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Elastic Autofocusing

C.A. da Costa* (University of Edinburgh), M. Ravasi (University of Edinburgh), G. Meles (University of Edinburgh) & A. Curtis (University of Edinburgh)

SUMMARY

The method of acoustic autofocusing is a technique to estimate the wave field that originates from a virtual source inside of a medium, with little knowledge of the medium parameters, and with no receivers (nor sources) located inside of the medium. Herein we present the first autofocusing algorithm modified for elastic media. Our algorithm is based on the elastic version of the receiver-side wave field extrapolation step performed in certain wave-equation migration algorithms. The elastic autofocusing algorithm has similar requirements to the acoustic case, namely reflection data measure at an acquisition surface (e.g., the Earth's surface), as well as estimates of the direct arrivals from the subsurface virtual source to the acquisition surface. We use two numerical experiments in varying-density, constant velocity, lossless elastic media to examine the accuracy of the method.
Introduction

A Green’s function between two points is the response that would be recorded by a receiver at one point if an impulsive source was placed at the other. Seismic interferometry is a technique that is commonly used to estimate Green’s functions provided that two sources (receivers) are located at the points, and at least one point is surrounded by a boundary of receivers (sources) (Wapenaar, 2004; Curtis et al., 2006). The lack of such a closed boundary may generate artifacts in the reconstruction (Snieder et al., 2006).

The more recent autofocusing method (Broggini and Snieder, 2012; Wapenaar et al., 2012, 2013) is not impaired by partial boundaries (e.g. by single-sided illumination), nor requires a receiver or source to be placed at one of the points in the medium. Autofocusing requires the standard reflection response to be measured at an acquisition surface, and an estimate of the direct waves from a “virtual source” inside of the medium to that surface (Figure 1). It produces the acoustic Green’s function from the virtual source to those receivers. Hitherto, autofocusing theory has been developed rigorously for 3D acoustic media (Wapenaar et al., 2013). We propose an extension to elastic media, and present two numerical examples: the first uses a layer-cake 1D model, and the second contains a synclinal subsurface interface.

Elastic autofocusing

The iterative scheme proposed by Wapenaar et al. (2013) to estimate the acoustic Green’s function from a virtual source comprises two steps to be performed at each iteration. First, the reflection response is convolved with the down-going field (initialized as the time-reversed direct wave from the virtual source to the acquisition surface) to create an up-going field. In the second step, the up-going field is windowed such that it only contains (non-physical) events before the first arrival, then time-reversed and summed to the initial down-going field to create a new down-going field for the next iteration. Note that the first step performed in the initial iteration is equivalent to injecting the reflection response backward in time through a reference medium that is consistent with the initial direct wave field. As such, it is equivalent to the receiver-side wave field extrapolation step in wave-equation migration algorithms such as RTM (Baysal et al., 1983). This leads us to conjecture that in the elastic case, the same step can be replaced to the receiver-side wave field extrapolation step in wave-equation migration algorithms such as RTM.

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\[
G_{k,M}^{-}(x_R, \omega) = \int \mathbf{R}(x_R, x_S, \omega) \mathbf{G}_{k,M}^{+}(x_S, \omega) \bigg|_{z=0} \, \text{d}x_S \quad k \in \mathbb{N}
\]

In Equation 1, the first superscript in the parentheses corresponds to the type of the measured field \((v \text{ for velocity and } \tau \text{ for stress}),\) the first subscript in parentheses details which of those components is measured; the second superscript indicates the source type \((\phi \text{ for potentials, } f \text{ for external volume force and } h \text{ for external deformation rate source})\) and the second subscript indicates the direction in which the source acts or, in case of a potential source, \(M\) indicates which potential. Outside of the parentheses, the superscript denotes up- or down- going fields by \(-\) and \(+\), respectively, and the subscript \(k\) denotes the \(k\)-th iteration. Thus, the \(\tau_{zz}\) component of the down-going field in \(k\)-th iteration initialized by a \(P\)-wave source is denoted by \(G_{k,zz,P}^{+(\tau, \phi)}\). Throughout, we implicitly assume \(x_{VS}\) as virtual source location unless otherwise indicated.
We extend the second step in the elastic formulation with
\[
G_{k,M}^+(x_R, \omega) = G_{0,M}^+(x_R, \omega) - W(x_R, \omega)G_{k-1,M}^-(x_R, \omega)^*, \quad k \in \mathbb{N}
\] (3)

where \(W(x_R, \omega)\) corresponds to a filter which retains only nonphysical events in \(G_{k,M}^-(x_R, \omega)\). After convergence, the full Green’s function estimated is \(G_{k,M}^+(x_R, t) = G_{k,M}^+(x_R, -t) + G_{k,M}^+(x_R, t)\).

Note that in order to recover the elastic Green’s function, stresses and particle velocities of reflections originating from force and deformation sources are needed. Assuming only the particle velocities from force sources are available, we use vertical component readings when reconstructing the elastic response originating from \(P\)-wave virtual source, and horizontal components for an \(S\)-wave virtual source. Thus to estimate \(G_{(\nu,\phi)}^{(\nu,\phi)}\), the first and second steps are given by
\[
G_{k,P}^-(x_R, \omega) = \int_\mathbb{R} (\nu,\phi) \left( x_R, x_S, \omega \right) G_{k,P}^+(x_S, \omega) \left. \left| \delta x_S \right. \right|_{z=0}, \quad G_{0,P}^+ \equiv G_{(\nu,\phi)}^{(\nu,\phi)}
\]
\[
G_{k,P}^+(x_R, \omega) = G_{0,P}^+(x_R, \omega) - w_P(x_R, \omega)G_{k-1,P}^-(x_R, \omega)^* \quad k \in \mathbb{N}
\] (4)

where the window \(w_P\) mutes anything after the first arriving \(P\)-wave energy, serving as an approximation to \(W\). The Green’s function is then reconstructed by \(\hat{G}_{k,P}(x_R, x_S, \omega) = G_{k,P}^+(x_R, \omega)^* + G_{k,P}^-(x_R, \omega)\) to reconstruct the \(\nu_x\) response of an \(S\)-wave source, \(G_{(\nu,\phi)}^{(\nu,\phi)}\), a similar iteration is employed:
\[
G_{k,S}^-(x_R, \omega) = \int_\mathbb{R} (\nu,\phi) \left( x_R, x_S, \omega \right) G_{k,S}^+(x_S, \omega) \left. \left| \delta x_S \right. \right|_{z=0}, \quad G_{0,S}^+ \equiv G_{(\nu,\phi)}^{(\nu,\phi)}
\]
\[
G_{k,S}^+(x_R, \omega) = G_{0,S}^+(x_R, \omega) - w_S(x_R, \omega)G_{k-1,S}^-(x_R, \omega)^* \quad k \in \mathbb{N}
\] (5)

where now \(w_S\) filters events after the direct \(S\)-wave \(G_{0,S}^+\).

\[\text{Figure 1} \quad \text{Density models. Triangles represent both receiver and source positions on the acquisition surface; black circles represent the virtual source positions.}\]

**Results**

The first numerical experiment uses a varying-density horizontally layered model (Figure 1a). The \(P\)- and \(S\)-wave velocities were 2.9 km/s and 1.2 km/s, respectively. A smooth model was used to compute the first arrival of the \(P\)-wave, whose \(\nu_x\) component was used as the initialization of \(G_{0,P}^+\). The result after ten iterations is shown in Figure 2b, compared with the real Green’s function in Figure 2a, and its first iteration in Figure 2c which is the retrieval using the backpropagation step of RTM. The solid black arrows indicate that all primary events have been recovered in the first iteration. However, internal multiples (red solid arrow) do not appear in Figure 2c. This is consistent with the acoustic autofocus-ing, in which internal multiples are created from the convolution of certain nonphysical events and the reflection response. Nonetheless not all events were reconstructed, as are pointed out by the red dashed arrows in Figure 2a. The top such arrow indicates the transmitted \(P\) to \(S\) conversion: this is not part of the
scattered field and hence should not be recovered unless present in the input. The second arrow indicates a converted reflected wave that was filtered out by $w_P$ because of its long travel time. Some artifacts are also introduced (dashed black arrows), though after a few iterations the strongest are eliminated.

![Graphs showing Green's functions for layer-cake model](image)

**Figure 2** True and recovered Green's functions for the layer-cake model: left column – true result; center – final estimate; right – first iteration; top – P-wave; bottom – S-wave.
The observations for S-wave estimates are similar. The nonconverted primaries (solid black arrows) are present in figures 2e and f but the internal multiples (solid red arrow) is only present in the former. The dashed red arrow in Figure 2d indicates the converted P-S reflection which is not accurately reconstructed at large offsets; though the event is physical energy, since its travel time is smaller than that of the direct S-wave arrival at large offsets, the window \( w_S \) filters it regardless. In the same way above, the strong acausal artifacts are attenuated after a few iterations.

The density model for the second experiment is displayed in Figure 1b. The P- and S-wave velocities were 2.7 km/s and 1.5 km/s, respectively. As before, a smoothed model was used for computing the direct transmission. The reflection response used was filtered to remove the Stoneley waves for which the theory in Wapenaar et al. (2013) does not account, and which otherwise cause strong artifacts. In Figure 3, the solid black arrows display primary events reconstructed by autofocusing and RTM backpropagation, the solid red arrows point to internal multiples only recovered by autofocusing, the dashed red arrows indicate events not recovered and the dashed black arrows indicate artifacts. Figures 3b and e show final estimates of the respective true Green’s function (figures 3a and d), and compare these to reconstructions based only on RTM backpropagation (figures 3c and f). They include some internal multiples and fewer artifacts, consistent with the layer-cake results. The missing events echo the fact that the windows \( w_P \) and \( w_S \) act only as approximations to \( W \). Nonetheless, both for P- and S-waves, elastic autofocusing presents a considerable improvement over standard receiver-side extrapolation, as it reduces artifacts, as well as introduces multiply-scattered events.

Conclusions

We present an iterative scheme for retrieving the elastic Green’s function from a point at which no physical receivers need be placed. It is a modification of the acoustic autofocusing method Wapenaar et al. (2013), that uses the elastic equivalent of the wave field extrapolation step performed at each iteration. The wave field components in the elastic autofocusing scheme requires information that is often unavailable, hence a simplified scheme is proposed, and is applied to estimate different components of Green’s functions through two models. The results demonstrate that even our simplified algorithm recovers the majority of events, and introduces fewer artifacts when compared to RTM backpropagation, even in the more complex model.

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References


