction needed for the amplitude attenuation calculation turns out to be quite easy for the 1-D case. Banik et al. (1985a,b) did the correction using a variable transformation from distance \( z \) to one-way traveltime \( T = |dz/v |, \) where \( v \) is the wave velocity. The ensemble average is taken at the same \( T \), not the same \( z \), for all the realizations. This procedure is equivalent to performing a phase correction for each realization before averaging, which removes totally the traveltime fluctuation caused by forward scattering. Therefore, the result is a mean amplitude attenuation, not a mean field attenuation in its original meaning; although the authors did not discuss this point. The formulation in fact is a traveltime-corrected mean field approach. From their result (Banik et al., 1985a), we can clearly see this point. The result,

\[
F_\tau = \frac{1}{2} Q^{-1} = \frac{1}{2} \omega M(\omega),
\]  

(12)

depends only on the spectrum of the impedance fluctuations \( M(\omega) \) and has no relation to the velocity structure of the medium. We know that forward scattering can be produced only by velocity fluctuation, and backscattering only by the impedance fluctuation (Wu and Aki, 1985a,b). Therefore the result of Banik et al. removed the influence of forward scattering. The apparent attenuation obtained is for the mean amplitude, not for the mean field. The amplitude decay is mainly caused by the energy loss due to backscattering. The accompanying time delays are caused by the interference of multiple backscattered waves. For the telegraph model of the layered medium, equation (12) in Banik et al. (1985b) is

\[
F_\tau = \frac{1}{2} Q^{-1} = \frac{M_0}{2} \frac{(f / f_c)^2}{1 + (f / f_c)^2},
\]  

(13)

where \( f \) is the wave frequency and \( f_c \) is the corner frequency of the medium, which is the inverse of the average scale length.
is inversely proportional to the average bed thickness. This is physically meaningful for the amplitude attenuation but opposite to the mean field attenuation in its original meaning.

The simplicity of the phase correction in the 1-D case came from the specially simple geometry: there are only two directions. For 2-D and 3-D problems, there exists no such simplicity.

References


Reply by the author to Ru-Shan Wu

Wu provides above a discussion of my paper (Lerche, 1986). Several points are relevant:

(1) The Born approximation around the incident field does not conserve energy, does not allow for multiple scatterings (Born and Wolf, 1959), nor does it satisfy the fundamental optical theorem (Jackson, 1975).

(2) The mean field approximation is a Born approximation around the locally coherent component of the field, and so does allow for multiple scattering, conserves energy on average to the level of third-order irreducible correlations (Frisch, 1968; Tatarskii, 1971), and satisfies the optical theorem to the same degree.

(3) In Wu’s (1982) treatment an angle $\theta_e$ is introduced in an ad-hoc manner. Wu sets $\theta_e$ at $45^\circ$; Sato (1982) finds $29^\circ$, numerical experiments using the full wave equation by Dubendorf and Menke (1986) find about $10^\circ$ for P waves, and about $15^\circ$ and $6^\circ$ for S waves; while the mean field theory gives $0^\circ$ (Lerche, 1986). This discrepancy alone is sufficient to indicate that the classical Born single-scattering approximation is invalid, all testostations to the contrary notwithstanding.

(4) The 1-D work of Banik et al. (1985a and b) use mean field theory and agrees considerably better with exact numerical wave equation computations than does the conventional Born approximation. In addition, the mean field results in 1-D were shown to be minimum-phase (i.e., causal).

(5) The exact 1-D results were computed analytically (Resnick et al., 1986) for both transmitted and reflected waves without recourse to either the Born approximation or the mean field approximation; and it was shown that the mean field was a significantly better approximation by direct calculation.

(6) If only the phase is allowed to vary and not the amplitude, as in equation (1) of Wu’s discussion, then (a) no reflections are possible (contrary to the second-order nature of the wave equation) and, more significantly, (b) the field fails to satisfy the Kramers-Kronig relations — a necessary requirement for a causal signal.

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References