THE TELESEISMIC MANIFESTATION OF pP: PROBLEMS AND PARADOXES

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Abstract. Energy radiated upward from underground nuclear explosions has a complex interaction with the free surface that strongly influences the seismic wavefields recorded at teleseismic and regional distances. This interaction, differing from that for earthquakes primarily due to the much higher strains and strain rates involved, is essential to understand for both explosion yield estimation and for discriminating earthquakes from explosions. Reduction of explosion P wave energy from the free surface, which produces the pP phase, involves frequency-dependent, non-linear processes that are intimately linked to surface spallation. Attempts to characterize the teleseismic pP arrival using a variety of time series analysis procedures have yielded seemingly inconsistent results, which can be attributed to a combination of limited bandwidth, neglected frequency dependence, and unresolved trade-offs with source time function, receiver and attenuation effects. Recovery of broadband ground displacement, now viable with modern instrumentation, is resulting in more robust recoveries of broadband ground displacement, now viable with modern instrumentation, is resulting in more robust recoveries.

The initial vertical peak ground velocities within the spall zone can actually be well explained by elastic theory (Burdick et al., 1985), which implies the existence of significant rock strength under compression; however, the subsequent tensile spallation phenomena clearly involve inelastic and nonlinear processes. At distances slightly beyond the spall zone, the free surface reflection involves a lenticular zone of spallation, in which rock failure occurs at depth when the downgoing tensional stress wave resulting from reflection at the free surface (pP) exceeds the sum of the upward compressional stress, the lithostatic stress, and the tensile strength of the rock. Spallation is commonly observed (Springer, 1974), and may involve several discrete surfaces of parting at depth (Eissler et al., 1966).

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Introduction

Seismic waves from underground nuclear explosions provide a reliable means for identifying and estimating the yield of such tests, which is critical for nuclear test treaty monitoring. The precise knowledge of explosion timing and location has also allowed explosion seismic waves to play a major role in imaging the deep interior structure of Earth. The characteristics of underground tests that are most distinctive relative to natural earthquakes are the shallow burial depths of explosions and the (ideally) spherical symmetry of the initial radiation from the source. The proximity to the surface and symmetry of radiation lead to strong interference between the downgoing P wave energy and the compressional wave reflected off of the surface (pP), which arrives within a second after the direct P. The free surface reflection reverses the sense of motion of pP relative to P, producing destructive interference between these signals for frequencies less than about 0.5 Hz and modulation of the higher frequency spectrum, which in turn provide many of the distinguishing characteristics of explosion seismic signals. However, this interference is complicated by nonlinear surface effects on the pP reflection. It has thus been a long standing problem to fully characterize the pP reflection and its complexity.

Understanding the pP surface reflection from nuclear explosions requires consideration of seismic wave interaction with the free surface above the source. The upcoming compressional wave produced by an underground nuclear explosion can produce remarkable accelerations and ground velocities at the free surface. For example, the ~5Mt explosion CANNIKIN produced surface vertical accelerations varying from 17 to 3.2 g at horizontal ranges of 0.3 to 3.4 km from the shotpoint, and corresponding peak ground velocities of from 946 to 233 cm/s [Burdick et al., 1984b]. The initial compressional pulse of acceleration in these close-in recordings (Figure 1a) is followed by a ballistic interval characterized by -1 g acceleration (corresponding to the surface rock breaking and flying into the air). This terminates with high-frequency pulses as the airborne material impacts (i.e. slapping) and slapping. This complex surface interaction involves a lenticular zone of spallation, in which rock seismogram has thus been a long standing problem to fully characterize the pP reflection and its complexity.

In these signals the major arrivals are the P wave turning below the source and the pP reflection from the free surface (the downward spike in the synthetic and observed waveforms in Figure 1b). For these records the pP reflection point is several kilometers horizontally from the shotpoint, and the distributed spall source does not appear to produce a coherent high-frequency arrival, which allows the successful elastic modeling. At regional distances, there is evidence for corresponding Pn arrivals [Burdick et al., 1989], and it does appear that spall contributes to Pn and Lg phases [Taylor and Randall, 1989]. At teleseismic distances, the pP arrival will move directly sample the zone just above the shotpoint, where the downgoing pP wave will encounter the disturbed medium around the explosion cavity, and where spallation is most pronounced and can potentially constructively interfere to give coherent teleseismic arrivals.

The upcoming P energy from a nuclear explosion is partitioned into pP, P, spallation and slapping phases, as well as surface wave excitation and anelastic losses. The upcoming radiation itself may deviate from an isotropic wavefront if there is significant pre-stress in the vicinity of the source, or if an earthquake is triggered by the explosion—effects which are considered elsewhere in this monograph. Understanding the pP phase is required for constraining the source depth, for appraising any bias on the body wave magnitude resulting from constructive or destructive interference, and for assessing how upgoing energy is partitioned in the teleseismic wavefield, which may reveal source region properties [e.g. Gupta and Blandford, 1987]. Systematic differences in pP delay times between source regions may also provide a means for characterizing the source

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medium, which is critical for yield estimation. The question that thus arises is what is the teleseismic manifestation of pP? This article will review the seismological investigations of teleseismic pP for underground explosions to synthesize our understanding of this complex free surface interaction.

Throughout the following discussion of teleseismic investigations of the pP phase, it is useful to keep in mind the linear filter representation of a teleseismic signal spectrum:

\[
U(\omega) = E(\omega) I(\omega) Q(\omega) G(\omega)
\]

(1)

where \(\omega\) is angular frequency, \(U(\omega)\) is the far field P wave displacement spectrum, \(E(\omega)\) is the far-field explosion source spectrum (generally assumed to not vary with take-off angle from the source, although departure from spherically symmetric radiation has sometimes been suggested), \(I(\omega)\) is the instrument response, \(Q(\omega)\) is the attenuation operator, and \(G(\omega)\) is the total Earth propagation response. The Earth response for teleseismic signals is often approximated by:

\[
G(\omega) = S(\omega) R(\omega)/r_0
\]

(2)

where \(S(\omega)\) is the source region transfer function, including the direct P arrival, pP, crustal reverberations near the source, and any secondary arrivals associated with spall; \(R(\omega)\) is the receiver region transfer function including crustal reverberations beneath the receiver; and \(r_0\) is a geometric spreading term. The transfer functions are expected to correspond to time domain spike trains for teleseismic distances, to the extent that crustal structure can be approximated by a set of horizontal layers. Of course, some distortion due to frequency dependent reflection coefficients accompanying non-linear effects or complex scattering structures may actually be required. It is critical to recognize the complete tradeoff that exists between the multiplicative filters. Any attempt to estimate \(S(\omega)\) is subject to limitations in our knowledge of \(E(\omega), R(\omega)\) and \(Q(\omega)\). All existing methodologies for estimating pP behavior, whether frequency domain or time domain, involve assumptions about one or more of these parameters, and much of the inconsistency in published pP characteristics reflects differing assumptions underlying, and sometimes obscured by, the processing.

Long-Period Constraints on pP Characteristics

There have been twenty years of extensive research on teleseismic P waves from underground explosions directed at quantifying the pP arrival, but unlike the situation for earthquakes, relatively little progress has been made by analyzing long-period body waves. Usually, when considering long-period body waves, simple assumptions about propagation effects are adequate to determine gross properties of the source. Ideally, a purely isotropic explosion should produce a teleseismic P wave comprised of only a direct compressional P phase, and a slightly delayed dilatational pP phase. Given the shallow burial depths of all explosions, the time between pP and direct P, \(\tau_0\), should be only 1 s or less, and at teleseismic distances the elastic pP surface reflection coefficient, \(\alpha\), should be close to

Fig. 1. (a) Surface vertical accelerations and velocity recordings from the spall zone for CANNIKIN. (b) Surface vertical velocity synthetics and observations for CANNIKIN from distances beyond the spall zone. [From Burdick et al., 1984b].
In this ideal case, we can assume $S(t) = \delta(t) + \alpha \delta(t - \tau_0)$. The destructive interference of these two arrivals should greatly reduce the amplitude and increase the dominant frequency content of the P waves recorded on long-period instruments (10–15 s pendulum periods) relative to earthquake signals, which tend to have deeper sources and strong additional S arrivals.

P arrivals on long-period WWSSN instruments for large explosions (Figure 2) are in fact very distinctive from P waves from earthquakes with comparable mb (measured near 1 s period). The explosion arrivals are low amplitude, resemble differentiated instrument responses, and are depleted in low-frequency content relative to earthquake signals, which serves as the basis for some discrimination procedures for large events [Molnar, 1971; Wyss et al., 1971; Hasegawa, 1972; Helmberger and Harkrider, 1972; Shumway and Blandford, 1978; Burdick and Helmberger, 1979; Burdick et al., 1984a]. In the frequency domain this is manifested as a peaking of the explosion P wave spectra at periods near 2–3 s for megaton size shots, with a rapid decrease in spectral levels at longer periods.

Peaking of the explosion spectra is readily explained by interference with a strong pP arrival, if we assume that the source time function (the time history of pressure applied on the source elastic radius) for long-period radiation is essentially a step-function. If the pP arrival has an elastic reflection, the teleseismic P spectrum will be modulated by a factor of $(1 + \alpha^2 + 2\alpha \cos \omega \tau_0)^{-1/2}$, where $\omega$ is angular frequency. For $\alpha = -1$, and $\tau_0 = 1.0$ s this modulation factor will have a maximum value at a period of about 2 s. For a step source time function, the far field radiation, with a strong derivative of the source time function convolved with the modulation term) is directly proportional to this modulating factor, and hence, proportional to $\alpha$ at low frequencies. The spectrum is thus expected to drop off at long periods from the peak near 2 s, as is observed.

However, peaking of the teleseismic explosion P wave spectra may also be attributed to overshoot of the source time function, which requires the pressure on the boundary of the elastic zone surrounding the explosion cavity to be more impulsive than step-like [Molnar, 1971; Wyss et al., 1971; Müller, 1973]. Overshoot of the source function has been suggested in many studies of near-field and even teleseismic data, and cannot be dismissed as a possibility. This remains a fundamental trade-off between $S(\omega)$ and $E(\omega)$. Some progress has been made by combining body wave and surface wave constraints on the broadband source spectral content, but difficulties remain in independently determining overshoot of the source function [Lay et al., 1984b]. It is likely that both overshoot and P interference contribute to the depletion of long-period energy in teleseismic P waves. Regardless of the precise mechanisms for the drop off in long-period spectral levels, the net result is that most explosion P wave observations are made using high-frequency instrumentation. Thus, the rest of this review will concentrate on pP results obtained using short-period and broadband seismograms.

High-Frequency pP Analysis Procedures

This review of teleseismic short-period P wave analyses is organized to roughly parallel the history of technique development and application. We will first consider procedures that utilize only the amplitude or power spectra, then time-domain waveform and differential waveform modeling procedures, and finally the variety of deconvolution techniques which are presently giving the most useful results. At their core, all methodologies exploit the spectral interference produced by multiple arrivals in the signal, but they vary widely with respect to assumptions about the source radiation, attenuation, and Earth transfer functions.

Explosions

| SHA | Faultless |
| OXF | Greeley |

Earthquakes

| Borrego Mtn | South America |
| ATL | LPA |

Fig. 2. Comparison of WWSSN long-period P recordings for three NTS events: GREELEY, BOXCAR, and FAULTLESS, and three earthquakes: Borrego Mountain, California (April 9, 1968), Peru (April 13, 1963), and Seattle (April 29, 1965), all having mb = 6.2–6.5. Arrows indicate minute marks. Note the high-frequency character of the P arrivals for the explosions relative to the earthquakes. [From Molnar, 1971].
**Power Spectrum Techniques**

The underlying assumption for most amplitude or power spectrum procedures is that the pP and any other secondary arrivals are delayed, undistorted echos of the direct P arrival with relative arrival times and amplitudes to be determined. For the two arrival approximation described in the previous section, the displacement power spectrum will have the modulation factor \(1 + \alpha^2 + 2\alpha\cos(\omega\tau_0)\), which for negative values of \(\alpha\) (as expected for pP) predicts that spectral nulls will occur at frequencies of \(f_n = n/\tau_0\), \(n = 0,1,2,3,...\), while positive values of \(\alpha\) (as might be associated with slapdown) predict spectral nulls at frequencies \(f_n = [(2n + 1)/2]/\tau_0\). Identification of spectral nulls and their frequency spacing is thus an obvious procedure by which to attempt to characterize the pP arrival.

For some events, such as CANNIKIN (Figure 3) the strong spectral scalloping can be well matched by a three arrival spike train, where the third spike has the same polarity as direct P and can be attributed to a slapdown arrival [Bakun and Johnson, 1973]. This modeling of the amplitude spectrum requires a parameterization of the source time function and attenuation filter. These spectra were fit with pP arrival values of \(\alpha\) ranging from -0.4 to -0.71 and \(\tau\) values from 1.12 to 1.18 s, and slapdown arrival values of \(\alpha\) ranging from 0.67 to 0.85 with delays of 1.92 to 1.94 s. Time domain comparisons based on the spectral fitting are shown on the right, indicating that omission of the phase spectra in the modeling has not led to significant loss of timing information. Note that the primary spectral scallop is well matched, but even the three source model provides a marginal fit to the higher frequency spectra. This, in part, stems from the simplified version of \(S(t)\) used, in which crustal reverberations near the source are ignored, as well as from ignoring receiver complexity.

An extension of the direct power spectral modeling technique that reduces the potential error from incorrect attenuation assumptions and unknown receiver complexity, involves ratioing the spectra from two nearby events recorded at a common station (Figure 4), and stacking the ratios from various stations to enhance the signal to noise ratio [King et al., 1972]. Assuming perfect cancellation of the attenuation, instrument, and receiver effects, the stacked ratios for events i and j give the following:

\[
U_j(\omega)/U_i(\omega) = [E_j(\omega)S_i(\omega)]/[E_i(\omega)S_j(\omega)]
\]

(3)

The ratios can then be modeled assuming spike trains for the two events simultaneously, with differences in the source functions explicitly being inverted for as well. The procedure clearly works best if the source functions and depths are very different, otherwise the information about each parameter is lost in the ratioing procedure. Any common attributes of the source such as overshoot tend to be lost as well.

Numerous applications of these power spectrum techniques have been performed [e.g. Cohen, 1970, 1975; King et al., 1972, 1974; Kulhanek, ...]
1971; Bakun and Johnson, 1973; Flinn et al., 1973; Shumway and Blandford, 1978), with it being quickly recognized that the implied pP delay times and amplitudes were inconsistent with the known overburden velocities and elastic free surface reflection coefficients. Systematically, the pP delay time is longer than expected and the amplitude is smaller. The presence of a clear third arrival for the two large Amchitka explosions (MILROW and CANNIKIN) led to the idea that the missing pP energy was being converted into the even more delayed “slapdown” arrival.

The most recent amplitude spectrum procedure is that of Murphy et al. (1989) and Murphy (1989), which attempts to achieve a separation of $E(\omega)$ and $R(\omega)$ by using a suite of events recorded by a suite of stations. A linear regression model is used to simultaneously determine average station correction factors and station-corrected, network averaged P wave spectra, under the constraint that the station correction factors at each frequency sum to zero. The procedure is to compute the spectral amplitude in a sequence of frequency bands, $\omega_k$, by using narrowband filters for station $j$ from event $i$. Then the regression models minimizes residual error, $e_i(\omega_k)$, in a least squares sense for the instrument corrected spectra:

$$ \log U_i(\omega_k) = \log [E_i(\omega_k)S_i(\omega_k)Q(\omega_k)] + \log R_j(\omega_k) + e_i(\omega_k) $$

(4)

where the station correction factors, $R_j(\omega_k)$, describe the systematic, frequency-dependent departures at station $j$ from the average propagation effects (such as average $Q(\omega_k)$) of the network. Once the receiver effects are separated, corrections for attenuation and modulation effects associated with $S_i(\omega_k)$ are removed to obtain $E_i(\omega_k)$. Examples of this procedure for NTS events are shown in Figure 5, and it is again apparent that low pP amplitudes (A) are obtained, along with large pP delay times relative to the expected values of 0.6-0.9 s. This procedure does not eliminate the problems arising from tradeoffs between the assumed attenuation and source models, but does help to statistically remove the station influence. Assuming that the station terms sum to zero projects any common effects onto the source model, so a large number of observations must be used in this technique. If pP does not have the same time dependence as P, or if other phases arrive within the time interval encompassed by P and pP, both the timing and amplitude estimates for pP can be biased, as is true of all modeling procedures. If there is significant variation in the pP timing between stations the spectral nulls in the network averaged spectra could be smeared out, leading to an underestimate of true pP amplitude, and further smoothing of the nulls results from using multiple narrow band filters to estimate the spectra.

Given the tendency for spectral modeling procedures to result in anomalously low amplitude pP phases which are delayed beyond the elastic predictions, one must question the model assumptions used in the various spectral scalloping procedures. While it is quite reasonable to anticipate that some pP energy has been lost to the spallation process, and the downgoing pP reflection will encounter a very disrupted medium, which may have lower average P velocities than the initial overburden, it is certainly not clear that the resultant pP waveform will any longer resemble a simple echo of the direct P arrival. Remember that this is a fundamental assumption in all of the spectral fitting procedures. While we will return repeatedly to this question, it is instructive to consider Figure 6. Two-dimensional finite difference calculations that attempt to account for nonlinear P reflection processes consistently predict a frequency dependent pP reflection coefficient that is significantly smaller than the elastic value (Bache 1982). Even small departures from elastic theory will obscure spectral nulls that the techniques described above are designed to find. The resulting time domain waveforms for the two calculations in Figure 6 are virtually identical, which suggests the difficulties to be encountered in the next section where waveform modeling procedures are described.

Waveform Modeling Techniques

A significant disadvantage of the power spectrum procedures is that they all require spectral carpentry on the signal, involving windowing, tapering, and transforming the signal. The degree of spectral scalloping is window dependent, thus high resolution of the pP parameters may be difficult to obtain. As a result, many studies have attempted to model the

![Fig. 6. Far-field displacement spectra for a one-dimensional finite difference model with elastic pP reflection processes, and a twodimensional (axisymmetric) model with non-linear pP reflection process.](image)
Fig. 7. Synthetic short-period and long-period explosion signals for a common source model and attenuation function, but with varying pP lag time and relative amplitude. [From Burdick et al., 1984a].

Fig. 8. Stacked envelopes of WWSSN short-period recordings for explosions in several different test sites. The complexity of the main peak for Pahute Mesa events indicates the delayed pP and strong spall arrivals for this test site relative to the Novaya Zemlya sites. Detailed consideration of the individual seismograms can ideally quantify the associated pP parameters, which then reflect the emplacement medium. [Lay and Welc, 1987].

Fig. 9. Comparison of observed short and long-period P waves for CANNIKIN with synthetics for a range of attenuation parameters (\(t^*\)). The synthetics were generated using a near-field source model, \(\alpha = -9\), and \(\tau_0 = 1.15\) s. [From Burdick et al., 1984a].

Fig. 10. Synthetic seismograms and amplitude spectra for two models for event MILROW, which illustrate the trade-offs between parameters. The synthetics on the left and the solid line spectra are for an \(\omega^{-2}\) source model, with \(t^* = 0.7\) s, and a pP reflection coefficient modified from the elastic model by a factor \(F = 0.5 + 0.5 \exp\left(-\omega/2\pi\right)^2\). The synthetics on the right, and the dashed spectra are for an \(\omega^{-3}\) source model, with \(t^* = 1.0\) s, and an elastic pP reflection coefficient. [From Cormier, 1982].
time domain waveforms directly, exploiting the phase information to emphasize the early time window of the signal containing the pP arrival. The synthetics in Figure 7 suggest the potential time domain resolution of pP parameters that could be obtained by comparison with observations, while Figure 8 demonstrates that time domain information does clearly contain gross information about different test site pP properties. Complete waveform modeling comes with the cost of having to specify many parameters including the transfer functions at the source and receiver, the source model, and the attenuation model, as well as requiring a measure of waveform fit that is sensitive to the pP parameters [e.g. Carpenter, 1966; Hasegawa and Whitham, 1969; Hasegawa, 1971; Bache et al., 1975; Bache et al., 1979; Burdick and Helmberger, 1979; Lundquist et al., 1980; Helmberger and Hadley, 1981; Burdick et al., 1984a; Mellman et al., 1985].

These waveform modeling studies differ primarily in the degree to which they utilize independent constraints on one or more of the various filters required to synthesize the time domain waveform. For example, Hasegawa [1971] and Mellman et al. [1985] utilize detailed crustal transfer functions to account for $R(\omega)$, while Helmberger and Hadley [1981] and Burdick et al. [1984a] constrain the source spectrum, $E(\omega)$, by modeling near-field records, and constrain $Q(\omega)$ by matching absolute amplitudes of teleseismic signals. Figure 9 shows synthetic and observed waveforms for event CANNIKIN from Burdick et al. [1984a], where the pP parameters were selected by matching the general shape of the P waveforms for a large set of stations, allowing for variation in attenuation between stations. No explicit accounting for receiver effects was involved in this analysis since a global set of stations was utilized. The P delay times inferred from this modeling are very compatible with spectral analysis results; however, the pP amplitudes are closer to the elastic prediction for this time domain modeling. It is not clear whether this inconsistency is a result of inadequate parameterization of the time domain modeling or biases in the spectral carpentry procedures.

Time domain modeling of the entire waveform is, of course, also subject to many trade-offs in the pP parameterization. Figure 10 shows a calculation by Cormier [1982], in which virtually identical waveforms are produced by trading off frequency dependence of the source model, the attenuation operator, and the pP reflection coefficient. In this noise-free example, only spectral analysis could differentiate between the models. Recognition of these strong trade-offs led to the development of higher resolution time domain techniques, which strive to remove receiver and propagation effects from the problem by determining inter-event transfer functions that exploit the differential waveform information [Filson and Frasier, 1972; Mellman and Kaufman, 1981].

The most extensively developed of the relative waveform procedures is called intercorrelation [Lay et al., 1984a, 1985; Lay, 1985; Burger et al., 1979].

![Fig. 11. Example of intercorrelation of seismograms recorded at WWSSN station ADE for Amchitka events MILROW and LONGSHOT. The observations are each convolved with $E(t)\ast S(t)$ for the other event to equalize the waveforms. $S(t)$ in this case involves just the P and pP arrivals, with the pP parameters being adjusted to optimize the equalized waveform agreement. [From Lay et al., 1984a).](image)

![Fig. 12. Equalized waveforms for the optimal MILROW:CANNIKIN intercorrelation for $S(t)$ with three spikes for each event. In this case the source functions were determined by modeling near-field records. The top trace in each pair is a MILROW observation convolved with the CANNIKIN $E(t)\ast S(t)$, which is shown below ($C_0$), and the lower trace is the CANNIKIN observation at the same station convolved with the MILROW $E(t)\ast S(t)$, which is also shown below ($M_0$). [From Lay et al., 1984a).](image)
In this procedure, seismograms from a given station for two events at the same test site are equalized by parameterizing $S(t)$ for each event as a spike train, and correcting for differences in the source functions arising from yield scaling. Figure 11 illustrates the convolution of each observed trace with $E(t)S(t)$ for the other event. The propagation effects in the mantle and near the receiver, along with the instrument response, are intrinsically accounted for by this procedure. The principal parameters are the spike train sequence, here chosen to involve only the P and pP arrivals, with the pP amplitude and delay time to be determined by making the intercorrelated seismograms as similar as possible. The choice of source function is not as important as for direct forward modeling, because it is the difference in source function between events which influences the equalization. The major limitation of this procedure is again in the specification of a spike train for the source region transfer function, along with the fact that the optimization of spike parameters is only viable with three or fewer spikes in each $S(t)$.

In practice, the intercorrelation procedure is applied to a large set of stations simultaneously for two or more events. Typical results are shown in Figure 12, where three spike versions of $S(t)$ have been used to equalize MILROW and CANNIKIN waveforms. These spike trains are shown after convolution with the respective source functions in the traces labeled $M_0$ and $C_s$. Note that the second spike, pP is comparable in size to the third, upward, spike, which corresponds to the 'slapdown' arrival. In this study [Lay et al., 1984a], the source functions were independently constrained by near-field modeling, to try to minimize the trade-offs with pP parameters. While the preferred pP delay times are in very close agreement with spectral results, especially those obtained by the network.

Fig. 13. Comparison of spectral ratios for pairs of Pahute Mesa events at several NORSAR channels with predicted ratios using pP parameters from intercorrelation [Der et al., 1979b]. Note the poor agreement at frequencies above 1 Hz.

Fig. 14. Short-period and derived broadband recordings for MILROW and CANNIKIN, from four UK array beams. In each case the top trace is the short-period event beam, the second trace is deconvolved ground motion, and the third trace is the ground motion corrected for attenuation assuming $t^* = 0.15$. $A_s$ corresponds to the third arrival which has positive polarity. [From Douglas et al., 1987].
Fig. 15. An example of L1 deconvolution of YKA broadband displacement data for a Shagan River event of August 4, 1979. The deconvolved wavelet used has a $t^* = 0.35$ and a von Seggern–Blandford time function. The resulting spike train is shown in the middle, and a reconstituted waveform is shown at the bottom. [From Mellman et al., 1985].

Fig. 16. Mean impulse trains (solid lines) and standard deviation (dashed lines) obtained by averaging 4 impulse trains deconvolved from LRSM recordings for MILROW (top) and CANNIKIN (bottom). [From Bakun and Johnson, 1973].
Fig. 17. Source functions obtained by combined and individual array multi-channel deconvolutions for events at the Degelen test site [Der et al., 1987a]. These functions should represent \( E(t)S(t) \) alone.

The functions obtained have some complexities, notably broadening, which suggest a frequency dependent arrival.

To further characterize the details of \( S(\omega) \), the broadband seismogram can be deconvolved by an assumed source model, \( E(\omega) \). Extracting the source wavelet can be done by a variety of procedures, one of which is shown in Figure 15, where L1 deconvolution of the source wavelet (along with instrument and attenuation) has been performed by linear programming, with the constraint that the resulting \( S(\omega) \) has a minimum number of spikes [Mellman et al., 1985]. Note the complex transfer function which is obtained, which is a combination of source and receiver effects. This procedure is only as reliable as the choice of \( E(\omega) \) and \( Q(\omega) \), and assumes intrinsic spikiness of the transfer functions. Autocorrelation and matched filtering are other procedures for characterizing the source and receiver spike trains [Cohen, 1970; Flinn et al., 1973, Douglas et al., 1972]. Another procedure for extracting the propagational impulse train is homomorphic deconvolution [Cohen, 1970; Bakun and Johnson, 1973]. Results of applying this procedure to remove instrument, source and attenuation effects for MILROW and CANNIKIN are shown in Figure 16. Note that the \( pP \) and 'slapdown' phases are very similar to the results in Figures 12 and 14.

The latest deconvolutional approach, which involves few assumptions about \( S(\omega) \), and explicitly strives to eliminate \( R(\omega) \) involves multi-channel maximum likelihood iterative deconvolution of a suite of events recorded at an array of stations [Der et al., 1983, 1987a,b, 1989;...
Fig. 20. Comparison of source functions \([E(t)S(t)]\) obtained by the ground motion restitution method of Lyman et al. [1986] (left) and the multi-channel deconvolution method of Der et al. [1987a] for four Pahute Mesa events recorded by the EKA array. The former method places greater weight on recovering the long-period component, and does not factor out frequency dependent receiver effects. Note that the pP arrival is more apparent in the longer period deconvolutions, and at higher frequencies is low amplitude (marked by the question marks).

Fig. 21. Bandpass filtered synthetic seismograms for an explosion signal with P and pP arrivals, with a frequency-dependent pP reflection coefficient. Note how the apparent pP amplitude, indicated by the overshoot, differs depending on the frequency band of the trace. [Der et al., 1989].

The deconvolutions for Pahute Mesa events in Figure 19 indicate little overshoot of the ground motion (small pP arrivals), and indeed Der et al. [1989] assign pP an amplitude of zero for these events. Intercorrelation for these events has suggested nearly elastic pP amplitudes [Lay, 1985], and the spectral stacking results of Murphy [1989] give intermediate values for pP amplitudes, but almost the same delay times as for intercorrelation. Can frequency dependence of pP reconcile these inconsistencies? The situation actually becomes more confused when Figure 20 is considered. This shows determinations of the broadband source functions \([E(t)S(t)]\) for four large Pahute Mesa events determined by the separate deconvolution procedures of Lyman et al. [1986], and Der et al. [1987a]. The results are from the same data at the EKA array, but the deconvolutions of Lyman et al. [1986] exhibit strong overshoots, consistent with significant pP arrivals, whereas the multi-channel...
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deconvolutions suggest no pP arrival at all. The latter results have higher
frequency content as well.

The path from NTS to EKA is known to be in a direction of strong
defocussing [Lynnes and Lay, 1988], and the bandwidth of the signals is
further limited by attenuation. As a consequence of the limited
bandwidth, many of the ground displacements are very similar, despite the
differences in source function and burial depth. It appears that these
common features between events have been assigned to the receivers in
the multi-station deconvolutions, which may or may not be correct. In
addition, the two deconvolution techniques emphasize the longer period
content in very different ways, with the multi-channel procedure placing
greater weight on the higher frequency spectra. Truly reliable separation
of the receiver functions appears to require more dramatic differences
between the depths and source functions in the population of sources than
are commonly observed for a set of explosions at a given test site.

It is also possible that the difference in passband of the deconvolutions
combined with a frequency dependent pP arrival is primarily responsible
for the inconsistent results for NTS events. Figure 21 shows several
bandpass filtered versions of a synthetic ground displacement which has a
pP reflection that depends on frequency [Der et al., 1989]. Note that the
bandwidth influences the strength of the apparent pP arrival. Thus, it
may actually be possible to reconcile all of the pP determinations for the
Pahute Mesa events by recognizing the varying frequency sensitivity of
the techniques, and invoking a physically reasonable frequency dependence
for the pP reflection process.

Even when very broadband source functions are available, care must be
taken in interpreting the pP parameters due to the uncertainty in the
source time function, as well as the interference effect between P and pP.

This is illustrated in Figure 22, which shows errors in pP lag time
resulting from the limited bandwidth of the synthetic pulses. Also note
how very short time delays can lead to a rapid variation in peak to peak
amplitude, which could bias magnitude measurements for small, shallow
events.

![Fig. 22. Demonstration of the biasing effect in pP lag time measurement
for very short lag times with bandlimited data. The actual pP arrival
times are shown by the solid line labeled pP, while the times inferred
from the trough overestimate the true time. The effect on the peak to
peak amplitude of the broadband data is shown at the top as a function of
pP lag time as well. [Der et al., 1989].]

![Fig. 23. A simple, momentum conserving, phenomenological model for
the coupled pP and spallation process. Opening and closing of the spall
source, taken as either a tensional crack or a conical distributed surface,
leads to additional arrivals at teleseismic distances. The spall opening
arrival destructively interferes with pP, leading to anomalously late
inferred pP arrival. In reality this would be a complex frequency
dependent interference. The geometry of the closure process can
concentrate the corresponding downgoing energy, producing the frequently
observed teleseismic "slapdown" phase. [From Burdick et al., 1984b].]
Model For The Effects Of Spall Radiation

**Fig. 24.** Application of the spall model in Figure 23 to the MILROW and CANNIKIN events. The predicted source functions and synthetic short-period seismograms for the model are compared with the results of intercorrelation analysis of the actual data by Lay et al. [1984]. [From Burdick et al., 1984b.]

\[
x \leq R : \text{Ref. Coeff.} = \cos\left(\frac{\pi x}{2R}\right) - 1
\]
\[
x > R : \text{Ref. Coeff.} = -1
\]

**WWSSN SYNTHETICS**

\[
\begin{align*}
R (\text{km}) & \quad \text{Max. Amp} \\
5 & \quad L: 0.193 \times 10^{-2} \quad S: 0.437 \times 10^{-2} \\
4 & \quad L: 0.255 \times 10^{-2} \quad S: 0.575 \times 10^{-2} \\
3 & \quad L: 0.349 \times 10^{-2} \quad S: 0.100 \times 10^{-1} \\
2 & \quad L: 0.471 \times 10^{-2} \quad S: 0.193 \times 10^{-1} \\
1 & \quad L: 0.606 \times 10^{-2} \quad S: 0.328 \times 10^{-1} \\
0 & \quad L: 0.703 \times 10^{-2} \quad S: 0.399 \times 10^{-1}
\end{align*}
\]

**Fig. 25.** Simulation of a frequency dependent pP reflection from a free surface with spatially varying reflection coefficient using the Kirchhoff-Helmholtz approach. The short-period and long-period synthetics for varying radius of the anomalous reflecting zone are shown at the bottom. Note the systematic delay of the peak energy as the weakly reflecting region grows, and the rapid decrease in the amplitude of the short-period reflection. [From Scott and Helmberger, 1983].

**Fig. 26.** Results of a hybrid finite-difference-Kirchhoff method used to model explosions in the complex crustal structure at the Yucca Flat test site, compared with teleseismic observations at station MAT. Complexity of the basic interactions strongly affect the early part of the waveform where pP arrives, as well as the later coda. [From Stead and Helmberger, 1988].
Thus far, we have emphasized the empirical analyses of pP parameters, and found frequent indication of additional arrivals that appear to be from the source region. What is the precise physics by which pP and spall processes are linked, and how does it affect the teleseismic manifestation of pP? This is a poorly understood topic, perhaps because of the many difficulties encountered in quantifying the pP arrival alone, as described above. Nonetheless, it is well recognized on physical grounds that spall and pP must be intimately linked, and a simple three-spike model is inadequate to represent the process [Day et al., 1983]. Burdick et al. [1984b] proposed a phenomenological model for the coupled pP and spall process which can explain some of the anomalous properties of pP, such as its apparent delay and additional arrivals. Their model, constructed to conserve momentum, is shown in Figure 23, where the spall process is initiated by the pP arrival, and produces both downgoing and upgoing waves during both spall opening and closing. The initial downgoing spall arrival will destructively interfere with the pP arrival, resulting in an apparent delay of the surface interaction. The spall source can be viewed as a tensional crack or a distributed source over a conical surface, for computational purposes. Utilizing a model of this type, synthetic seismograms can be constructed which are quite consistent with the results of intercorrelation, deconvolution, and spectral methods for the Amchitka events, as shown in Figure 24. While clearly a simplification of the non-linear spallation process, this approach provides a parameterization of the complete free surface interaction that can be used to synthesize signals at all distance ranges [Burdick et al., 1984b]. Further development of parameterized free surface interaction models is required to enable a more complete interpretation of the source functions that are being obtained by deconvolution procedures.

Current Numerical Modeling Procedures

Along with the many developments in pP waveform analysis, there have been substantial advances in numerical modeling procedures that are revealing the physics of the free surface interaction and its teleseismic manifestation. An informative example is provided by the implementation of the Kirchhoff-Helmholtz wave theory to assess simple models of frequency dependent pP reflection from the free surface [Scott and Helmberger, 1983]. Figure 25 shows the result of a spatially varying pP reflection coefficient, decreasing in amplitude just above the shot point. The three-dimensional wave theory predicts a pP reflection which will be delayed and decreased in amplitude in proportion to the size of the anomalous zone of low reflection, which physically may correspond to the spall zone. This model can qualitatively account for the anomalous delay, decreased amplitude and frequency dependence of the actual pP observations. Accounting for the missing energy requires more complete modeling procedures, such as the two-dimensional non-linear finite-difference calculations of McLaughlin et al. [1988], in which an attempt is made to include much of the physics of the actual spallation and pP reflection process. These axisymmetric calculations tend to actually underpredict the pP arrival, so it is clear that all of the pertinent physics has not yet been included, and possibly the assumption of axisymmetry is inadequate to explain actual pP reflection processes. The common observation of offset of collapse craters from the shotpoint suggests that asymmetry may be an important factor in pP radiation.

Numerical modeling procedures are also useful for addressing heterogeneity in the shallow crustal velocity structure in the vicinity of the shotpoint. Even purely elastic finite difference calculations for complex regions such as the Yucca Flat Test Site at NTS exhibit very complex P coda, initiating with the pP arrival [Stead and Helmberger, 1988; McLaughlin et al., 1986]. In this calculation [Stead and Helmberger, 1988] of teleseismic waveforms, a hybrid two-dimensional finite difference and Kirchhoff-Helmholtz procedure was used to account for the shallow crustal reflections and wave conversions near the source. This level of modeling is critical for appraising the complexity apparent in source function deconvolutions like those in Figure 20. When the source codv is as strong as in Figure 26, methods invoking simple assumptions of 2 or 3 spike source functions will clearly give erroneous results for pP. Another situation in which numerical modeling is necessary is when there is significant surface topography near the test site [a common occurrence]. Figure 27 shows two dimensional finite-difference calculations for a line source [McLaughlin et al., 1987], that illustrate how the upgoing explosive wavefield can be disrupted by topography. Future evaluation of three dimensional effects and broadband data will help to assess whether the pP phase actually has significant azimuthal variations, as suggested by Figure 17.

Discussion and Conclusions

The current level of understanding of the teleseismic pP phase from underground nuclear explosions is far frorn complete. This review has illustrated the diversity of procedures and results which have been obtained over the past twenty years of seismological investigations. A summary...
of the current state of pP parameter estimation is provided by Table 1, which lists published pP characteristics from a variety of methodologies for the three Amchitka explosions. This table demonstrates the relative consistency in pP timing, and the variability of pP amplitude estimates between different techniques. The parameters for a systematic 'spall' arrival, Ps, are also tabulated. While the discrepancy in timing is actually not very large for this test site, there is much greater variation in delay times for source regions with shallow burial depths such as Novaya Zemlya. In such cases, only the high-frequency multi-channel deconvolution procedures obtain realistic pP delays, while other spectral and time domain techniques give much larger delays requiring acute frequency dependence of the pP arrival.

There is general agreement that the actual pP phase is influenced by frequency dependent reflection, with longer period energy having higher reflection coefficients and a tendency toward longer delay times. The estimated delay of a pP phase may be biased late when the phase is assumed to be a reflected impulse. All estimates of pP parameters are influenced by the bandwidth of the technique being used as well as the assumptions about the frequency content. As a general rule, many of the contradictory pP parameters in the literature could be reconciled by specifying the frequency band most emphasized in the processing. The greatest stability appears to accompany the largest bandwidth procedures.

Given the direct trade-offs between source and propagation effects for teleseismic signals, especially when frequency dependence is involved, it appears that the most reasonable approach to analyzing the pP phase is simple broadband ground motion restitution. This involves removing the instrument response effects to extend the bandwidth of the signal. The resulting signals can then be interpreted for a variety of assumed attenuation and source models, and stacked to suppress receiver effects. The latter processing should always acknowledge the direct trade-offs that exist, and should fully explore assumptions about the source model and attenuation model before placing any weight on the resulting interpretations of the pP parameters, depth, coupling, etc. Current spectral factoring techniques that separate source and receiver transfer functions tend to emphasize high-frequency content, and appear to be unstable with respect to partitioning of common spectral characteristics. A suite of sources spanning a wide range of burial depths and yield is required to stabilize these procedures.

There is a need for more numerical analysis of the pP-spall process, as interpretation of the broadband ground motions requires a parametric description of this energy partitioning. In addition, continued development of numerical models to elucidate the complexity of pP and subsequent coda arising from complex near-source structure and surface topography is very important. The numerical studies performed to date suggest that even the elastic processes accompanying the pP reflection are very complex, and possibly azimuthally variable.

### Table 1. Comparison of pP and Ps Parameters Determined by Different Procedures for the Amchitka Tests

<table>
<thead>
<tr>
<th>Event/Method</th>
<th>pP-P (s)</th>
<th>pP/P</th>
<th>Ps-P (s)</th>
<th>Ps/P</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LONGSHOT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Near Field</td>
<td>0.4–0.41</td>
<td>0.45</td>
<td>1.76</td>
<td></td>
<td>Overburden/Elastic Prediction Springer [1974]</td>
</tr>
<tr>
<td>Spectral Methods</td>
<td>0.55</td>
<td>0.55</td>
<td>0.87</td>
<td>0.3</td>
<td>King et al. [1972]</td>
</tr>
<tr>
<td></td>
<td>0.43</td>
<td>-0.6</td>
<td></td>
<td></td>
<td>Marshall [1972]</td>
</tr>
<tr>
<td></td>
<td>0.49</td>
<td>-0.3</td>
<td></td>
<td></td>
<td>King et al. [1974]</td>
</tr>
<tr>
<td>Intercorrelation</td>
<td>0.55</td>
<td>-0.8–1.0</td>
<td>0.8–0.9</td>
<td>0.3</td>
<td>Lay et al. [1984a]</td>
</tr>
<tr>
<td>Deconvolutions</td>
<td>0.4–0.5</td>
<td>-0.8–1.0</td>
<td></td>
<td></td>
<td>Bakun and Johnson [1973]</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>-0.8–1.0</td>
<td></td>
<td></td>
<td>Douglas et al. [1987]</td>
</tr>
<tr>
<td><strong>MILROW</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Near Field</td>
<td>0.64–0.71</td>
<td>0.74</td>
<td>2.49</td>
<td></td>
<td>Overburden/Elastic Prediction Springer [1974]</td>
</tr>
<tr>
<td>Spectral Methods</td>
<td>0.55</td>
<td>0.55</td>
<td>1.35</td>
<td>0.4</td>
<td>King et al. [1972]</td>
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<tr>
<td></td>
<td>0.67</td>
<td>-0.5</td>
<td></td>
<td></td>
<td>Marshall [1972]</td>
</tr>
<tr>
<td></td>
<td>0.81</td>
<td>-0.5</td>
<td></td>
<td></td>
<td>King et al. [1974]</td>
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<tr>
<td></td>
<td>0.83</td>
<td>-0.54</td>
<td></td>
<td></td>
<td>Murphy &amp; O'Donnell [1988]</td>
</tr>
<tr>
<td>Intercorrelation</td>
<td>0.8–0.85</td>
<td>-0.7–0.9</td>
<td>1.35</td>
<td>0.5</td>
<td>Lay et al. [1984a]</td>
</tr>
<tr>
<td>Forward Modeling</td>
<td>0.8</td>
<td>-0.9</td>
<td></td>
<td></td>
<td>Burdick et al. [1984a]</td>
</tr>
<tr>
<td>Deconvolutions</td>
<td>0.75–0.8</td>
<td>-0.6</td>
<td>1.3–1.4</td>
<td>0.7</td>
<td>Bakun and Johnson [1973]</td>
</tr>
<tr>
<td></td>
<td>0.7–1.0</td>
<td>-0.5</td>
<td></td>
<td></td>
<td>Douglas et al. [1988]</td>
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<tr>
<td><strong>CANNIKIN</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Near Field</td>
<td>0.92–1.06</td>
<td>1.0</td>
<td></td>
<td></td>
<td>Overburden/Elastic Prediction Springer [1974]</td>
</tr>
<tr>
<td>Spectral Methods</td>
<td>0.48</td>
<td>0.4</td>
<td>1.85</td>
<td>0.4</td>
<td>King et al. [1974]</td>
</tr>
<tr>
<td></td>
<td>1.12</td>
<td>-0.4</td>
<td></td>
<td></td>
<td>Lay et al. [1984a]</td>
</tr>
<tr>
<td>Intercorrelation</td>
<td>1.1–1.2</td>
<td>-0.6–1.0</td>
<td>1.95</td>
<td>0.6</td>
<td>Burdick et al. [1984a]</td>
</tr>
<tr>
<td>Forward Modeling</td>
<td>1.15</td>
<td>-0.9</td>
<td></td>
<td></td>
<td>Bakun and Johnson [1973]</td>
</tr>
<tr>
<td>Deconvolutions</td>
<td>1.1–1.2</td>
<td>-0.5</td>
<td>1.9–2.0</td>
<td>0.5</td>
<td>Bakun and Johnson [1973]</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
<td>Burdick and Helmberger [1979]</td>
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<tr>
<td></td>
<td>1.0</td>
<td>-0.8–0.9</td>
<td></td>
<td></td>
<td>Douglas et al. [1987]</td>
</tr>
</tbody>
</table>
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