Methods for Evaluating Earthquake Potential and Likelihood in and around California

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INTRODUCTION

From the outset, the vision of the Regional Earthquake Likelihood Models (RELM) project recognized that the best way to come to grips with the full impact and uncertainty in earthquake hazard estimates is to compare a wide range of independent, well-documented, and physically defensible hazard models that produce identically formatted output. Ideally, these models should be rooted in the complete spectrum of geophysical input. Toward this end, I offer testable earthquake potential maps based on geodesy, geology, historical seismicity, and computer simulations of earthquakes.

GEODETIC HAZARD MODELS

Motivation. Until recently, earthquake rate estimation was entirely the domain of geologists and seismologists. With well-defined faults and sufficiently frequent earthquakes, geology, historical seismicity, and paleoseismology can furnish fairly reliable earthquake statistics. More commonly, questionable fault geometries, fault slip rates, fault rupture modes, and scattered seismicity characterize the situation, and earthquake statistics do not reveal themselves readily. For much of the world, historical seismicity and paleoseismology cannot constrain earthquake statistics to the degree necessary for an acceptable rate assessment. Today, information from space geodesy patches some of these voids. Space geodetic monitoring quantifies potential earthquake activity within a network even if that activity occurs on faults that are unknown, too slowly slipping, or too deep to study by conventional geological or seismological techniques. Geodesy’s most valuable contributions in this arena spring from its ability to:

1. provide rates of earthquakes on faults that are undocumented or unobservable by traditional methods;
2. provide independent verification of the rates of deformation in regions where geologists have documented faults; and
3. provide a means to judge the consistency of the contemporary deformation field and the historical earthquake record.

Technical description. Geodetic earthquake potential maps require few inputs. This feature is both the beauty and the value of the approach. The steps in computing the maps include:

1. Compile a GPS velocity map for the California/Nevada region. I employ ~1,500 GPS site velocities taken from Southern California Earthquake Center (SCEC) Crustal Motion Map 3 (Shen et al. 2003) and the Scripps Orbit and Permanent Array Center/Southern California Integrated GPS Network (SOPAC/SCIGN) (SOPAC 2005; figure 1, left).
2. Invert GPS horizontal site velocities into maximum geodetic strain rate

\[ \varepsilon_{\text{max}} = \max \left( |\lambda_1|, |\lambda_2| \right) \] (1)

using a variable-sized smoothing window and taking care to remove rotations in the process (figure 1, right). Ward (1998) gives details for this mapping. \( \varepsilon_{\text{max}} \) is the largest eigenvalue (\( \lambda_1 \) or \( \lambda_2 \) in absolute value) of the horizontal strain-rate tensor. In general, \( \lambda_1 + \lambda_2 \neq 0 \), so \( \varepsilon_{\text{max}} \geq \text{abs}(\lambda_1 - \lambda_2)/2 \), the maximum horizontal shear strain rate. \( \varepsilon_{\text{max}} \) offers a good representation of total deformation rate from horizontal motions. Moreover, as an indicator of total strain rate, \( \varepsilon_{\text{max}} \) behaves more stably than do the eigenvalues or eigenvectors of \( \varepsilon \) individually. The stability of equation (1) enables reliable estimates of strain rate even from sparse geodetic coverage.
3. Translate the maximum geodetic strain rate into geodetic moment rate density

\[ M_{\text{geod}}(r) = 2 \mu H \varepsilon_{\text{max}} \] (2)

using Kostrov’s formula per unit area and an assumed seismogenic thickness, \( H \). Maps of \( H \) may be available; if not, otherwise regionally constant selections suffice. The \( H \) of 11 km used in these calculations gives a total southern California geodetic moment rate budget of \( 1.64 \times 10^{19} \text{N m/yr} \).
4. Finally, moment rate density becomes earthquake rate density (or earthquake potential) under the assumption that equation (2) distributes into earthquake sizes that follow a truncated Gutenberg-Richter (Gutenberg and Richter 1954) distribution of given \( b \)-value and \( M_{\text{max}} \). The mean recurrence interval for magnitude \( M+ \) events is thus (Ward 1994)
Figures 2(A), (B), and (C) map geodetic earthquake potential as $\log_{10}$ rate of events per year $M > 5.5$ or $M > 6.5$ predicted in 100-km by 100-km boxes around each location. To get earthquake rate per km$^2$, multiply by $10^{-4}$. Grid-based earthquake potential maps like these represent a general means to formulate earthquake hazard, and all of the models presented here follow this format. The maps in figure 2 assume $b = 0.9$ and $M_{\text{max}} = 8.1$ or $M_{\text{max}} = 8.5$ for the entire region. If desired, uncertainty in the GPS velocity vectors can be translated formally into uncertainties of strain rate and then into uncertainty in hazard. It is likely, however, that the variation in hazard due to different selections of $H$, $b$-value and $M_{\text{max}}$ will dwarf the formal uncertainties.

Figure 2(D) shows a plot of the total rate of southern California seismicity predicted from the geodetic models (red lines) and compares them with observed seismicity (yellow squares). In geodetic earthquake potential models, geodetic moment rate is conserved rather than earthquake rates, so the selection of $M_{\text{max}}$ plays strongly into earthquake rates. If $M_{\text{max}} = 8.5$, the earthquake rates predicted by geodesy fit the observed rates over the entire magnitude range.

**Required Information.** (1) GPS site positions, horizontal velocities, and errors. (2) A map of seismogenic thickness. (3) A $b$-value and $M_{\text{max}}$.

**Advantages.** (1) Being purely strain-rate based, geodetic models require few subjective constraints. (2) The maps have well-defined error bounds. (3) The approach applies over wide geographical coverage even where little fault information is known. (4) Geodetic strain rates vary slowly with time, so hazards mapped from them might better represent the near future than do geological methods.

**Drawbacks.** (1) Mapping of geodetic strain to seismic strain is not unique. (2) Some strain may be aseismic or non-tectonic, for example due to water or magma injection or withdrawal. (3) Geographic patterns of instantaneous geodetic strain may not reflect patterns of seismic moment release. (4) Rates of earthquakes are fixed indirectly by Gutenberg-Richter relation (equation 3).
GEOLOGICAL HAZARD MODELS

Motivation. One member in RELM’s hazard library has to include a purely geological map. Estimation of earthquake potential from direct interpretation of active faults is still the most widespread approach—even in California where we have alternatives. Geological-only rate estimates differ from geodetic-based ones in that only locations near a specified fault have earthquake potential. Accordingly, geological models require a system of faults. The current system (figure 3) encompasses all of the faults that slip faster than 0.25 mm/yr. This amounts to 5,500 km of faults in southern California and 7,500 km for the entire state.

Technical description. The steps in computing geological earthquake potential include:

Figure 2. Geodetic earthquake potential models for: (A) $M > 5.5$ assuming $b = -0.9$ and $M_{\text{max}} = 8.1$; (B) $M > 6.5$ assuming $b = -0.9$ and $M_{\text{max}} = 8.1$; or (C) 8.5. Because moment is conserved, the choice of $M_{\text{max}}$ affects predicted earthquake rates by about a factor of two (D). $M_{\text{max}} = 8.5$ fits the historical earthquake rates (yellow squares) well.
1. Turn small patches of every fault in the system into moment rate point sources \( M(\mathbf{r}_j) \) using the specified fault slip rate, down-dip extent and a rigidity value. I employ fault elements of about 3-km size along strike. The geological moment rate for the faults of figure 3 within southern California as defined in figure 1 sums to \( 1.18 \times 10^{19} \text{Nm/yr} \).

2. Smooth the moment rate of these point sources into maps of moment rate density by area-averaging with an isotropic Gaussian filter that spans several tens of kilometers. Specifically, if \( \dot{M}(\mathbf{r}_j) \) is the moment rate at fault element points \( \mathbf{r}_j \), the geological moment rate density at grid points \( \mathbf{r}_i \) is

\[
\dot{M}_{\text{geologic}}(\mathbf{r}_i) = \sum_j \dot{M}(\mathbf{r}_j) \exp\left( -\left[ \frac{||\mathbf{r}_i - \mathbf{r}_j||}{\Delta} \right]^2 \right) / \pi \Delta^2
\]

where \( \Delta \) is an averaging distance that may vary with grid point location \( \mathbf{r}_i \). Naturally, smoothing conserves the rates

\[
\int dA \dot{M}_{\text{geologic}}(\mathbf{r}_i) = \sum_j \dot{M}(\mathbf{r}_j).
\]

Smoothing merges the earthquake potential of adjacent faults, accounts for possible fault location uncertainties, and makes a fault-based estimate into a grid-based estimate.
3. Translate the mapped moment-rate density into the rate density of earthquakes of various magnitudes. For well-known faults, distributing seismic moment into earthquake rates could be based on actual rupture mode statistics. For all of the faults here however, I use the Gutenberg-Richter relation (equation 3) again, with given $b$-value and $M_{\text{max}}$ replacing $M_{\text{geod}}$ with $M_{\text{geol}}$. In geological earthquake potential models, as in geodetic models, moment rate is conserved rather than earthquake rates, so the choice of $M_{\text{max}}$ impacts earthquake rates by about a factor of two. The maps in figure 4(A), (B), and (C) show geological earthquake potential as log rate of earthquakes per year per 100 km$^2 \times 100$ km$^2$ for $M > 5.5$ or $M > 6.5$ assuming $b = 0.9$ and $M_{\text{max}} = 8.1$ or $M_{\text{max}} = 8.5$ for the entire region.

**Required Information.** (1) Map of fault traces with a resolution of a kilometer or two. (2) Down dip seismogenic depth. (3) Geological fault slip rates. (4) Rupture mode statistics if known, otherwise a $M_{\text{max}}$ and $b$-value for the Gutenberg-Richter relation.

**Advantages.** (1) Earthquake potential falls near known faults. (2) Gives proper account to well-constrained fault slip rate information and to historically quiet faults. (3) Conforms well with traditional approaches to earthquake hazard. (4) Maps fault-based hazard to grid-based hazard.

**Drawbacks.** (1) Geologists will never be able to specify every fault location and slip rate. (2) Geological rates do not necessarily reproduce historical earthquake rates. (3) Rates of earthquakes are fixed indirectly by Gutenberg-Richter relation. (4) Geological methods provide no obvious means to include time dependence.

### SEISMIC HAZARD MODELS

**Motivation.** Predicting future earthquake hazard based on past seismicity seems like a “no brainer,” but even the basics of the connection are not agreed upon by seismologists. For instance, does the occurrence of Earthquake A decrease or increase the likelihood of another earthquake in the vicinity? Expert Bill says, “Earthquake A shadowed the stress in the area so other earthquakes should be less likely there.” Expert Tom says, “No, Earthquake A evidences accelerating moment release rate in the area so other events will surely follow.” Eventually, physically based earthquake simulations might shed light on these issues (see “Synthetic Earthquake Hazards Models” below), but in the meantime, a conservative view must assume that “Locations where earthquakes have happened in the past are locations where earthquakes will likely happen in the future.” If nothing else, hazard models based on seismicity can be used to test models derived from geology and geodesy.

**Technical description.** In seismic earthquake potential models, earthquake rates $N_r(M_{\text{min}})$ above a minimum magnitude $M_{\text{min}}$ are conserved and all earthquakes are given equal importance.

1. Catalog epicenters down to an $M_{\text{min}}$ where significant geographic coverage can be obtained. $M_{\text{min}}$ however, should not be so low that catalog completeness becomes an issue. I take two catalogs from Kagan (2005) and Kagan *et al.* (2006). One catalog spans 1850–2003 and gives a rate of $N(M_{\text{min}})$ earthquakes of 1.00 × 10$^4$ Nm/yr with a seismic moment rate of 1.05 × 10$^{19}$ Nm/yr. The second catalog spans 1925–2003 and gives a rate of $N(M_{\text{min}})$ earthquakes of 1.15/yr with a seismic moment rate of 0.76 × 10$^{19}$ Nm/yr. The former is not complete to $M = 5.5$, but the better geographic sampling of seismicity in the longer catalog seems to compensate.

2. Catalog earthquake locations are turned into earthquake potential at $M_{\text{min}}$ by area averaging with an isotropic Gaussian filter that spans several, to many, tens of kilometers depending on density of seismicity. Specifically, earthquake rate density at grid point $r$ is

$$\rho(r) = R_{\text{lat}}^{-1} \sum_{j} \frac{\exp(-[(r - r_j)/\Delta]^2)}{\pi\Delta^2}$$

where $R_{\text{lat}}^{-1}$ is the inverse of the catalog duration and the sum is taken over all quakes larger than $M_{\text{min}}$. This step blends the potential of adjacent events and accounts for possible epicenter location uncertainties.

3. Rescale the smoothed rates to reproduce given $N_r(M_{\text{min}})$ for the region of interest—in this case, southern California. I do this rescaling because smoothing equation (5) might push seismicity into adjacent regions and this method specifically conserves $N_r(M_{\text{min}})$.

4. Extrapolate earthquake rates at $M_{\text{max}}$ to higher magnitudes using a Gutenberg-Richter relation with given $b$-value and $M_{\text{max}}$. Figure 5 plots seismic earthquake potential for $M_{\text{max}} = 5.5, 6.5,$ and $7.5$ assuming $M_{\text{min}} = 5.5$, $b = -0.9$, and $M_{\text{max}} = 8.1$ or 8.5.

**Required Information.** (1) A catalog of earthquake locations, dates, and magnitudes. (2) An estimate of catalog completeness level (minimum magnitude) versus catalog length.

**Advantages.** (1) Earthquake potential concentrates near locations that actually had earthquakes. (2) Straightforward estimator. (3) Arguably a time-dependent indicator—as the catalog evolves, so does the hazard. (4) Always reproduces observed seismicity rates at $M_{\text{min}}$ (5) Serves as reality check on other estimators.

**Drawbacks.** (1) Seismicity sample limited. Because direct predictions deteriorate for larger magnitudes, rates of large earthquakes must be extrapolated from rates of small ones. (2) Historically quiet faults give no hazard.
Figure 4. Geological earthquake potential models for: (A) $M > 5.5$ and $6.5$ assuming $b = -0.9$, (B) $M_{\text{max}} = 8.1$ or (C) $M_{\text{max}} = 8.5$. Now, $M_{\text{max}} = 8.1$ fits the historical earthquake rates (D).
Figure 5. Seismic earthquake potential models for $M > 5.5, 6.5,$ and $7.5$ (A, B, and C panels). Assuming $M_{\text{min}} = 5.5$, $b = -0.9$ and $M_{\text{max}} = 8.1$. The figures use Kagan’s earthquake catalog in the interval 1850–2003 (left) and 1925–2003 (right). Because $N_{\text{i}}(M_{\text{ref}})$ is conserved, the choice of $M_{\text{max}}$ has little effect on the rates of all but great earthquakes (D panels).
COMBINED HAZARD MODELS

Motivation. As I have pointed out above, each of the seismic, geodetic, and geological models has its strengths and weaknesses. While it is not RELM’s goal to select a consensus or best model, the strongest single estimate of earthquake rates might be a blend of the three approaches. Because all of the models have identical output, blending is easy. Figure 6(D) shows a plot of combined earthquake potential derived from a straight average of the seismic, geological, and geodetic models.

SYNTHETIC EARTHQUAKE HAZARD MODELS

Motivation. A fundamental problem that makes the prediction of earthquake behaviors more difficult than other types of predictions in the physical sciences is lack of samples. Large earthquakes recur too infrequently for us to glean a reliable understanding of their characteristics. Because of this fundamental undersampling, scientists must turn to alternative approaches to advance the field. One alternative that holds promise is earthquake simulation; that is, physically based computer models of earthquake recurrence.

All seismicity simulations involve a balance between fault driving stress and fault frictional resistance. When the scale tips in favor of fault driving stress, an earthquake occurs to reestablish balance. On faults, physical laws govern both stress accumulation and stress rebalancing by earthquakes. Keeping track of, and occasionally rebalancing, fault stresses in obedience with physical laws is the essence of earthquake simulation.

Earthquake simulation requires five elements: a regional Earth structure; a set of faults to embed in this structure; a means to apply tectonic loading stresses to these faults between earthquakes; a means to transfer stresses from fault to fault during earthquakes; and a means to decide when and where an earthquake should start and how big it should grow to when it occurs. (1) Earthquake potential falls near known faults. (2) Time-consuming to compute. (3) Does not necessarily reproduce historical earthquake rates. (4) Earthquake simulators have many parameters like fault strength and friction laws. (5) Some scientists question the worth of earthquake simulation.

Advantages. (1) Geological model employs the same sets of faults and slip rates, they have identical moment rate density distribution along the faults. The primary difference in the models is how the moment rate partitions into earthquakes. In the geological models, the partitioning was prescribed artificially using a Gutenberg-Richter relation. In computer simulations the magnitude distribution of seismicity falls off automatically from the physics of the fault system. Current models have a sufficiently rich set of fault behaviors to produce a near power law distribution of quakes (figure 8). Comparisons between predicted and observed bulk seismicity speak favorably to the model’s effectiveness.

Drawbacks. (1) Geologists will never be able to specify every fault location, slip rate, and recurrence interval. (2) Partition of moment rate into earthquake rate is determined from physical laws—i.e., b-value and $M_{max}$ not fixed a priori. (3) Potential to supply time-dependent statistics.

Technical description.

1. Run a several-thousand-year simulation of earthquakes on the fault system using known fault slip rates with guidance provided by recurrence and slip-per-event information.
2. Smooth the computed rupture catalog into synthetic earthquake rate density maps taking into consideration the finite extent of ruptures as

$$\hat{\rho}(r) = T_{\text{sim}}^{-1} \sum_j \sum_{k=1}^{N_k} \frac{\exp\left(-\frac{1}{2} \left[\frac{r - r_j}{\Delta r}\right]^2\right)}{\pi k \Delta^2}.$$  

In equation (6) the first sum is overall $j$ ruptures greater than given magnitude, $M$. The second sum is overall $k_j$ fault elements that ruptured in the $j$th event.

In synthetic earthquake potential models, geological moment rate is loosely conserved rather than earthquake rates. Note that because the earthquake simulation model and the geological model employ the same sets of faults and slip rates, they have identical moment rate density distribution along the faults. (1) Geologists will never be able to specify every fault location, slip rate, and recurrence interval. (2) Time-consuming to compute. (3) Does not necessarily reproduce historical earthquake rates. (4) Earthquake simulators have many parameters like fault strength and friction laws. (5) Some scientists question the worth of earthquake simulation.

SHAKING HAZARD CALCULATION

With maps of earthquake potential, together with an attenuation relation, the calculation of time-independent shaking hazard is straightforward. For grid-based rate density estimates (figures 2, 4, 5, 6, and 8) earthquakes may be considered as point sources occurring at the stated rates and magnitudes at the grid coordinates. The mean rate of exceedance of shaking measure $A_{\text{bin}}$ at any location $r$ is found by summing over grid points

$$\bar{N}(A_{\text{bin}}, r) = dA \sum_j \hat{\rho}(r) P_j(A_{\text{bin}}, r).$$  

(7)
Figure 6. Combined earthquake potential model (D) derived from a straight average of seismic (A), geological (B), and geodetic (C) models.
Figure 7. Six frames of animation from a 1,500-year run of the earthquake simulator. See: http://es.ucsc.edu/~ward/simulation9_pga.mov and http://es.ucsc.edu/~ward/allcal.mov. The animation plots all earthquakes $M > 5$. For events $M > 6$, PGA is contoured in bluish tones around the rupture and a magnitude is shown. The graph to the left plots the cumulative number of $M$ quakes (red dots) overlaid on the actual rates (black dots) from 1850 to 2002.
Figure 8. Synthetic earthquake potential models for $M > 5.5$, $6.5$, and $7.5$ (A, B, and C panels). Over long intervals, the simulation should reproduce geological fault slip rates and geological moment rates. Unlike the geological models, synthetic seismicity models do not employ a Gutenberg-Richter relation. Thus, large earthquakes do not fall in proportion to small ones. The simulation fits the historical earthquake rates quite well (D panel).
In equation (7), $\dot{\rho}(r_j)$ is the rate density of events at the $j$th grid point greater than some magnitude as read from the earthquake potential map. $P_j(A_{\text{crit}}, r)$ is the probability that those events generate a shaking measure greater than $A_{\text{crit}}$ at $r$, and $dA$ is the area represented by each grid point. Poissonian hazard maps (figure 9) for any time interval fall immediately from equation (7). Alternatively for rupture-based approaches (figure 7), the mean rate of exceedance of shaking measure $A_{\text{crit}}$ at $r$ can be estimated by summing over ruptures

$$N(A_{\text{crit}}, r) = T_{\text{crit}}^{-1} \sum_j P_j(A_{\text{crit}}, r)$$

where $P_j(A_{\text{crit}}, r)$ is the probability that the $j$th rupture in the catalog generates a shaking measure greater than $A_{\text{crit}}$ at $r$.

Figure 10 plots 30-year likelihood of exceeding Peak Ground Accelerations (PGA) of 10% g and 20% g as predicted directly from the computer simulation in figure 7. I argue that hazard estimates from earthquake simulators are as defensible as those generated by any other technique and will, in time, embody the “best available science” for this purpose.

**Vision for the Future.** I see computer simulations opening avenues to time-dependent hazard estimates or even operational earthquake forecasts. Merely by viewing the earthquake animation links above, it should be clear that with the simulator’s...
ability to produce earthquake catalogs of virtually unlimited duration, any number of conditionally dependent probabilities lie exposed for query. For instance, “Map the probability of all events greater than M 6 that follow within 10 years of an M 7.5 San Andreas fault event at Coachella” or “Under the San Gabriel Mountains, how likely is it that an M 6 quake precedes an M 7 quake by less than six months?”

CONCLUSIONS

Approaches to earthquake potential estimation for areas in and around California are many and varied. I have outlined several well-documented and plausible estimators based on independent data and distinct assumptions. Although it is not the charge of this exercise to compare, evaluate, or defend any of the specific predictions, the means to confront the field has been supplied in that all estimators produce functionally identical output. In RELM’s next phase, members of the suite can be positioned and tested by quantitative means.

REFERENCES

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