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Time-domain full waveform inversion using instantaneous phase information with damping

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Abstract

In time domain, the instantaneous phase can be obtained from the complex seismic trace using Hilbert transform. The instantaneous phase information has great potential in overcoming the local minima problem and improving the result of full waveform inversion. However, the phase wrapping problem, which comes from numerical calculation, prevents its application. In order to avoid the phase wrapping problem, we choose to use the exponential phase combined with the damping method, which gives instantaneous phase-based multi-stage inversion. We construct the objective functions based on the exponential instantaneous phase, and also derive the corresponding gradient operators. Conventional full waveform inversion and the instantaneous phase-based inversion are compared with numerical examples, which indicates that in the case without low frequency information in seismic data, our method is an effective and efficient approach for initial model construction for full waveform inversion.

Keywords: time-domain FWI, instantaneous phase, multi-stage inversion, damping factor

(Some figures may appear in colour only in the online journal)

1. Introduction

Full waveform inversion (FWI) can retrieve subsurface model parameters from the recorded dataset (Tarantola 1984, Pratt 1999a, Pratt and Shipp 1999b). However, it depends strongly on the initial model because of the highly nonlinear problem (Mora 1987). In order to avoid convergence to the local minima, a good initial model is needed, that is the initial model needs to be not far from the true model. Bunks et al (1995) and Boonyasiriwat et al (2009) tried to overcome this problem using multi-scale methods, in which low frequency information in the data is firstly used to recover long-wavelength structures, then high frequency information is used for fine structures. The key point for the success of the multi-scale method is that low frequency information needs to exist in the seismic data. However, it is always difficult to record low frequency information for most acquisition systems. So the problem is how to retrieve model structures effectively without low frequency.

Several approaches have been proposed to solve this problem. Shin and Cha (2008, 2009) proposed the Laplace domain and Laplace–Fourier domain waveform inversion, where large-scale model structures are obtained using low frequency information from the damped wavefields. Biondi and Almonim (2012), and Biondi and Almonim (2013) developed the tomographic full waveform inversion, where the problem is solved in the extended domain. Warner and Lluis (2014), and Warner et al (2015) proposed the adaptive full waveform inversion, which converted the problem of retrieving large-scale model parameters into measuring the coefficients of a Wiener filter. Wu et al (2014) and
Chi et al. (2014) developed the envelope inversion method, where low frequency information extracted from seismic trace envelopes is used to get the large-scale background model structures. Yuan et al. (2015) used well log data to obtain the long-wavelength component of seismic parameters. Zhang et al. (2017) used a sparse blind deconvolution to reconstruct the low frequency seismic data for inversion. Wang et al. (2017) used the transmission matrix to solve the nonlinear problem. Some researchers solved the problem by decomposing the scattering angles. For example, Alkhalifah (2015) analyzed the influence of scattering angles on velocity updating gradients. Xie (2015) developed a time-domain multi-scale inversion strategy where the angle-domain wavenumber filter is accomplished using the Poynting vector method. Luo and Xie (2017) presented frequency-domain FWI with an angle-domain wavenumber filter using the slant-stacking method.

The phase information contains the kinematics property of wave propagation, thus it has great potential to be used in FWI. However it suffers from the phase wrapping problem, which limits its applications. Some approaches have been proposed to overcome this problem. Choi and Alkhalifah (2013) introduced the instantaneous travel time in the inversion, which is obtained from the phase derivative with respect to the angular frequency. Shah et al. (2012) and Choi and Alkhalifah (2015) both unwrapped the phase by constructing the 2D phase map during the inversion.

The above approaches about using phase information in the inversion are all implemented in the frequency domain, as the complex frequency-domain seismic data intrinsically contains and separates the phase and amplitude information. However, the phase information (the instantaneous phase) can also be extracted and used from time-domain data, which makes it possible for the time-domain FWI to benefit from the great potential of phase information. In time domain, the instantaneous phase can be extracted from the analytic signals, or the complex seismic traces, which has already played a great role in time-domain seismic data processing (Barnes 1996, Marfurt et al. 1998, Gao et al. 1999, Wang et al. 2014, 2016). Bozda et al. (2011) measured the instantaneous phase misfit function and constructed the adjoint kernels for global seismology. In their work, they used the raw wrapped phase information, which is directly obtained from the arc-tangent calculation. However, this would not work well for full waveform inversion in seismic exploration, because of the severe cycle skipping problem. Sophisticated phase unwrapping methods are needed in this situation. However, they are always complicated and time consuming. In order to avoid the phase unwrapping process, we choose to use the exponential phase in this paper. We also combine the exponential phase with the damping method to further improve the inversion result. This time-domain instantaneous phase-based inversion method is fast and easy to accomplish. Numerical examples prove the validity of this approach in initial model construction in time-domain FWI for the situation when low frequency information is missing from the seismic data.

The rest of the paper is organized as follows: in section 2, we first show the basic idea of this method. Then, in section 3, we introduce the inversion method using the instantaneous phase only, as well as some numerical examples. This leads us to the combined instantaneous phase and damping approach, which will be described in section 4. Numerical examples of the combined method will also be shown in this section. Finally in section 5, we give some conclusions.

2. Basic idea

In the time domain, the instantaneous phase of the seismic data can be extracted from the analytic signal \( \hat{f}(t) \), which is a complex seismic trace without negative frequency components, and can be obtained from the original trace \( f(t) \) and its Hilbert transform \( f_H(t) \),

\[
\hat{f}(t) = f(t) + i \cdot f_H(t).
\]

In the following sections, the Hilbert transform will also be written as \( H\{ \}. \) The complex data trace in equation (1) can be rewritten in the following form

\[
\hat{f}(t) = E(t)e^{i\phi(t)},
\]

where \( E(t) \) is the instantaneous amplitude or data envelope, which was used in envelope inversion (Wu et al. 2014, Luo and Wu 2015) and can be calculated using the following equation

\[
E(t) = \sqrt{f^2(t) + f_H^2(t)}.
\]

In equation (2), \( \phi(t) \) is called the instantaneous phase, and can be obtained as follows

\[
\phi(t) = \arctan \frac{\Re\{\hat{f}(t)\}}{\Im\{\hat{f}(t)\}} = \arctan \frac{f_H(t)}{f(t)}.
\]

From the above equations we see that the analytic seismic trace contains both the instantaneous amplitude and the instantaneous phase information. The phase information is closely related to the kinematics property in the wavefield, so it has great potential to be used in FWI for overcoming the local minima problem. However, the phase suffers from the wrapping problem, which is introduced by numerical calculation using equation (4). In order to effectively use the instantaneous phase, we need to find some way to avoid the phase wrapping problem.

Note that the phase wrapping problem comes from the arc-tangent operator in numerical calculation; in order to reduce this problem, we try to avoid the arc-tangent calculation. For this purpose, we choose to use the exponential phase, which can be obtained from equations (1), (2) and (3),

\[
e^{i\phi(t)} = \frac{\hat{f}(t)}{E(t)} = \frac{f(t) + i \cdot f_H(t)}{\sqrt{f^2(t) + f_H^2(t)}}.
\]
In this way the amplitude is peeled off from the signal with only the phase information left, and without the arc-tangent calculation, phase jumps are eliminated, so the phase unwrapping process can be avoided in this way.

3. Inversion with instantaneous phase

If we consider the acoustic wave situation with constant density, then the wave equation has the following form

\[
\frac{1}{v^2(x)} \frac{\partial^2 u(x, t)}{\partial t^2} = -\frac{\partial^2 u(x, t)}{\partial x^2} + s(x, t),
\]

where \( u(x, t) \) represents the wavefield at space location \( x = (x, z) \) and time \( t \), \( v(x) \) means the velocity at location \( x \), and \( s(x, t) \) is the source function. For simplicity, the space coordinate \( x \) will be omitted from the equations in the rest of this paper.

In full waveform inversion, subsurface parameters are retrieved by fitting the observed dataset and the synthetic dataset. The most commonly used least squares objective function is given by

\[
\sigma(v) = \frac{1}{2} \sum_{sr} \int_0^T [d(t) - u(t)]^2 dt,
\]

where \( d(t) \) is the synthetic dataset and \( u(t) \) is the observed dataset. The nonlinear problem is solved in an iterative way using the steepest-descent method in this paper, and the velocity can be updated in the following form,

\[
v_{k+1} = v_k - \alpha_k \Delta J_k,
\]

where \( \Delta J_k \) is the gradient at the \( k \)th iteration with respect to the velocity \( v \) and \( \alpha_k \) is the step length (a positive scalar), which is obtained through line search. The gradient can be obtained as follows

\[
\Delta J = \frac{\partial \sigma}{\partial v} = \sum_{sr} \int_0^T [d(t) - u(t)] \frac{\partial d(t)}{\partial v} dt.
\]

The above expression can be rewritten in vector form as follows

\[
\frac{\partial \sigma}{\partial v} = J^T \eta,
\]

where

\[
J = \frac{\partial d(t)}{\partial v}, \quad \eta = d(t) - u(t).
\]

In the above equation, \( J \) is the Jacobian vector (or the linear Frechet derivative operator), \( \eta \) is the data residual vector. The gradient in equation (10) can be calculated from the zero-lag correlation between the forward source wavefields and the backward residual wavefields, with \( \eta \) acting as the source function for the backward wavefields (Lailly 1983, Tarantola 1984).

In order to adopt the instantaneous phase information in full waveform inversion, we use the exponential instantaneous phase as expressed in equation (5) and define the following objective function

\[
\sigma_{\text{phase}}(v) = \sum_{sr} \int_0^T |e^{\phi_0}_{\text{syn}}(t) - e^{\phi_0}_{\text{obs}}(t)|^2 dt
\]

\[
= \sum_{sr} \int_0^T \left[ \frac{d(t)}{E_{\text{syn}}(t)} - \frac{u(t)}{E_{\text{obs}}(t)} \right]^2 dt
\]

\[
= \sum_{sr} \int_0^T \left[ \frac{d(t)}{E_{\text{syn}}(t)} - \frac{u(t)}{E_{\text{obs}}(t)} \right]^2 dt
\]

where \( d(t) \) is the synthetic data and \( u(t) \) is the observed data. \( E_{\text{syn}}(t) \) and \( E_{\text{obs}}(t) \) represent their corresponding envelopes. \( A_1(t) \) and \( A_2(t) \) are introduced to simplify the expression of the equation, where

\[
\begin{align*}
A_1(t) &= \frac{d(t)}{E_{\text{syn}}(t)} - \frac{u(t)}{E_{\text{obs}}(t)} \\
A_2(t) &= \frac{d_H(t)}{E_{\text{syn}}(t)} - \frac{u_H(t)}{E_{\text{obs}}(t)}
\end{align*}
\]

The gradient for the velocity updating can be obtained by calculating the derivative of the objective function with respect to the velocity,

\[
\Delta J_{\text{phase}} = \frac{\partial \sigma_{\text{phase}}}{\partial v} = \sum_{sr} \int_0^T dt \left[ \frac{A_1(t)}{E_{\text{syn}}(t)} \right] - \frac{A_1^2(t) d_H^2(t) + A_2(t) d(t) d_H(t)}{E_{\text{syn}}^3(t)} \frac{d(t)}{E_{\text{obs}}(t)}
\]

\[
+ H \left[ \frac{A_2(t) d_H^2(t) + A_2(t) d(t) d_H(t)}{E_{\text{syn}}^3(t)} - \frac{A_2(t)}{E_{\text{obs}}(t)} \right] \right] \frac{\partial d(t)}{\partial v}
\]

By introducing the Jacobian vector \( J \) and the following residual vector \( \eta_{\text{phase}} \)

\[
\eta_{\text{phase}} = \frac{A_1(t)}{E_{\text{syn}}(t)} - \frac{A_1(t) d_H(t) + A_2(t) d(t) d_H(t)}{E_{\text{syn}}^3(t)} \frac{d(t)}{E_{\text{obs}}(t)}
\]

\[
+ H \left[ \frac{A_2(t) d_H^2(t) + A_2(t) d(t) d_H(t)}{E_{\text{syn}}^3(t)} - \frac{A_2(t)}{E_{\text{obs}}(t)} \right] \right] \frac{\partial d(t)}{\partial v}
\]

equation (14) can be rewritten in the vector form similar to equation (10),

\[
\frac{\partial \sigma_{\text{phase}}}{\partial v} = J^T \eta_{\text{phase}},
\]

where \( \eta_{\text{phase}} \) is the adjoint source that acts as the effective residual for the instantaneous phase inversion method. This new approach can also be accomplished with the back propagation method. The difference is that \( \eta_{\text{phase}} \) is the new source function for the backward wavefields.
Figure 1. The overthrust model.

Figure 2. The linear initial model.

Figure 3. (a) Inversion results with instantaneous phase only and starting from the linear initial model; (b) conventional full waveform inversion result with the result in (a) as the new starting model.

Figure 4. Conventional FWI result starting directly from the linear initial model.
3.1. Examples

In this part we first try the instantaneous phase inversion method and investigate the properties of this new approach. We use the overthrust model as shown in figure 1. The model has 800 grids in the horizontal and 160 grids in the vertical, with the spatial sampling intervals as 12 m in both directions. There are 50 shots and 200 receivers, which are both fixed and equally spaced on the surface of the model. The shot interval is 192 m and the

Figure 5. Flowchart for the multi-stage damped instantaneous phase inversion approach.

Figure 6. Instantaneous phase inversion with damping. (a) $\alpha$ is 5; (b) $\alpha$ is 3; (c) $\alpha$ is 1.
The receiver interval is 48 m. The source function is the Ricker wavelet with 10 Hz as the dominant frequency. There are 3000 time samples with the sampling interval as 1 ms, so the total recording time is 3 s. We assume that low frequencies below 5 Hz are missing from the seismic data, so frequency components below 5 Hz are removed from the data. The initial model is a 1D linear model shown in figure 2.

We use the instantaneous phase information to conduct the objective function as expressed in equation (12), and perform the inversion using the linear initial model. Figure 3(a) shows the inversion result after ten iterations. We see that some of the model structures can already been observed from the result. We use this inverted result to replace the initial model and perform the conventional FWI.

The final result after 290 iterations for the conventional FWI is shown in figure 3(b). We can see that this inversion result is basically close to the true model, except that there is a small area that is not well recovered in the center of the model. However if we compare this result with the conventional FWI result in figure 4, which is obtained with 300 iterations by directly using the linear initial model, we can see that although the inversion result from the instantaneous phase method is not perfect, it is much improved from the conventional FWI. This tells us that the instantaneous phase leads the inversion in the correct direction.

Looking into the inversion results, we see that the shallow parts of the instantaneous phase inversion are almost correct, which is obviously the opposite situation in the conventional FWI. Because the correct inversion for the deep layers of the model relies strongly on the well recovery of the shallow layers, if we embed in a layer stripping strategy in the inversion, the result will be improved. Based on this analysis, we introduce the damping factor (Choi and Alkhalifah 2013, Chen et al 2015) into the instantaneous phase objective function, and hence propose the damped instantaneous phase inversion method.

4. Inversion using the instantaneous phase with damping

In this section we describe the multi-stage inversion strategy with the damped instantaneous phase. We define the new objective function as follows

$$\sigma^2_{\text{dphase}}(v) = \sum_{sr} \int_0^T \left[ \frac{d(t)}{E_{\text{syn}}(t)} \cdot e^{-\alpha t} - \frac{u(t)}{E_{\text{obs}}(t)} \right]^2 dt$$

$$= \sum_{sr} \int_0^T \left[ \frac{d(t)}{E_{\text{syn}}(t)} \cdot e^{-\alpha t} - \frac{u(t)}{E_{\text{obs}}(t)} \right]^2 dt$$

$$+ \sum_{sr} \int_0^T \left[ \frac{d(t)}{E_{\text{syn}}(t)} \cdot e^{-\alpha t} - \frac{u(t)}{E_{\text{obs}}(t)} \right]^2 dt$$

$$= \sum_{sr} \int_0^T [A_1(t)e^{-\alpha t}]^2 dt + \sum_{sr} \int_0^T [A_2(t)e^{-\alpha t}]^2 dt$$

Figure 7. Conventional FWI result with the result in figure 6(c) as the new starting model.

Figure 8. Comparisons of shot gathers and residual waveforms. (a) Three sets of traces from models inverted using the conventional FWI and the damped phase FWI, and the true velocity model, respectively. (b) Two sets of residual waveforms generated by subtracting the synthetics obtained in the true velocity model from those obtained in models inverted using the conventional FWI and the damped phase FWI, respectively.

In order to obtain better results, we introduce the damping factor (Choi and Alkhalifah 2013, Chen et al 2015) into the instantaneous phase objective function, and hence propose the damped instantaneous phase inversion method.
where $\alpha$ is a nonnegative number and $e^{-\alpha t}$ acts as a damping factor. The new gradient can be obtained by calculating the derivative of the new objective function with respect to the velocity,

$$
\Delta_v \eta_{\text{phase}} = \frac{\partial \sigma_{\text{phase}}}{\partial v} = \sum_s \int_0^T dt \left[ \frac{A_1(t)e^{-2\alpha t}}{E_{\text{syn}}(t)} - A_1(t)d^2(t)e^{-2\alpha t} + A_2(t)d(t)d(t)e^{-2\alpha t} \right]

+ H \left\{ \frac{A_1(t)d(t)d(t)e^{-2\alpha t}}{E_{\text{syn}}(t)} + A_2(t)d(t)\overline{d}(t)e^{-2\alpha t} \right\} \frac{\partial d(t)}{\partial v}. \tag{18}
$$

The above equation can be expressed in a similar vector form,

$$
\frac{\partial \sigma_{\text{phase}}}{\partial v} = J^T \eta_{\text{phase}}, \tag{19}
$$

with the effective residual as

$$
\eta_{\text{phase}} = \frac{A_1(t)e^{-2\alpha t}}{E_{\text{syn}}(t)} - A_1(t)d^2(t)e^{-2\alpha t} + A_2(t)d(t)d(t)e^{-2\alpha t}

+ H \left\{ \frac{A_1(t)d(t)d(t)e^{-2\alpha t}}{E_{\text{syn}}(t)} + A_2(t)d(t)\overline{d}(t)e^{-2\alpha t} \right\} \frac{\partial d(t)}{\partial v}. \tag{20}
$$

The above equations are similar to equations (14)–(16). The difference is that the damping factor is included. We see that the damping factor is directly applied to the exponential phase, not to the original data trace. A larger value of $\alpha$ means heavier damping, which constrains the inversion to the shallow layers. So with the decrease in the value of $\alpha$, the inversion goes deeper and deeper, and will finally provide a good initial model for FWI. This comprises the multi-stage damped instantaneous phase inversion approach. Figure 5 shows the flowchart for this multi-stage strategy.

**4.1. Examples**

In this section, we test the performance of the damped instantaneous phase inversion approach. The velocity model used is still the overthrust model shown in figure 1. The model parameters, the acquisition geometry and source parameters are the same as previous examples.

The starting model is the 1D linear initial model in figure 2. During the phase inversion stage, $\alpha$ for the damping factor is gradually chosen as 5, 3 and 1. There are ten iterations for each damping factor. An inverted result from using one damping factor is set as the new starting model for the inversion using the next damping factor. Figure 6(a) shows the result when using $\alpha$ as 5. Because a large value of $\alpha$ corresponds to heavy damping, which constrains the inversion in the shallow part, we can observe that only the shallow part of the model is updated. Comparing this result with that in figure 3(a), we see that because the inversion is constrained in the shallow part, the updating in this part becomes clearer. Figure 6(b) shows the result when using $\alpha$ as 3, and with the result from damping factor 5 as the initial model. Figure 6(c) shows the result when using $\alpha$ as 1, and with the result from damping factor 3 as the initial model. We can see that when the damping becomes lighter and lighter, the updating for the model becomes deeper and deeper. However, in the mean time, the updating for the shallow part never stops, and makes it clearer and clearer.

![Figure 9. Result for the conventional FWI with damping starting directly from the linear initial model.](image)

![Figure 10. Misfit curves from different inversion approaches.](image)
The result in figure 6(c) is the final result for the instantaneous phase inversion stage, and we will use it as the new starting model for the next stage FWI. We want to clarify that the damping factor is applied only during the instantaneous phase inversion stage, but not included during the conventional inversion stage. Figure 7 shows the final FWI result, which uses the result in figure 6(c) as the new starting model and is obtained with 270 iterations for the

![Figure 11. Comparisons of velocity traces at distances (a) 3 km, (b) 4.92 km.](image)

![Figure 12. Instantaneous phase inversion with damping using the Ricker wavelet dominant at 20 Hz. (a) α is 5; (b) α is 3; (c) α is 1.](image)
conventional FWI. Compared with the conventional inverted result in figure 4, we see that the damped instantaneous phase method provides a much improved inversion result. This can also be observed from the shot gather comparison in figure 8, where figure 8(a) compares the synthetic shot gathers and figure 8(b) compares the waveform residuals. We can see that the damped phase inversion method recovers most of the features generated from the true velocity model, while the conventional FWI causes prominent amplitude and travel time errors.

If we compare the results from the damped phase method in figure 7 with the results from the phase-only method in figure 3, we see that the damping factor plays an important role in the whole process, which makes the combined method provide better results than the phase-only method, with both fewer artifacts in the final result and clearer recovery for the shallow layers.

In order to compare our method with the conventional damping factor method, we apply the damping factor directly to the conventional FWI without using the instantaneous phase information. The inversion still starts from the linear initial model. The values for \( \alpha \) are still chosen as 5, 3 and 1, and then go to 0, which equals the conventional FWI. There are also ten iterations for damping factors 5, 3, and 1, respectively, and 270 iterations for the conventional FWI stage. Figure 9 shows the final inversion result. We can see that although this result is better that the conventional inversion result in figure 4, the overthrust structures are still not well recovered. In addition, compared to the phase-only inversion result in figure 3, the damped method causes more velocity errors.

Figure 10 compares the misfit reductions with iterations for different approaches. The misfit curves are for the entire inversion including all inversion steps. In the above tests, the total numbers of iterations for different approaches are all 300 to make it fair. From the misfit curves we can see that the phase-only method and the damping-only method are better than the conventional inversion method, and the phase-only method has slightly less data misfit than the damping-only method. The damped instantaneous phase inversion method combines their properties together and thus has the least data misfit. Figure 11 compares two sets of selected velocity traces at distances 3 km and 4.92 km. We choose these two sets of traces because they are located at positions where the velocity structures are not well recovered by the phase- and damping-only approaches, respectively. From the comparison we see that in both situations, the damped instantaneous phase method approaches the true velocity model effectively. Our tests show that the combined instantaneous phase information with damping is able to provide us with pretty good initial models, from which we can achieve good full waveform inversion results.

In previous tests, the seismic data are generated using the Ricker wavelet dominant at 10 Hz. To test the robustness of this method to frequency, we conduct inversion where the seismic data are generated with higher frequencies, i.e. using the Ricker wavelet dominant at 20 Hz. The model used is still the overthrust model in figure 1. The model parameters and the acquisition geometry are the same as for previous tests. There are 3000 time samples with the sampling interval as 1 ms, so the total recording time is 3 s. Low frequencies below 5 Hz are removed from the seismic data. The starting model is the 1D linear initial model in figure 2. During the phase inversion stage, \( \alpha \) for the damping factor is still gradually set as 5, 3 and 1. There are ten iterations for each damping factor. Figure 12 shows the inversion results for the phase inversion stage, where 12(a), (b), and (c) are the inverted model with \( \alpha \) set as 5, 3, and 1, respectively. We see that compared to figure 6, the inverted models in figure 12 show slightly finer structures because of the higher frequency components in the seismic data; however, the overall features are similar. Figure 13 shows the final full waveform inversion result, which uses the result in figure 12(c) as the new starting model and is obtained with 270 iterations for the conventional FWI. We see that this result is also promising and close to the true model. This test shows that the damped instantaneous phase method is somehow robust to data frequencies. More work may be required in future to further test the robustness of this method.

5. Conclusions

In this paper we presented a time-domain FWI method using instantaneous phase information to reduce the local minima problem. We used the exponential phase to avoid the complex phase unwrapping process. We constructed the objective functions and showed the corresponding gradient calculations for the new method. We showed performances of both the pure instantaneous phase inversion and instantaneous phase inversion combined with damping factors. Numerical examples showed that the instantaneous phase inversion with
damping is able to provide good initial models and helps to achieve good full waveform inversion results. This method is an effective and efficient approach for initial model construction in time-domain FWI.

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References

Alkhalifah T 2015 Scattering-angle based filtering of the waveform inversion gradients Geophys. J. Int. 200 363–73
Alkhalifah T 2015 Conditioning the full-waveform inversion gradient to welcome anisotropy Geophysics 80 R111–22
Barnes A E 1996 Theory of two-dimensional complex seismic trace analysis Geophysics 61 264–72
Biondi B and Almomin A 2012 Tomographic full waveform inversion (TFWI) by combining full waveform inversion with wave-equation migration velocity analysis 82nd Annual Int. Meeting SEG Expanded Abstracts 1074–80
Biondi B and Almomin A 2013 Tomographic full waveform inversion (TFWI) by extending the velocity model along the time-lag axis 83rd Annual Int. Meeting, SEG Expanded Abstracts 1031–6
Bozda E, Trampert J and Tromp J 2011 Misfit functions for full waveform inversion based on instantaneous phase and envelope measurements Geophys. J. Int. 185 845–70
Choi Y and Alkhalifah T 2013 Frequency-domain waveform inversion using the phase derivative Geophysics. J. Int. 195 1904–16
Choi Y and Alkhalifah T 2015 Unwrapped phase inversion with an exponential damping Geophysics 80 R251–64

Lailly P 1983 The seismic inverse problem as a sequence of before stack migration Conf. on Inverse Scattering, Theory and Application, Society For Industrial and Applied Mathematics Expanded Abstracts pp 206–20
Mora P 1987 Nonlinear two-dimensional elastic inversion of multi-offset seismic data Geophysics 52 1211–28
Pratt R G 1999a Seismic waveform inversion in the frequency domain: I. theory and verification in a physical scale model Geophysics 64 888–901
Pratt R G and Shipp R M 1999b Seismic waveform inversion in the frequency domain: II. fault delineation in sediments using crosshole data Geophysics 64 902–14
Shin C and Cha Y H 2009 Waveform inversion in the Laplace-Fourier domain Geophys. J. Int. 177 1067–79
Tarantola A 1984 Inversion of seismic reflection data in the acoustic approximation Geophysics 49 1259–66