Velocity and Density Reconstruction Based on Scattering Angle Separation

JINGRUI LUO1,3 and RU-SHAN WU2

Abstract—The density of the medium under investigation is important for seismic interpretation and rock property analysis. However, accurate estimation of density is difficult, because seismic responses to density and velocity are coupled, and there exits a trade-off problem. By analyzing the properties of their radiation patterns, we propose a scattering angle-based inversion strategy for simultaneous estimation of density and velocity. Both the source-side and receiver-side wavefields are decomposed into local angle domain, and then local scattering angles are calculated. Small scattering angles correspond to forward scattering, which are caused by velocity perturbation, so small scattering angles are used only for velocity estimation. Density perturbation is only responsible for backward scattering, which are related to large scattering angles. Therefore, by filtering out small scattering angles during density inversion, the crosstalk between velocity and density is greatly reduced. Numerical examples prove that the scattering angle-based inversion method provides much improved density inversion results. Furthermore, the multi-stage strategy also helps velocity to overcome the cycle skipping problem in highly non-linear full waveform inversion.

Key words: Scattering angle, angle filter, density, velocity, inversion.

1. Introduction

Ever since full waveform inversion was proposed (Lailly 1983; Tarantola 1984), it has been an important tool for subsurface parameter estimation. Within the acoustic territory, Gersztenkorn et al. (1986) presented a robust iterative inversion strategy for one-dimensional inversion. Gauthier et al. (1986) numerically implemented the two-dimensional acoustic inversion. Pica et al. (1990) applied this method to a real marine seismic data set. Zhou et al. (1995) formulated and applied an acoustic waveform inversion method to the real crosshole data. The time domain full waveform inversion was extended and applied to the frequency domain by Pratt (1999) and Pratt and Shipp (1999). The frequency domain method is superior and less time consuming. However, it requires a high memory capacity. With the development in both hardware and algorithm, the full waveform inversion has been able to be applied to the 3D situations (Benhadjali et al. 2008; Sirgue et al. 2008; Pyun et al. 2011; Kim et al. 2013; Operto et al. 2015). In addition to the acoustic case, the full waveform inversion was also extended and applied to media including elasticity (Tarantola 1986; Mora 1987, 1988; Crase et al. 1990; Forgues and Lambar 1997; Choi et al. 2008; Jeong et al. 2015; Raknes et al. 2015), attenuation (Tarantola 1988; Charara et al. 2000; Brossier 2011) and anisotropy (Barnes et al. 2008; Warner et al. 2013; Debens et al. 2015; Masmoudi and Alkhalifah 2016; Oh and Alkhalifah 2016, 2017; Guitton and Alkhalifah 2017; Qu et al. 2017).

Within all the subsurface parameters, density is an important one, which is very useful for seismic interpretation and rock property analysis (Connolly 1999), however, although full waveform inversion has great potential in retrieving all kinds of parameters, accurate estimation of density is always difficult. Many researchers have studied this problem and indicated that density cannot be adequately inverted, no matter what kind of parameterization is chosen during the inversion (Choi et al. 2008; Khn et al. 2012; Alkhalifah and Plessix 2014). In many cases, the density is simply assumed as a fixed value or obtained from the empirical equation relating density.

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1 School of Automation and Information Engineering, Xi’an University of Technology, Xi’an 710048, China. E-mail: bluebirdjingrui@gmail.com
2 Institute of Geophysics and Planetary Physics, University of California, Santa Cruz, CA 95064, USA.
3 School of Automation and Information Engineering, Xi’an University of Technology, 5 South Jinhua Road, Xi’an 710048, China.

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Several strategies were proposed to improve the density estimation from full waveform inversion. Kolb and Canadas (1986) used the velocity and the reflection coefficients to describe the acoustic media, and simultaneously adopted an increasing frequency bandwidth inversion strategy and a downward identification technique to increase the inversion accuracy. Sambridge et al. (1991) developed the subspace scheme for multi-parameter estimation including p-wave and s-wave impedances and density. In their method, they split the descent vector into different parts with each part depending on a different parameter type, and the misfit function was minimized in the subspace. Jeong et al. (2012) recovered the Lamé constants and density through a two-stage strategy in frequency domain elastic FWI, where the Lamé constants are firstly recovered with fixed density in the first stage, and density is updated along with Lamé constants in the second stage. Abubakar et al. (2012) presented a joint inversion approach where they employed a regularization function for enforcing the structural similarity between the resistivity and the velocity and density. Prieux et al. (2013) firstly used the large offset data for velocity estimation, and then used the whole dataset along with the updated velocity for simultaneous velocity and density estimation. This strategy improved the stability in density estimation. Considering that velocity and density have different sensitivities to the wavefields, Bai and Yingst (2014) developed a simultaneous inversion using preconditioned velocity and density gradients. Xu and McMechan (2014) assigned them different step lengths for updating according to their gradients so as to decouple multi-parameters. Qin and Lambar (2016) developed a shot-domain preserved-amplitude inversion workflow to delineate both velocity and density to reduce the crosstalk. Besides, the inverse Hessian operator is also applied to reduce trade-offs among multiple parameters and to improve the accuracy in density estimation (Mtivier et al. 2015; Yang et al. 2016).

The difficulty in accurate density estimation lies in the overlap of the parameter responses between velocity and density, so there is the trade-off problem, and the density inversion is always influenced by the velocity information. However, under velocity–density parameterization, the forward scattering comes from velocity perturbation and is related to small scattering angles, while the density perturbation contributes only to backscattering, which corresponds to large scattering angles. So if small scattering angle information can be filtered out from the density inversion, influence from the velocity can be reduced. The angle information is very useful and was successfully applied in several areas, such as migration velocity analysis (Rickett and Sava 2002; Biondi et al. 2004), reverse time migration (Yoon and Marfurt 2006; Yan and Xie 2012; Wang et al. 2014), and full waveform inversion (Alkhalifah 2015a, b; Luo and Xie 2017). In this paper, we propose the scattering angle-based inversion strategy for velocity and density estimation. Both the source-side and receiver-side wavefields are decomposed into the local angle domain and the related scattering angles are calculated. The velocity is firstly updated starting from small scattering angles with fixed density. Then the density is recovered with large scattering angles based on the updated velocity. This makes a multi-stage inversion strategy, and trade-off from the velocity estimation can be greatly reduced during density inversion. We tested this strategy with numerical examples. The results prove that this scattering angle-based inversion method not only provides a much improved density model, but the multi-stage strategy also helps the velocity inversion in overcoming the cycle skipping problem in highly nonlinear full waveform inversion.

The rest of the paper is organized as follows. In Sect. 2, we introduce the theory background for this method, including scattering angle separation for multi-parameters, the scattering angle-based inversion strategy, and the scattering angle calculation. Then in Sect. 3, we show some numerical examples to show the validity of this method. Discussions are covered in Sect. 4, followed by conclusion remarks in Sect. 5.
2. Theory

In this section, we introduce the theory for this method and show the details on how to accomplish this approach.

2.1. Scattering Angle Separation for Velocity and Density

Assume there is a monochromatic wave radiated from source \( x_s \) and incident on a small target region \( V(x) \) in the subsurface medium neighboring \( x \). The monochromatic acoustic wave equation in the heterogeneous medium has the following form,

\[
\frac{\omega^2}{K(x)} \tilde{u}(x) + \nabla \left( \frac{1}{\rho(x)} \nabla \tilde{u}(x) \right) = 0,
\]

where \( \tilde{u}(x) \) is the frequency domain pressure wavefield and \( \omega \) is the angular frequency. The medium is described by the incompressibility field \( K(x) \) and the density \( \rho(x) \). The wave propagation velocity \( c \) is related to \( K \) and \( \rho \) as \( K = \rho c^2 \).

After interacting with heterogeneities within \( V \), the scattered wave at receiver \( x_g \) can be obtained by using the Born approximation (Hudson and Heritage 1981; Wu 1989) within region \( V \) as,

\[
u'(x, x_s, x_g) = k^2 \int_V \tilde{e}_K(x, x') G(x', x_s) G(x' ; x_g) dx' - \int_V \tilde{e}_p(x, x') \nabla G(x' ; x_s) \cdot \nabla G(x' ; x_g) dx',
\]

where \( k = \omega/c_0(x) \) is the background wavenumber. The angular frequency \( \omega \) is omitted from the equation for simplicity. \( G(x' ; x_s) \) and \( G(x' ; x_g) \) represent the background Green’s function for wave propagating from source \( x_s \) to \( x' \) and receiver \( x_g \) to \( x' \), respectively. \( \tilde{e}_K \) and \( \tilde{e}_p \) mean the perturbation of the incompressibility field \( K \) and density \( \rho \), respectively, with

\[
\tilde{e}_K(x, x') = \frac{K_0(x)}{K(x')} - 1;
\]

\[
\tilde{e}_p(x, x') = \frac{\rho_0(x)}{\rho(x')} - 1,
\]

where \( K_0 \) and \( \rho_0 \) are the background incompressibility and density, respectively. From the relation \( K = \rho c^2 \), the effect of velocity perturbation can be seen as \( \tilde{e}_c = \tilde{e}_p + 2\tilde{e}_s \), with

\[
\tilde{e}_c(x, x') = \frac{c_0(x)}{c(x')} - 1.
\]

Applying the local plane wave decomposition to the Green’s functions (Xie et al. 2005), the scattered wave in the local wavenumber domain can be obtained as,

\[
u'(x, x_s, x_g) = k^2 \int V \tilde{e}_K(x, k_x - k_i) G(k_x, x; x_s) G(k_x, x_s; x_g) dk_x dk_i - \int V \tilde{e}_p(x, k_x - k_i) \nabla G(k_x, x; x_s) \cdot \nabla G(k_x, x_s; x_g) dk_x dk_i
\]

\[
= k^2 \int V \left( \tilde{e}_K(x, k_x - k_i) + 2\tilde{e}_p(x, k_x - k_i) \right) G(k_x, x; x_s) G(k_x, x_s; x_g) dk_x dk_i - \int V \tilde{e}_p(x, k_x - k_i) \nabla G(k_x, x; x_s) \cdot \nabla G(k_x, x_s; x_g) dk_x dk_i.
\]

where vectors \( k_i \) and \( k_g \) are the local transforms with respect to \( x' \), which represent the incident and scattering wavenumbers, respectively. The integrals for \( k_i \) and \( k_g \) are between negative and positive Nyquist wavenumbers. \( \tilde{e}_K \), \( \tilde{e}_p \) and \( \tilde{e}_s \) are the wavenumber domain perturbations, with

\[
\tilde{e}_K(x, k_x - k_i) = \int V \tilde{e}_K(x, x') e^{i(k_x - k_i)x'} dx',
\]

\[
\tilde{e}_p(x, k_x - k_i) = \int V \tilde{e}_p(x, x') e^{i(k_x - k_i)x'} dx',
\]

\[
\tilde{e}_s(x, k_x - k_i) = \int V \tilde{e}_s(x, x') e^{i(k_x - k_i)x'} dx'.
\]

In the literature, \( k_g = -k_i \) is also used (see Wu and Toksöz (1987) for the definition). Equation (6) links the scattering geometry with the subsurface structures, which is shown in Fig. 1a. \( k_g - k_i \) is known as the exchange wavenumber. \( \theta_i \) and \( \theta_g \) represent the directions of the incident and scattered wave propagation, respectively. \( \Theta = \theta_g - \theta_i \) is the scattering angle. \( \Theta' = 180^\circ - \Theta \) is the opening angle between the incident and scattered waves.

In this paper, we consider the velocity–density parameterization. Under velocity–density
parameterization, the radiation patterns have the following form (Forgues and Lambar 1997),

\[ R_{c,\rho} = -2[1, \cos^2(\Theta'/2)], \]  

where \( \Theta' \) is the opening angle, and \( R_{c,\rho} \) represents the radiation pattern pair for velocity and density, which is illustrated in Fig. 1b (see also Wu and Zheng 2014). We can see that the velocity has an isotropic radiation pattern, while the density only causes backscattering, which corresponds to small opening angles or large scattering angles.

From Fig. 1, we can say that inversion without angle decomposition will introduce a trade-off between velocity and density for backscattered angles, which will heavily influence the density estimation. To clearly show the trade-off effect, we look into the gradient updating for velocity and density based on a simple contrast model. As shown in Fig. 2, the model size is 5 km in distance and 2 km in depth. The velocity has a constant background of 3000 m/s, and the density has a constant background of 2000 kg/m³. Each of them has a small round-shaped contrast area with 10% perturbation, while the contrast for the velocity and density is centered at distance 1.7 and 3.1 km, respectively, and both at depth 0.9 km. We uniformly placed 50 sources and...
100 receivers on the surface of the model, and use the background models as the initial models for the velocity and density; then model gradients can be calculated based on the scattered waves. The calculated gradients for the velocity and density are shown in Fig. 3a, b. From the results we can see that both the velocity perturbation and density perturbation cause gradient updating to the other, i.e., the trade-off problem. However, trade-off from the velocity to the density is significantly stronger than that from the density to the velocity. We see that in the density gradient, trade-off from the velocity is even stronger than the density itself. Thus, the density can hardly influence the velocity updating, yet it is strongly affected by the velocity perturbation.

By analyzing the scattering geometry and radiation patterns, we can propose the angle-based multi-stage inversion strategy to reduce the trade-off effect. Firstly, both the source-side and receiver-side wavefields are decomposed into the angle domain, and the scattering angles are calculated according to the incident and scattered wave directions. Then in the first inversion stage, the density is fixed, while the velocity is updated from small scattering angles to large scattering angles, which makes a series of sub-stages within the first inversion stage. Because small scattering angles, which correspond to forward scattering, are independent of the density perturbation, the velocity inversion is not affected by the density scattering. In the second inversion stage, only large scattering angles are picked out for density estimation. In the meantime, velocity is also updated in this stage with all angle information used. By using the angle decomposition, influence from the velocity to density can be greatly reduced. From Fig. 1 we can see that smaller scattering angles relate to smaller $|\mathbf{k}_i - \mathbf{k}_s|$, i.e., longer scale lengths. Therefore, inversion using small scattering angles tends to retrieve large-scale background models. So the decomposition of scattering angles also helps to avoid the cycle skipping problem in velocity estimation (Alkhalifah 2015a; Xie 2015; Luo and Xie 2017; Wu and Alkhalifah 2017).

2.2. Scattering Angle-Based Inversion Algorithm

Equation (1) shows the frequency domain acoustic wave equation. The corresponding time domain wave equation characterized by the velocity and density can be expressed as,

$$\frac{1}{c^2(x) t^2} \frac{\partial^2}{\partial t^2} u(x, t) + \rho(x) \nabla \cdot \left( \frac{1}{\rho(x)} \nabla u(x, t) \right) = \delta(x - x_s) f(t),$$

where $u(x, t)$ is the time domain wavefield, and $f(t)$ is the source function. Define the misfit function as

$$\sigma(m) = \frac{1}{2} (\Delta u^T \Delta u^*),$$

where $\Delta u$ is the difference between the synthetic data and observed data. $T$ is the transpose and $*$ denotes the complex conjugate. $m$ represents model...
parameters, i.e., velocity and density in this work. We use the steepest-descent method for the iteration. The model parameters can be retrieved by minimizing the misfit function and updated in the following form,

\[ \mathbf{m}^{k+1} = \mathbf{m}^k - \zeta_m^k \mathbf{m}, \]

where \( \zeta_m^k \) is the gradient of the misfit function with respect to \( \mathbf{m} \) in the \( k \)th iteration. \( \zeta_m^k \) is the step length which is a positive scalar. The gradients can be obtained by calculating the derivative of the misfit function,

\[
\zeta_v(x) = \sum_{sr} \frac{2}{\rho(x)c^3(x)} \int_0^T \overline{u}(x, t) \frac{\partial}{\partial t} \overline{u}(x, t) dt,
\]

\[
\zeta_p(x) = \sum_{sr} \frac{1}{\rho^2(x)c^2(x)} \int_0^T \nabla \overline{u}(x, t) \nabla \overline{u}(x, t) dt
+ \sum_{sr} \frac{1}{\rho^2(x)c^2(x)} \int_0^T \overline{u}(x, t) \frac{\partial}{\partial t} \overline{u}(x, t) dt,
\]

where \( \overline{u} \) and \( \frac{\partial}{\partial t} \overline{u} \) represent the forward propagated wavefield from the source-side and backward propagated wavefield from the receiver-side, respectively. \( \overline{u} \) and \( \frac{\partial}{\partial t} \overline{u} \) represent time derivatives with respect to \( \overline{u} \) and \( \frac{\partial}{\partial t} \overline{u} \), respectively.

Model updating using gradients in Eq. (11) involves wavefields from all angle information, which causes trade-off between multi-parameters as can be seen from Fig. 1. To reduce the trade-off and update the velocity and density with different scattering angle information, both the source-side and receiver-side wavefields are decomposed into local plane waves,

\[
\overline{u}(x, t) = \sum_{\theta_i} \overline{u}(x, t, \theta_i),
\]

\[
\overline{u}(x, t) = \sum_{\theta_g} \overline{u}(x, t, \theta_g),
\]

where \( \theta_i \) and \( \theta_g \) indicate the local propagation directions for the source-side wavefield \( \overline{u} \) and the receiver-side wavefield \( \overline{u} \), respectively (see Fig. 1). Substituting Eq. (12) into Eq. (11), the angle domain gradients for the velocity and density are,

\[
\zeta_v(x) = \sum_{sr} \frac{2}{\rho(x)c^3(x)} \int_0^T \frac{\partial}{\partial t} \overline{u}(x, t, \theta_i) \overline{u}(x, t, \theta_i) F_c(\theta),
\]

\[
\zeta_p(x) = \sum_{sr} \frac{1}{\rho^2(x)c^2(x)} \int_0^T \nabla \overline{u}(x, t, \theta_i) \nabla \overline{u}(x, t, \theta_i) F_p(\theta),
\]

\[
+ \sum_{sr} \frac{1}{\rho^2(x)c^2(x)} \int_0^T \overline{u}(x, t, \theta_i) \frac{\partial}{\partial t} \overline{u}(x, t, \theta_i) F_p(\theta),
\]

where \( F_c(\theta) \) and \( F_p(\theta) \) represent the angle domain filter for velocity and density estimation, respectively, which control the angle bands that enter the inversion. By controlling the angle filter, wavefields with different angle information are allowed to enter the model updating. Thus, the decoupling of velocity and density can be accomplished by assigning different angle bands to \( F_c \) and \( F_p \).

### 2.3. Calculation for Scattering Angles

In the scattering angle-based inversion strategy, angle filters \( F_c(\theta) \) and \( F_p(\theta) \) are needed, which requires the calculation of scattering angle \( \Theta \) during the inversion. There are several approaches to accomplish the angle filter (Alkhalifah 2015b; Choi and Alkhalifah 2015; Xie 2015; Luo and Xie 2017). In this paper, we use the Poynting vector method in time domain full waveform inversion due to its high efficiency (Xie 2015).

The Poynting vector indicates the energy flux density of the wavefield, which reveals the wave propagation direction. It can be calculated using the following equation (Yoon and Marfurt 2006),

\[
P_i = -\nabla \cdot \mathbf{\hat{u}} \frac{\partial \mathbf{\hat{u}}}{\partial t},
\]

\[
P_s = -\nabla \cdot \mathbf{\hat{u}} \frac{\partial \mathbf{\hat{u}}}{\partial t},
\]

where \( P_i \) and \( P_s \) represent the Poynting vector for the incident and scattered wave, respectively. Given the
Poynting vectors, the local propagation directions $\theta_i$ and $\theta_g$ in 2D can be obtained as,

\[ \theta_i = \arctan \frac{P_{ix}}{P_{iz}}, \]
\[ \theta_g = \arctan \frac{P_{gx}}{P_{gz}}, \]

where the superscripts $x$ and $z$ represent $x$ component and $z$ component of a vector, respectively. The scattering angle can also be obtained directly as,

\[ \Theta = \arccos \frac{P_i \cdot P_g}{|P_i||P_g|}. \]

To improve the robustness, we introduce the time integral technique during the scattering angle calculation (Yoon et al. 2011).

We summarize the whole process of the above algorithm for the scattering angle-based strategy as follows,

Step 1. For a certain time step, we generate the source-side forward propagated wavefields and the receiver-side backward propagated wavefields for each grid point.

Step 2. At each grid point, we calculate the Poynting vector for the source-side wavefields and receiver-side wavefields according to Eq. (14), respectively. We integrate the Poynting vectors for a short time period to get a robust result.

Step 3. We calculate the scattering angle for each grid point based on the Poynting vectors according to Eqs. (15) or (16).

Step 4. For each grid point, we decide if the scattering angle is included in the preset angle band as indicated in Eq. (13). If included, then the corresponding wavefields are allowed for the gradient calculation. Otherwise, the corresponding wavefields are ignored and not used for the gradient calculation.
Step 5. We repeat the above steps until the maximum time step, and the final gradients are obtained.

3. Numerical Examples

In this section, we test the validity of the proposed method. We first use the simple contrast models as shown in Fig. 2 to clearly show the effect of the angle separation on reducing the trade-off problem, then we apply this method to the Marmousi model.

3.1. Simple Contrast Model

The true models for the velocity and density are shown in Fig. 2. The initial models for the inversion are chosen as the constant background models. The model size is 500 grid points in the horizontal direction and 200 grid points in the vertical direction. The grid interval is 10 m. Sources and receivers are both fixed and uniformly spaced on the surface of the model. There are 50 sources and 100 receivers with the source interval as 100 m and the receiver interval as 50 m. The source function is the Ricker wavelet with peak frequency of 10 Hz. The time sampling interval is 1 ms and the total recording time is 3 s.

We first perform a regular full waveform inversion without using angle decomposition. The inversion results are shown in Fig. 4. We see that both the velocity and density suffer from the trade-off problem with other unexpected contrast areas shown in the results. If we look more into the results, we see that trade-off from the density to the velocity is slight, while the trade-off from the velocity to the density is overwhelming compared to the true density contrast.
To reduce the trade-off problem, we apply the angle separation into the inversion. The source-side and receiver-side wavefields are both decomposed into the angle domain. The scattering angles are calculated, and the velocity and density are inverted with different angle information.

We first invert for the velocity and keep density as the constant background. Scattering angles up to $120^\circ$ are used for the inversion, while larger scattering angles corresponding to sharp backscattering are avoided. The inversion result is shown in Fig. 5a. We see that because the contribution from the density perturbation in the sharp backscattering part is filtered out, the slight trade-off in the velocity model is avoided. Next, we invert for the density based on the obtained velocity in Fig. 5a. This time, only large scattering angles $120^\circ$–$180^\circ$ are used for density inversion. The inverted model is shown in Fig. 5b. We can see that with only large scattering angles used, the strong trade-offs from the velocity are significantly reduced, and only the true density perturbation area is well recovered as expected. We see that for the simple contrast model, the angle-based method greatly improved both the velocity and density inversion results.

3.2. The Marmousi Model

The true Marmousi models for the velocity and density are shown in Fig. 6. The initial models for the inversion are shown in Fig. 7. The initial model for the velocity inversion is a 1-D linear model, and initial model for the density inversion is a highly smoothed version of the true density. The model size is 2300 grid points in the horizontal direction and 750 grid points in the vertical direction. The grid interval is 4 m. Sources and receivers are both fixed and...
uniformly spaced on the surface of the model. There are 46 sources and 230 receivers with the source interval as 200 m and the receiver interval as 40 m. The source function is the Ricker wavelet with peak frequency of 10 Hz. Low frequencies below 5 Hz are removed from the data. The time sampling interval is 1 ms and the total recording time is 3 s.

We use the angle-based multi-stage inversion strategy as stated in Sect. 2.1. The source-side and receiver-side wavefields are both decomposed into the angle domain. The scattering angles are calculated, and the velocity and density are inverted with different angle information.

In the first stage, the velocity is inverted, with density kept as the smooth initial model. The inversion starts from the 1-D linear model. At the beginning, small scattering angles are used, then medium scattering angles and large scattering angles

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**Figure 8**

Inverted velocity model with different scattering angle bands in the first inversion stage. **a** Small scattering angles; **b** medium scattering angles; **c** all scattering angles.
are included gradually. Inversion results during this stage are shown in Fig. 8. Figure 8a shows the result with small scattering angles up to 30°. We can see that large-scale structures are well retrieved. Figure 8b shows the subsequent inversion with medium scattering angles up to 90° included. We can see that some medium-scale structures are retrieved. Then large scattering angles up to 180° are included in the inversion, and the inversion result is shown in Fig. 8c. We can see that finer scale structures are added to the model. After the progressive inversion from small scattering angles to large scattering angles, the velocity is well retrieved. The result in Fig. 8c is already very close to the true model.

In the second stage, the density and velocity are simultaneously updated, while only large scattering angles are used for the density estimation and the whole scattering angle band is used for the velocity updating. The starting model for the density inversion is the heavily smoothed initial model in Fig. 7b. The starting model for the velocity updating is the result from the first stage in Fig. 8c. Scattering angles from 120° to 180° are used for density inversion. The final inverted models are shown in Fig. 9. We can see that the inverted density is close to the true model with most of the features retrieved. Besides, the velocity model is further updated. As comparison, another test is conducted for the second inversion stage, where the whole angle band including all angle information is used for the density inversion, when the initial model for the density is the highly smoothed model and the starting model for the velocity is still the result from the first stage in Fig. 8c. Firstly, the gradients for the density updating at the first iteration are compared in Fig. 10. We can see that compared with using only large scattering angles, using all angle information introduces strong trade-off from the velocity. Figure 11 shows the final inverted density with all angle......
information. We see that the result is heavily influenced and distorted.

For a further comparison, we perform the inversion without using angle decomposition at all for both velocity and density, and start the inversion directly from the initial models in Fig. 7. The final results are shown in Fig. 12. We see that without the large-scale structure information obtained from the small scattering angles, the velocity suffers from the cycle skipping problem and is trapped in a local minimum.
The density is strongly affected by the velocity and converges to an awful result. Figures 13 and 14 show trace comparisons for the velocity and density, respectively, which compare the results in Figs. 9 and 12 at distances 2 and 7 km. From these comparisons, we can also see that the angle-based inversion result is much better than the conventional result which does not use angle information.

The above tests prove that the scattering angle-based inversion method greatly reduces the trade-off problem during velocity and density inversion, and provides much improved inversion results.

4. Discussion

In this work, we introduced the scattering angle-based inversion strategy for velocity and density inversion, which aims to reduce the trade-off between multi-parameters and improve the density estimation. In the current work, the scattering angles are calculated in the time domain using the Poynting vector method owing to its high efficiency. We see that the calculation of the Poynting vectors indicated in Eq. (14) requires the time derivatives and space derivatives of the wavefields, which are all incidental intermediate results of the forward modeling process and desire no additional calculation. So there is almost no extra computational time consumption compared to the conventional inversion method. However, the Poynting vector gives the average energy flux direction. For situations where the wave propagation is very complex and involves multiple directions for a certain location, the Poynting vector method may limit the actual effect implied in the scattering angle information. More sophisticated
approaches are desired for future study. In this work, the velocity inversion starts from a linear initial model. As the density is responsible only for backscattering, smooth background including large-scale structures are difficult to be obtained during the inversion; thus, a highly smoothed initial model is used for the density inversion. Initial model construction is also an important aspect of density estimation, which needs elaborate research in the future. This work considers the acoustic situation. Elastic parameters are very important for exploration geophysics; therefore it is valuable to include elasticity into this method in the future.

5. Conclusions

We presented a scattering angle-based inversion strategy for velocity and density estimation. Both the source-side and receiver-side wavefields are decomposed into the angle domain, and then the scattering angles are calculated and selected for the inversion. Based on the radiation patterns, velocity is first inverted from small scattering angles to large scattering angles in the first stage. Then velocity and density are simultaneously updated in the second stage, with only large scattering angles contributing to density. In this way, the influence from the velocity is reduced. We used the Marmousi model to test the validity of this method. Numerical examples prove that the scattering angle-based inversion method not only provides much improved density inversion result, but the multi-stage strategy also helps velocity to overcome the cycle skipping problem in the highly nonlinear full waveform inversion.
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