AVERAGE Q AND YIELD ESTIMATES FROM THE PAHUTE MESA TEST SITE

BY R. W. BURGER, T. LAY, AND L. J. BURDICK

ABSTRACT

Attenuation models, with and without frequency dependence, have been developed through analysis of time-domain amplitude measurements and teleseismic spectral shape data from Pahute Mesa nuclear explosions. The time-domain analysis is based on a near-field to far-field amplitude comparison. The near-field amplitude information is incorporated in two parameterized explosion source models (Mueller-Murphy and Helmberger-Hadley) based on analyses of near-field data. The teleseismic amplitude observations are from a large data set of WWSSN short-period analog recordings. For the narrow-band time-domain data, the various source and attenuation models are indistinguishable. We utilize the spectral shape data in the 0.5- to 4-Hz band as a constraint on the source-attenuation models at higher frequencies, concluding that either source model, when convolved with the appropriate frequency-dependent Q model, can be consistent with both the near-field and far-field time-domain amplitudes and the spectral shape data. Given the trade-off between source and attenuation models and the similarity of the different source models in the 0.5- to 4-Hz band, it is difficult to prefer clearly one source model over the other. The Mueller-Murphy model is more consistent with surface wave amplitude measurements because of larger predicted long-period energy levels. Whether or not frequency dependence is included in the attenuation model, the value of $t^*$ near 1 Hz is about 1.0 sec (assuming the Mueller-Murphy source model) or 0.8 sec (assuming the Helmberger-Hadley source model). This 0.2 sec difference results from greater 1-Hz energy levels for the Mueller-Murphy source model. Adopting an average attenuation model, predicted amplitudes and yields are shown to be within the uncertainty of the data for all the events analyzed.

INTRODUCTION

Understanding the nature and effect of the earth's anelasticity on seismic body waves is an important problem in seismology. Detailed analysis of the earthquake source or earth structure requires a characterization of an attenuation operator. Furthermore, reliable estimates of yield based on network-averaged, short-period $m_b$ require detailed knowledge of P-wave attenuation in the earth. Because of lateral variations of the earth's anelasticity, it is necessary to estimate the deviation of $t^*$ from the global average in the short-period body wave band at a given test site to predict the $m_b$-yield relation baseline for that site. Bache et al. (1985, 1986) and Der et al. (1985) have used spectral shapes and decay rates of teleseismic P waves from explosions to estimate the level and frequency dependence of Q. However, spectral methods for estimating Q have several potential sources of error (Cormier, 1982). Most important is that spectral decay methods require a priori knowledge of the source spectrum. Errors in the assumed source spectral fall-off rate translate into a bias in the attenuation measurement. This presents a problem in the spectral decay studies of explosion signals given that explosion source models with different spectral fall-off characteristics have been proposed at various times (e.g., Haskell, 1967; Mueller and Murphy, 1971; von Seegern and Blandford, 1972; Helmberger and Hadley, 1981).
A common misconception is that these source models can be effectively characterized by their high-frequency spectral fall-off rates \( (f^{-2}, f^{-3}, \text{or} f^{-4}) \). However, we show in this work that this is not necessarily the case. For the wide range of yields investigated in this study (155 to 1300 kt), the Helmberger and Hadley (1981) source, which has an asymptotic spectral decay rate of \( f^{-3} \), has spectral properties in the band of interest (0.5 to 4 Hz) similar to the Mueller and Murphy (1971) source, which has an \( f^{-2} \) asymptotic decay. We also show that these source models, when convolved with an appropriate attenuation model, are indistinguishable in WWSSN short-period data. It is likely that none of these source models is completely accurate. The spectrum may initially decay at one rate and then, at a higher frequency, begin to decay at a faster rate.

Time-domain modeling of explosion signals provides an alternate method to the spectral shape procedures for estimating attenuation. However, separate analysis of time-domain and spectral decay information may produce incompatible results, given the different intrinsic sensitivity of each procedure (Cormier, 1982). In this investigation, we have measured the attenuation of \( P \) waves from explosions at the Pahute Mesa test site utilizing time-domain amplitude data in the 0.5- to 2-Hz band (WWSSN short-period) and spectral decay information as a constraint on the source-attenuation models at frequencies up to 4 Hz.

The optimal procedures to measure attenuation are those in which source canceling comparisons of different phases can be performed. Those include experiments using multiple \( ScS \) (Jordan and Sipkin, 1977; Sipkin and Jordan, 1979, 1980; among others), \( ScS/ScP \) (Burdick, 1985), \( sP/sS \) (Burdick, 1978), and \( P/PP \) (Shore, 1983). Phase pairs for which source canceling studies might be performed are not commonly observed from explosions. However, nuclear sources have been well recorded in the near-field and near-regional field. This allows an alternate procedure of determining the initial source excitation from the near-field data and estimating \( Q \) by matching the observed teleseismic amplitudes. This type of investigation has been made previously by Frasier and Filson (1972), Helmberger and Hadley (1981), and Burdick \emph{et al.} (1984).

Near-field records from explosions at Pahute Mesa have been studied extensively. Yield-scaled source models have been developed from this data using two different approaches. Mueller and Murphy (1971) compared yield scaling exponents as a function of frequency for events of various sizes recorded at a common station. Murphy (1977) later refined their representation into what is referred to in this paper as the Mueller-Murphy source model. Helmberger and Hadley (1981) developed their \( f^{-3} \) model by forward modeling of a small set of observations, but they did not present yield-scaling relations. Pahute Mesa yield-scaling relations for the latter source representation were later developed by Barker \emph{et al.} (1985) along with source models for a wider range of events. These two different types of studies account for the complex process of near-field wave propagation in very different ways and assume different parametric forms for the source-time history. We use both the Mueller-Murphy (1971) and Helmberger-Hadley (1981) source models in our modeling to illustrate the trade-offs between source and attenuation models in the 0.5- to 4-Hz band, and to appraise the various claims in the literature that one model is superior to the other.

It is widely recognized that the effective value of \( Q \) for the mantle varies with frequency within the body wave band (0.2 to 20 sec). However, it is common practice to assume that, within the frequency band of any one type of narrow-band data, \( Q \) is independent of frequency. The variation of \( Q \) with frequency is inferred by
comparing the results of several different types of studies (e.g., Burdick, 1985). In this work, we have considered both frequency-independent (Futterman, 1962) and frequency-dependent models for attenuation filters. The frequency-dependent attenuation model that we use is the standard linear solid model described by Minster (1978a, b). Our goal was to find whether either source model or either attenuation model did a superior job of fitting the combined teleseismic spectral and amplitude data sets from Pahute Mesa. Our conclusion is that any of the four source-attenuation model pairs can satisfy short-period WWSSN observations. But, when using spectral shape data in the 0.5- to 4-Hz band as an additional constraint, we find it necessary to include a frequency-dependent attenuation model. Furthermore, for the Minster Q models, both source models are consistent with the amplitude and spectral shape data in this band.

**MODELING APPROACH**

The basic approach taken in this investigation is to attempt to match simultaneously observed teleseismic amplitudes and magnitudes by computing synthetic seismograms using source models which fit near-field data. The parameters that we adjust to fit the data are the free parameters in two standard attenuation models. The reason for matching both the period-corrected amplitude (magnitude) and the uncorrected amplitude is to obtain a reasonable measure of how well the source and attenuation models predict the average period of the observed P waves. We also insure that the synthetic waveforms agree with the data.

**Observations.** The observations that we attempt to match are of two types. The first type is the teleseismic short-period P-wave amplitude and magnitude observations as measured from WWSSN film chips. A large data base of such measurements from 25 Pahute events was processed in a thorough and uniform fashion by Lay (1985). He measured over 1200 amplitude values and obtained from them average log-amplitudes and average magnitudes. The amplitude and magnitude values to be modeled here are given in Table 1. The amplitude that we model is that of the first upswing to first downswing of the explosion waveform. Although the $pP$ arrival does not influence the amplitude measurement for these events (Lay, 1985), it may influence the period (of the first cycle) measurement. Thus, the synthetic magnitudes may be somewhat affected by incorrect $pP$ delay time estimates. The effective band of WWSSN short-period data is 0.5 to 2 Hz. Therefore, the time-domain amplitude data discussed in this paper (and any possible spectral data from these instruments) are sensitive only to the spectral level in this band.

**Table 1**

<table>
<thead>
<tr>
<th>Pahute Mesa Events Studied*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Event</strong></td>
</tr>
<tr>
<td>ALMENDRO</td>
</tr>
<tr>
<td>BENHAM</td>
</tr>
<tr>
<td>BOXCAR</td>
</tr>
<tr>
<td>GREELEY</td>
</tr>
<tr>
<td>HALFBREAK</td>
</tr>
<tr>
<td>INLET</td>
</tr>
<tr>
<td>MAST</td>
</tr>
<tr>
<td>SCOTCH</td>
</tr>
</tbody>
</table>

* The amplitudes and magnitudes are from Lay (1985). The Helmberger-Hadley source parameters $K$ and $B$ are from Barker et al. (1985).
The second type of observations that we wish to match are spectral shape observations in the 0.5- to 4-Hz band. Unfortunately, body wave spectra from hand-digitized, analog records are not reliable at high frequencies, so we cannot use the spectra of the $P$ waves studied by Lay (1985). Der et al. (1985) analyzed the spectral shapes of Pahute Mesa events using several different broadband recording systems. They assumed the von Seggern and Blandford (1972) source function and determined a frequency-independent value of $t^*$ in the 0.5- to 4-Hz band of 0.4 to 0.5 sec for Pahute Mesa. We have computed a synthetic spectrum using this information, and we treat it here as an observation. We will also consider the results from the spectral studies of Bache et al. (1986). They obtained Pahute Mesa spectra and magnitude data from the UKAEA and NORSAR arrays and modeled them using the Mueller and Murphy (1971) source model. They solved for frequency-independent and frequency-dependent values of $Q$, concluding that the data require a frequency-dependent attenuation operator. The spectral shape data from these two studies are quite consistent with each other, but there is a general difference in the interpretation (Bache et al., 1986). We will constrain the models developed here from the time-domain amplitude data to match the spectral shape data as well.

**Attenuation models.** Two of the most commonly used attenuation models in body wave seismology are the frequency-independent Futterman (1962) and the frequency-dependent standard linear solid (Minster, 1978a, b) models. The Futterman model is parameterized by a single value, $t^*$, obtained by integrating the slowness divided by $Q$ over the raypath. The standard linear solid attenuation model is parameterized by three values. These are the minimum value of $Q$, the high-frequency characteristic relaxation time, $\tau_M$, and the low-frequency relaxation time, $\tau_m$. $\tau_M$ controls the increase of $Q$ at short periods and $\tau_m$ controls the increase at long periods. Since we will be modeling short-period observations, we simply fix $\tau_M$ at 1000 sec for all calculations. The attenuation model is then reparameterized in terms of a long-period value of $t^*$, designated by $t_0^*$, and $\tau_m$.

A reasonable range for $t_0^*$ can be established from normal mode and long-period multiple $ScS$ data. Anderson and Given (1982) give values of $t_0^*$ based on normal mode data that range from 0.9 to 1.2 sec between distances of 30° and 80° at a period of 100 sec. Sipkin and Jordan (1979) found that good quality multiple $ScS$ data give slightly lower values of $Q$ than the mode data. Burdick (1985) found that their observed average value for whole mantle $Q$ translates to a $t_0^*$ of about 1.3 sec. Because we have no independent evidence of the local value of $t_0^*$, we consider $t_0^*$ values ranging from 0.9 to 1.3 sec.

**Source models.** The explosion source models which we will use in our study are the Mueller-Murphy tuff source and the Helmberger-Hadley source representations. Each of the source models is subject to uncertainty due to the uncertainty of the source structure and the complexity of near-field propagation. Murphy (1977) provides the yield-scaling relations appropriate for Pahute Mesa. The form of these relations is based on theoretical and observational considerations. They provide an analytic expression for the source as a function of yield, burial depth, and material properties. We assume that the material properties are relatively constant at Pahute Mesa, so that as long as yield is known, we can derive the source-time history and its spectrum. Five events in the Pahute Mesa data set had announced yields: SCOTCH; HALFBEAK; GREELEY; BOXCAR; and BENHAM (Table 1). In the following, we use the seismic observations from these five events to obtain average Futterman and Minster $Q$ operators.

The Helmberger-Hadley source model is based on forward modeling studies of
near-field data from Pahute Mesa events (Hartzell et al., 1983; Barker et al., 1985). The band of their data is 1 to 5 Hz, so that their source models are appropriate to use in our investigation. Yield-scaling laws were developed, but are empirical in nature and based on few data points. It is thus most reasonable to use the source models developed for particular events whenever possible and we shall do so in the following. Six events were forward-modeled with good success in these studies: BOXCAR; SCOTCH; ALMENDRO; HALFBEAK; INLET; and MAST. The Helmberger-Hadley source parameters $K$ (corner frequency) and $\Psi_\infty$ (DC level) are given in Table 1. Again, we use these six events to establish the attenuation operators.

Three events with announced yields have been forward modeled; BOXCAR; SCOTCH; and HALFBEAK. We concentrate primarily on them in the following. A comparison of the Helmberger-Hadley (H-H) and Mueller-Murphy (M-M) source models for these three events is shown in Figure 1. Even though the two formalisms are mathematically different, they have very similar corner frequencies and spectral slopes near 1 Hz. The similarity of these features extends over a wide range of yield, from 155 kt (SCOTCH) to 1300 kt (BOXCAR). This similarity suggests that Mueller and Murphy (1971) and Barker et al. (1985) were resolving the same yield-scaling information but simply modeling it with different formalisms. It appears that the best agreement between the two source models would be obtained if the Helmberger-Hadley model were given less overshoot. The Mueller-Murphy source models have larger spectral displacement amplitudes than the Helmberger-Hadley source models for all three events. As we show in the following, this means that utilization of the Mueller-Murphy source requires slightly higher values of $t^*$. It is also important to note that, even though the Helmberger-Hadley source ultimately has a higher spectral fall-off rate, it does not become apparent until frequencies exceed 4 Hz. Thus, this difference between the sources is not significant for modeling data limited to frequencies lower than this.

The differences between the two source formalisms are further illustrated in Figure 2. The frequency dependence of the spectral slope is displayed at the top. The two source models have almost identical slopes between 0.5 and 3 Hz. The spectral slope of the Helmberger-Hadley model does not drop below that of the

![Figure 1. Predicted far-field source displacement spectra from BOXCAR, HALFBEAK, and SCOTCH. The Mueller-Murphy (1971) source models are based on the announced yields. The Helmberger-Hadley (1981) source models are based on the near-field modeling results of Barker et al. (1985).](image-url)
Mueller-Murphy source until frequencies of 2 to 4 Hz. The slope of the Helmberger-Hadley spectrum does not reach $-3$ until 10 Hz. This illustrates the danger of attempting to correct spectra for source effects by simply multiplying by $f^n$. The spectral ratios of the source functions are shown at the bottom of Figure 2. They illustrate again that the Mueller-Murphy model consistently predicts slightly higher amplitudes at all frequencies. The spectral ratios also demonstrate that the ratio of the source models varies with frequency by no more than a factor of 3, even up to 10 Hz. Furthermore, the ratio of the source models for HALFBEAK and BOXCAR varies by less than a factor of 2 between 0.5 and 4 Hz, thus making these two source models difficult to distinguish from one another using observed data in this band.

In the spectral shape and decay analysis, absolute amplitude information is not utilized, so the baseline difference between the two source models has no effect. It is apparent in Figure 2 that given spectral data in the 0.5- to 4-Hz band, the difference between correcting the observed spectrum for the Mueller-Murphy source model versus the Helmberger-Hadley model will depend somewhat on the yield of the event, being negligible for large events and increasing slightly for small events. Also, for small events, the Helmberger-Hadley source correction will actually lead to lower $Q$ values, just the opposite effect of simply correcting for $f^{-3}$ instead of $f^{-2}$. 

---

**Fig. 2.** (Top) Spectral slope for the displacement spectra shown in Figure 1. (Bottom) The spectral ratio of the Helmberger-Hadley to Mueller-Murphy source spectra in Figure 1.
RESULTS

Time-domain amplitudes. We begin this modeling study by considering the amplitude and magnitude data of Lay (1985). Since we do not have accurate spectra for the WWSSN seismograms, we model them in the time domain. We compute synthetic seismograms at a distance of 50° using the near-field source models convolved with the appropriate attenuation model and the WWSSN short-period instrument response. The synthetic seismograms include only the direct $P$ and $pP$ arrivals. The $pP$ delay times and amplitudes were measured using a complete waveform analysis called intercorrelation (Lay, 1985). Figure 3 illustrates the procedure for determining the attenuation models from the amplitude data for the event HALFBEAK. In the case shown, we found values for $t^*$ in the Futterman operator (top) and $\tau_m$ in the Minster operator that match the observed average amplitude. We consider three values of $t_0^*$ in the Minster model (1.0, 1.15, and 1.3 sec) as illustrated in the bottom three rows of the figure.

The synthetic waveforms for the Helmberger-Hadley source are different enough to allow us to select a preferred value of $t_0^*$. For comparison, shown at the right of

![FIG. 3. Synthetic teleseismic WWSSN short-period seismograms for HALFBEAK. The source functions are shown in Figure 1. The attenuation models are those that produce synthetic amplitudes which match the observed amplitudes. The synthetic seismograms include a $pP$ arrival (given by Lay, 1985). Also given for each synthetic seismogram is the synthetic magnitude. Shown on the right are representative observed WWSSN short-period teleseismic waveforms for HALFBEAK.](image-url)
Figure 3 are representative observed waveforms for the event HALFBEAK. For the Helmberger-Hadley source with the Minster Q model, the first synthetic waveform \((t_0^* = 1 \text{ sec})\) resembles the observations most closely. The period of the first upswing is too short and its relative amplitude too high in the lower two synthetic waveforms. Also, the shoulder in the second upswing, which is associated with the \(pP\) arrival, is not clear in the data. There are trade-offs here with the \(pP\) parameters, but we are assuming that the values measured by Lay (1985) are correct. For the Mueller-Murphy source, it is clear that there is a strong trade-off between the two parameters in the Minster Q operator. There are no substantial differences between the predicted waveforms. Fortunately, a preferred value of \(t_0^*\) can be established through consideration of the magnitude combined with the amplitude information.

The analysis of the magnitude data of Lay (1985) is performed in the same way as that of the amplitude values. We determine values for the parameters in the attenuation models that match \(m_b\), given by

\[
m_b = \log(A/T) + P(\Delta)
\]  

(1)

where \(A\) is amplitude in millimicrons, \(T\) is period, and \(P(\Delta)\) is the distance correction given by Veith and Clawson (1972).

Modeling studies like the one illustrated in Figure 3 were performed for the five events with announced yield using the Mueller-Murphy source and for the six events with near-field models using the Helmberger-Hadley source. The results were averaged and are presented in Table 2. The average Futterman \(t^*\) values for the Helmberger-Hadley source are 0.79 and 0.75 sec based on the amplitude and magnitude observations, respectively. The corresponding values for the Mueller-Murphy source are 0.99 and 1.05. The error bounds given in the table represent one standard deviation for the set of estimates. The standard deviation is given to indicate the relative consistency of the various models and is not intended as a formal uncertainty estimate. The result that the \(t^*\) estimates are about 0.2 sec larger for the Mueller-Murphy source compared to the Helmberger-Hadley source model is related to the fact that the Mueller-Murphy source model contains significantly more 1 Hz energy than the Helmberger-Hadley source model for events with similar yield (see Figures 1 and 2). The scatter in the average \(t^*\) measurements are generally smaller for the Mueller-Murphy than for the Helmberger-Hadley source model. Also, the scatter is generally smaller for the amplitude observations than for the magnitude observations.

### Table 2

#### Average Attenuation Models

<table>
<thead>
<tr>
<th></th>
<th>Murphy Source</th>
<th>Helmberger Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Futterman Attenuation Models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t^*) from log (A)</td>
<td>0.99 ± 0.04 (M•F)</td>
<td>0.79 ± 0.06 (H•F)</td>
</tr>
<tr>
<td>(t^*) from (m_b)</td>
<td>1.05 ± 0.08</td>
<td>0.75 ± 0.08</td>
</tr>
<tr>
<td><strong>Minster Attenuation Models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tau_m) from log (A)</td>
<td>0.051 ± 0.008 (M•M)</td>
<td>0.044 ± 0.011 (H•M)</td>
</tr>
<tr>
<td>(\tau_m) from (m_b)</td>
<td>0.052 ± 0.016</td>
<td>0.060 ± 0.016</td>
</tr>
</tbody>
</table>
It is still necessary to select a preferred value of \( t_0^* \) appropriate for Mueller-Murphy source model. We found that on average over all the events studied the value of \( t_0^* = 1.25 \) sec made it easiest to simultaneously fit the amplitude and magnitude observations with the Mueller-Murphy source. We therefore selected it as our preferred value. The average \( \tau_m \) that matched the log-amplitude data (0.051) is almost identical to the value that matched the magnitude data (0.052). This indicates that this source-attenuation model predicts both the observed amplitude and period of the teleseismic waveforms. The average values and standard deviations of \( \tau_m \) are given in Table 2. The \( \tau_m \) measurements (with \( t_0^* = 1.0 \) sec), assuming the Helmberger-Hadley source model, are slightly different for the log-amplitude (0.044) and magnitude (0.060) data. The result that the magnitude observations give a larger \( \tau_m \) than the amplitude data indicates that, for the Minster \( Q \) model that predicts the observed amplitudes, the period is too short (i.e., the Helmberger-Hadley waveforms are too high in relative frequency content). We choose \( t_0^* = 1.0 \) sec as the preferred value for the Helmberger-Hadley source model because that value represents a lower bound for \( t_0^* \). Larger values will only result in synthetic waveforms that are higher frequency, since a larger \( t_0^* \) will require a larger \( \tau_m \) to match the observed amplitudes.

The Futterman \( t^* \) measurements also indicate that the Helmberger-Hadley synthetic waveforms are higher in relative frequency content than the Mueller-Murphy waveforms in the 0.5- to 2-Hz frequency band. (This result is most clearly shown in Figures 1 and 2 and will be further illustrated in the next section.) If the measured \( t^* \) from log-amplitudes is larger than that from the magnitudes (as for the Helmberger-Hadley source model), then the synthetic seismograms are too high in relative frequency. For the Mueller-Murphy source model, the Futterman \( Q \) model gives synthetic waveforms which are too low in frequency. However, the Minster attenuation model, which does not attenuate high frequencies as strongly, matches both the observed amplitude and period.

We believe that the models based on the log-amplitude observations are more accurate than those based on the magnitude observations. This is because the amplitude data involve only one measurement while the magnitude data requires two, with the signal period occasionally being difficult to measure in the data. Furthermore, the period measurement is somewhat influenced by \( pP \) delay time. Errors in the \( pP \) delay time may alter the attenuation models that fit the magnitude data.

Synthetic seismograms for the two source models and the average attenuation models (based on the log-amplitude observations) for the BOXCAR, HALFBEAK, and SCOTCH events are shown in Figure 4. Also given are the log-amplitude and magnitude residuals. These are the differences between the observed and predicted values (a positive residual means that the predicted value is smaller than the observed). These residuals indicate how well the various average attenuation models match the observed amplitudes and magnitudes. The residuals of the log-amplitudes are shown in Figure 5 for each attenuation and source model. The events are ordered by estimated yield. No trend of the residuals with yield is apparent. In general, the Mueller-Murphy residuals are slightly smaller than the Helmberger-Hadley residuals.

To illustrate how well the attenuation models based on the log-amplitude data predict the magnitudes, the magnitude residuals for those models are given in Figure 6. These tend to be larger than the log-amplitude residuals. This is not a surprising result since the average attenuation models are based on the amplitude data, and
the observed magnitudes are subject to errors in period measurements. Again, no trend in residual with yield is apparent.

This analysis indicates the deviation of the observed amplitude measurements from the predicted values based on the true yield and the average attenuation models. The residuals for each source-attenuation model are generally much less than the standard deviations of the observed amplitude measurements given in Table 1. The rms of the residuals of the log-amplitudes are 0.04 (M+F), 0.05 (M+M), 0.07 (H+F), and 0.06 (H+M). The rms for the magnitude residuals are 0.07 (M+F), 0.07 (M+M), 0.08 (H+F), and 0.11 (H+M). These are less than the rms standard deviations of the observed log-amplitude measurements (0.10) and magnitude measurements (0.11). Thus, the scatter of observed amplitudes and magnitudes about the predictions of our models is less than the scatter in the raw amplitude and magnitude measurements.

The fundamental conclusion of our time-domain amplitude and waveform modeling is that from WWSSN data either of the two source models (Mueller-Murphy
Fig. 5. Residuals of the log-amplitude for each of the attenuation-source models. The average attenuation models are from log-amplitude observations and are given in Table 2. The source models are based on the announced yields (Mueller-Murphy source) or on near-field modeling (Helmberger-Hadley source).

Fig. 6. Residuals of the magnitudes for each of the attenuation-source models. The average attenuation models are derived from matching amplitude observations and are given in Table 2.

or Helmberger-Hadley) and either of the two attenuation models (Futterman or Minster) can produce acceptable results. By varying the free parameters in the two attenuation models within reasonable ranges, we can obtain essentially identical predictions of the amplitudes and waveforms. The values of the free parameters which fit the data (Table 2) are significant new results. However, because of the
inherent trade-offs between source and attenuation models, there are many other parameter combinations that also fit the time-domain amplitude data.

Spectral shape constraints. The amplitude spectra for the HALFBEAK synthetic seismograms from Figure 4 are shown in Figure 7. The amplitude spectra for the various source-attenuation models are very similar to each other. It would certainly be difficult to distinguish them with the band-limited data from the WWSSN short-period instrument, even if reliable spectra could be obtained. The maximum amplitude of the central peak and its initial fall-off rate in the data are well-resolved, since we obtain almost identical results for all four models. Close inspection of Figure 7 shows that the Helmberger-Hadley synthetic seismograms have a higher relative frequency content than the Mueller-Murphy waveforms in this band. Also, the Minster Q model synthetic seismograms are higher frequency than the Futterman Q model synthetic seismograms. Based on the result that the Minster Q model with $t_0^* = 1.25$ and $\tau_m = 0.051$ for the Mueller-Murphy source model predicts both the amplitude and period of the observed waveforms, we infer that the shapes of the observed spectra should most closely resemble the corresponding spectrum. However, when considering the amount of scatter in the amplitude observations, the scatter in the attenuation measurements, and the similarity of the various attenuation-source models, we feel that each of the four models is acceptable.

To ultimately select the best attenuation model, we must consider broadband spectral observations (e.g., Der et al., 1985; Bache et al., 1986). Der et al. (1985) computed synthetic spectral templates and compared them to observed body wave spectra of Pahute Mesa events to find the attenuation model that best matched the

![Graph](image-url)
spectral shape. They assumed the von Seggern-Blandford (1972) granite source model. Although many similar studies (e.g., Der et al., 1982a; Der and Lees, 1985) indicate that a frequency-dependent attenuation model is in general necessary, Der et al. (1985) found that the spectral shape in the 0.5- to 4-Hz band can also be fit with a frequency-independent Q model. They obtain apparent $t^*$ values ranging from 0.4 to 0.5 sec. In general, the apparent value of $t^*$ appears to be a stable parameter and differs only slightly from the absolute value of $t^*$ (Der et al., 1985). However, for the frequency-independent Q model, the apparent value of $t^*$ is equivalent to the absolute value of $t^*$. We therefore assume that a frequency-independent $t^*$ of 0.45 sec matches the spectral shape in the 0.5- to 4-Hz band and apply it as a constraint to our time-domain amplitude models.

Bache et al. (1986) corrected their Pahute Mesa spectral data for the Mueller-Murphy source spectrum and then solved for the attenuation operator directly. They presented several frequency-dependent attenuation models that match the spectral data in the 0.5- to 6-Hz band. One is a Minster Q model (single absorption band) with $t_{0^*} = 1.0$ sec and $\tau_m = 0.04$ (designated by QN2). The other is a Minster Q model with $t_{0^*} = 1.2$ sec and $\tau_m = 0.06$ convolved with a Futterman Q model with $t^* = 0.1$ sec (designated by QN1). In both of the spectral data studies, the goal was to find the attenuation model which best explained the spectral fall-off over a broad range of frequencies. As we show, the spectral shapes of Pahute Mesa events measured in the two studies are essentially the same. However, the absolute spectral level is quite different.

The left portion of Figure 8 shows the spectral shapes of the source models (for HALFBREAK as an example) convolved with the attenuation models presented in this study. The spectra have been multiplied by frequency squared. The four source-attenuation models described in this paper necessarily have the correct spectral amplitude at around 1 Hz since they are based on matching the time-domain

![HALFBREAK * Q MODELS](image1)

![ATTENUATION MODELS](image2)

**Fig. 8.** (Left) Comparison of the amplitude spectrum of HALFBREAK source model convolved with the attenuation model of the four source-attenuation models presented in this paper with the results of Der et al. (1985) and Bache et al. (1986). (Right) Comparison of the attenuation model spectrum of the results of this study (only Minster Q models) with the results of Der et al. (1985) and Bache et al. (1986).
amplitudes on the WWSSN short-period instrument. Shown as dotted curves are models which fit the spectral shape in this band from Der et al. (1985) and Bache et al. (1986). The curve from Der et al. (1985) is a von Seggern-Blandford granite source with a frequency-independent $t^*$ of 0.45 sec. The curve from Bache et al. (1986) is a Mueller-Murphy source with the attenuation model QN1. All models are plotted at their true absolute levels. We find that the results of Der et al. (1985) and Bache et al. (1986) have similar spectral shapes, but have different absolute levels. If we use the spectral shape data as a constraint for our source-attenuation models, we find that either source model convolved with the appropriate Minster Q model satisfies the spectral shape as well as the amplitude data. The slope of the source-attenuation models between 1 and 4 Hz as determined from linear regressions are similar for the four models: 0.47 for QN1; 0.56 for Der et al. (1985); 0.44 for M•M; and 0.41 for H•M. Each of these fits has a regression coefficient exceeding 0.98. The differences in slope are partially due to differences in the source spectral slopes of the von Seggern-Blandford, Mueller-Murphy, and Helmberger-Hadley source models.

The right portion of Figure 8 shows the attenuation model spectrum for the results of this study (only the Minster Q models) compared with the results of Der et al. (1985) and Bache et al. (1986). The five models presented here have similar spectral shapes over the 0.5- to 4-Hz band; thus, each model is considered to satisfy the spectral shape data. It is encouraging that the attenuation models of this study and of Bache et al. (1986) are so similar considering that the data analyzed are in different forms and from different networks. This is somewhat fortuitous, since our amplitude observations provide no resolution at higher frequencies. However, the inclusion of the spectral shape data as a constraint to this analysis confirms that our determination of the frequency dependence of Q is correct.

**YIELD ESTIMATION**

If we assume that the average attenuation models given here are accurate and that the observed log-amplitudes are accurate, we can estimate the yields for this set of explosions. The yields for the eight events were estimated from each of the four source-attenuation models from the log-amplitude data and are given in Table 3. To estimate the yield using the Helmberger-Hadley source model requires that we define yield-scaling relations for values of $K$ and $\Psi_\infty$. These were based on theoretical and empirical results outlined by Barker et al. (1985)

$$K = C_1 Y^{-7/36},$$

(2)

and

$$\Psi_\infty = C_2 Y^{0.91},$$

(3)

where $Y$ is yield and $C_1$ and $C_2$ are constants. The constants are obtained by substituting the announced yield and near-field values for BOXCAR into equations (2) and (3). Hence, $C_1$ is 26.21 and $C_2$ is $1.76 \times 10^8$.

The residuals of the log-yields (estimated minus announced) for the five events with announced yields are given in Figure 9. Combining the log-amplitude residuals shown in Figure 5 with the log-yield residuals in Figure 9, we can estimate the correlation between the error in estimated yield and the error in observed amplitude. For the Mueller-Murphy source model with the Minster attenuation model,

$$\Delta \log Y = -1.3 \Delta \log A.$$  

(4)
Table 3: Yield Estimates (KT) from the Observed Log-Amplitudes

<table>
<thead>
<tr>
<th>Event</th>
<th>Announced Yield</th>
<th>Futterman</th>
<th>Minster</th>
<th>Futterman</th>
<th>Minster</th>
<th>Q Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALMENDRO</td>
<td>769</td>
<td>778</td>
<td>655</td>
<td>670</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BENHAM</td>
<td>1150</td>
<td>1158</td>
<td>1218</td>
<td>973</td>
<td>1013</td>
<td></td>
</tr>
<tr>
<td>BOXCAR</td>
<td>1300</td>
<td>1068</td>
<td>1100</td>
<td>891</td>
<td>922</td>
<td></td>
</tr>
<tr>
<td>GREELEY</td>
<td>870</td>
<td>910</td>
<td>929</td>
<td>767</td>
<td>789</td>
<td></td>
</tr>
<tr>
<td>HALFBEAK</td>
<td>365</td>
<td>436</td>
<td>452</td>
<td>403</td>
<td>407</td>
<td></td>
</tr>
<tr>
<td>INLET</td>
<td>359</td>
<td>349</td>
<td>325</td>
<td>324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAST</td>
<td>445</td>
<td>449</td>
<td>404</td>
<td>406</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCOTCH</td>
<td>155</td>
<td>141</td>
<td>134</td>
<td>143</td>
<td>139</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9. Residuals of the log-yields for the five events with announced yields for each of the source-attenuation models. The log-yield residuals are the announced subtracted from the predicted.

With the exception of SCOTCH, the yield residuals tend to become more positive with increasing yield for each of the source-attenuation models. However, these residuals are quite small and may be insignificant considering the amount of scatter in the observations and the attenuation measurements.

Using the estimated yields from Table 3, we computed the predicted source spectra for these events. Figure 10 shows the comparison between the estimated source spectra using the Minster Q model with the Mueller-Murphy source and the predicted source spectra from the announced yield. The differences between the announced and the estimated spectra are shown to be minimal. It would certainly be difficult to distinguish between the predicted and estimated source spectra from observations. For the other source-attenuation models, the differences between the predicted and estimated source spectra are on the same level as those in Figure 10.

Discussion

There has been much recent discussion as to the value of $t^*$ at higher frequencies. The typical value of $t^* = 1.0$ sec at long periods results in extremely high attenuation...
at higher frequencies for teleseismic observations. Der et al. (1982a) state that 4-Hz amplitudes would be reduced by four orders of magnitude relative to 1-Hz amplitudes as a result of this $t^*$ value. However, numerous investigations show that there are substantial amounts of 4- to 10-Hz energy, far greater than predicted by a $t^*$ of 1 sec. Because of these observations, $t^*$ is thought to be much smaller at higher frequencies.

Using the preferred source-attenuation models presented in Figure 8, we can compute the value of $t^*$ as a function of frequency (Figure 11). Shown are the values of $t^*$ for the four source-attenuation models presented in this study, along with the results of Der and Lees (1985) and Bache et al. (1986). The Futterman $Q$ models from this study ($M^*F$ and $H^*F$) are appropriate only near 1 Hz. The attenuation models $M^*M$, $H^*M$, $QN1$, and $QN2$ are considered to match both the amplitude and spectral shape data over this band. We see from this that the value of $t^*$ around 1 Hz is between 0.8 and 1.0 sec, depending on the choice of source-attenuation model. Der and Lees (1985) proposed a frequency-dependent attenuation model for the Western United States that satisfied spectral shape and time-domain waveform constraints. The events they studied were deep earthquakes, so the data were not subject to upper mantle attenuation on the source side of the raypath. Their preferred model (QP S-T) is shown for comparison in Figure 11. (S-T refers to shield-to-tectonic paths). Although Der and Lees (1985) argue that previous studies (e.g., Der et al., 1982b) demonstrate that the apparent $t^*$ from deep earthquakes is similar to that obtained from shallow events in shield regions, it is reasonable to expect that the QP S-T curve represents a lower bound. If we were to assume some $t^*$ contribution from the source side of the raypath, then the QP S-T curve would become consistent with the other attenuation models.

Values of $t^*$ of 0.8 to 1.0 sec near 1 Hz are quite consistent with other estimates of $t^*$. Burdick et al. (1984) obtain a value of 0.9 sec from a near-field to far-field amplitude comparison using Amchitka nuclear explosions. Burdick (1978) conducted a source-canceling experiment comparing $sP$ with $ss$ phases and obtained a value of $t^* = 1.3$ sec at about 0.5 Hz for an event in southern California (presumably similar to NTS). This value is consistent with the $M^*M$ and $QN1$ models. Burdick (1985) estimated a global average $t^*$ of about 1 sec near 1 Hz from a comparison of

---

**Fig. 10.** Comparison of the announced yield and predicted yield source displacement spectra for the Mueller-Murphy source model (assuming the Minster $Q$ model results).
Fig. 11. Comparison of the value of $t^*$ as a function of frequency for the four source-attenuation models from this paper and the two attenuation models from Bache et al. (1986). Also shown is the QP S-T attenuation model for the Western United States from Der and Lees (1985).

ScP and ScS phases. Finally, Burdick and Grand (1985) obtained $t^* = 0.99$ sec at around 1 Hz from the comparison of $sP$ and $sS$ waves from Asia.

To illustrate the effect of the absolute value of $t^*$ on the observed amplitudes, we computed synthetic seismograms for the Mueller-Murphy source model with various $Q$ models with similar spectral shapes in the 0.5- to 4-Hz band. We assumed the M*N source-attenuation model (this paper), along with $t_0^* = 0.75$ sec and $\tau_m = 0.025$, and a frequency-independent $t^* = 0.42$ sec (similar to Der et al., 1985). Each of these $Q$ models have the same spectral slope (0.57) in the 0.5- to 4 Hz band (linear regression coefficients exceed 0.97). The results for the HALFBEAK event are presented in Figure 12. The predicted amplitude using $t^* = 0.42$ sec is nearly a factor of 5 larger than observed, while the predicted amplitude for $t_0^* = 1.25$ and $\tau_m = 0.051$ is similar to the observed value. Furthermore, the synthetic seismogram with $t^* = 0.42$ sec is too high in relative frequency content compared to representative observations (shown at the right in Figure 3). This example illustrates that the use of spectral shape data without absolute amplitude information can lead to incorrect $Q$ models. We have previously shown that the use of absolute amplitude information without spectral shape data can also lead to incorrect results. For example, the M*M and M*F models have the same absolute amplitude level near 1 Hz, but the spectral slopes in the 0.5- to 4 Hz band are very different. It is thus necessary to use both absolute amplitude information and spectral shape or decay information to obtain reliable attenuation models.

A direct application of our results is that we are able to predict the relation between yield and magnitude based upon the appropriate attenuation models for a given site. We do so here assuming the Mueller-Murphy source and the Minster $Q$
model (M•M). Synthetic seismograms were computed as in the above analysis for yields ranging from 50 to 1300 kt. The $pP$ delay times were determined from the average $pP$ delay—depth relation determined from the results of Lay (1985) and assuming the predicted depth—yield dependence from Mueller and Murphy (1971). The predicted magnitudes are given in Figure 13. The linear relation (with a regression coefficient of 1) is given by

$$m_b = 3.695 + 0.8019 \log Y. \quad (5)$$

For the predicted amplitudes, the relation is given by

$$\log A = 0.3862 + 0.7252 \log Y. \quad (6)$$

The difference in slopes in equations (5) and (6) is due to the period correction included in the magnitude calculation. Equation (5) is almost identical to that obtained by Lay (1985) from a regression of observed magnitudes for the five events with announced yields. The observed magnitudes are also given in Figure 13 and are in good agreement with the predicted $m_b$-yield relation.
Predicted $mb$–Yield Relation

$$mb = 3.695 + 0.8019 \log Y$$

**FIG. 13.** The predicted $mb$–yield relation for WWSSN short-period determinations for the Mueller-Murphy source and Minster $Q$ model. Shown are the observed magnitudes for the five events with announced yields.

**TABLE 4**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ALMENDRO</td>
<td>17.8</td>
<td>19.0</td>
<td>(20.0)</td>
<td>5.7</td>
</tr>
<tr>
<td>BENHAM</td>
<td>--</td>
<td>--</td>
<td>26.9 (9.6)</td>
<td></td>
</tr>
<tr>
<td>BOXCAR</td>
<td>--</td>
<td>--</td>
<td>29.5</td>
<td>12.0</td>
</tr>
<tr>
<td>GREELEY</td>
<td>--</td>
<td>--</td>
<td>21.7 (7.6)</td>
<td></td>
</tr>
<tr>
<td>HALFBEAK</td>
<td>--</td>
<td>--</td>
<td>11.2</td>
<td>3.8</td>
</tr>
<tr>
<td>INLET</td>
<td>--</td>
<td>--</td>
<td>(10.9)</td>
<td>3.2</td>
</tr>
<tr>
<td>MAST</td>
<td>15.5</td>
<td>12.0</td>
<td>(13.2)</td>
<td>4.7</td>
</tr>
<tr>
<td>SCOTCH</td>
<td>8.9</td>
<td>9.1</td>
<td>5.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

* The numbers in parentheses are estimated from the predicted yields assuming the Minster $Q$ models.

We have not yet addressed the possibility of constraining the source models with longer period observations. As apparent in Figure 1, very precise constraints on 10-sec spectral levels could serve to discriminate between the source models. However, the long-period levels for the Helmberger-Hadley source could be increased simply by decreasing the overshoot parameter $B$, which was arbitrarily set to 1 in the near-field modeling. The near-field waveforms are essentially unaffected if the value of $B$ is set to 0.5, which gives $\Psi_o$ a factor of 1.7 larger. Smaller values of $B$ will increase $\Psi_o$ further, but will begin to affect the spectrum in the near-field band (1 to 5 Hz), and thus change the waveform matches. For now we will limit our discussion to $B > 0.5$, but acknowledge the possibility of even smaller values of $B$. Setting the value of $B$ at 0.5 will increase the WWSSN short-period (0.5 to 2 Hz) amplitudes by less than 10 per cent; thus, our high-frequency attenuation operator modeling will be relatively unchanged by any long-period modifications.

Several studies have obtained estimates of long-period source spectral levels for Pahute Mesa tests from analysis of surface waves (Given and Mellman, 1985;
Stevens, 1986). These estimates are compared with the predictions of the Mueller-Murphy source model, as well as the near-field modeling results of Barker et al. (1985) with $B = 1.0$ in Table 4. On average, the surface wave results are consistent with the Mueller-Murphy source predictions, but are a factor of 3 to 4 larger than the Helmberger-Hadley source predictions. Allowing for a factor of 1.7 increase in $\Psi_m$ by decreasing $B$ in the near-field models still leaves a factor of about 2 discrepancy between the surface wave estimates and the results of Barker et al. (1985). While this favors the Mueller-Murphy source model, further work is needed to establish the confidence levels on the various estimates before a definitive conclusion can be drawn.

CONCLUSIONS

Attenuation models for short-period $P$ waves from Pahute Mesa nuclear explosions are developed by time-domain modeling of teleseismic waveforms. Trade-offs between source and attenuation models are explored using two explosion source representations and two attenuation models, one of which includes frequency dependence. For the Mueller-Murphy source model, which has an asymptotic high-frequency spectral fall-off of $f^{-2}$, yield scaling is used to determine the source models. This source leads to frequency-independent $t^*$ values of about 1.0 sec or frequency-dependent absorption band models with $t_0^* = 1.25$ sec and $\tau_m$ (high-frequency roll-off) of about 0.05. The Helmberger-Hadley source, which has a high-frequency spectral decay of $f^{-3}$, is constrained by near-field waveform modeling. This source results in frequency-independent models with $t^*$ values of about 0.8 sec, or frequency dependent models with $t_0^* = 1.0$ sec and $\tau_m$ of about 0.04. However, spectral shape information from higher frequency data is better matched by the absorption band attenuation models for either source representation. Thus, the use of both spectral shape data and absolute amplitude information is necessary to determine reliable attenuation models. Also, simple correction of observed spectra by assuming the high-frequency asymptotic spectral decay rates of these source models in the short-period band can lead to erroneous conclusions regarding the attenuation operator. The two source models are sufficiently similar in the 0.5- to 4-Hz band that it is not possible to select the more appropriate model. However, long-period measurements tend to favor the higher long-period spectral amplitudes predicted by the Mueller-Murphy source model.

ACKNOWLEDGMENTS

We thank Susan Schwartz, Christopher Lynnes, and the anonymous reviewer for their comments on the manuscript. T. L. acknowledges support from the Alfred P. Sloan Foundation and a Shell Faculty Career Initiation Grant. This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Air Force Geophysics Laboratory under Contract F19628-85-C-0036.

REFERENCES


R. W. BURGER, T. LAY, AND L. J. BURDICK


WOODWARD-CLYDE CONSULTANTS
566 EL DORADO STREET
PASADENA, CALIFORNIA 91101 (R.W.B., L.J.B.)

MANUSCRIPT RECEIVED 27 JUNE 1987