Effects of 2D Random Velocity Heterogeneities in the Mantle Lid and Moho Topography on $P_n$ Geometric Spreading

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Abstract  $P_n$-wave energy refracts through the uppermost mantle, with the first seismic wave arrival at distances of $\sim 200$ to $\sim 1500$ km from crustal sources. The $P_n$ phase provides important constraints on source type, location, and magnitude, but its propagation is complicated by frequency-dependent sensitivity to the Earth’s sphericity and lithospheric velocity structure. Converging on an acceptable $P_n$ geometric spreading correction and specifying its uncertainties, a requirement for accurately determining frequency-dependent attenuation models for $P_n$, depends on improved understanding of the behavior of $P_n$ geometric spreading for various heterogeneous models. We investigate the effects of radial mantle lid velocity gradients, mantle lid random volumetric velocity heterogeneities, and Moho topography on $P_n$ geometric spreading using reflectivity and two-dimensional (2D) finite-difference 1-Hz wave propagation calculations for elastic Earth models. Mantle lid velocity gradients systematically modify the frequency-dependent geometric spreading from that found for models with constant velocity but retain the same overall functional form. $P_n$ amplitudes are also sensitive to the presence of modest 2D random lateral velocity heterogeneities within the uppermost mantle, with geometric spreading approaching a power-law behavior as the root mean square strength of heterogeneity increases. 2D Moho topography introduces scatter into the amplitude of $P_n$, but the overall behavior remains compatible with that for a laterally homogeneous model. Given the lack of knowledge of specific small-scale structure for any particular $P_n$ path, the preferred geometric spreading parameterization is the frequency-dependent model for a constant mantle lid velocity structure unless $P_n$ travel-time branch curvature can constrain the radial gradient in the mantle lid.

Introduction

Distinguishing small earthquakes from low-yield underground nuclear explosions (given a typical dearth of teleseismic observations) requires that regionally detected seismic phases ($P_n, S_n, P_g, L_g$) be understood in detail. Several source discrimination techniques have been developed that measure the relative amount of $P/S$ energy in regional signals (e.g., Walter et al., 1995; Hartse et al., 1997), with regional travel-time tomography and attenuation models being used to suppress the effects of wave propagation in the heterogeneous crust. The contribution of geometric spreading to the regional seismic signals must also be quantified and accounted for, particularly for use in developing attenuation models. Geometric spreading can be very difficult to distinguish from anelastic attenuation, especially for $P_n$ and $S_n$ phases, which are known to have frequency-dependent sensitivity to the Earth’s sphericity, upper-mantle velocity heterogeneity, and lower-crustal structure (Buldyrev and Lanin, 1965; Hill, 1973; Sereno and Given, 1990; Taylor et al., 2002). Morozov (2010) gives a detailed discussion on the definition of geometric spreading, in which he explains that geometric spreading can be defined as amplitude decay due to the expansion of a single wavefront, or that geometric spreading can be considered in such a way as to include the elastic effects of the Earth’s structure on amplitude decay. The single-wavefront definition requires that any amplitude variations from this simple model be attributed to scattering attenuation. On the other hand, the fully inclusive definition of geometric spreading would consider “scattering attenuation” merely as geometric spreading in a heterogeneous Earth. Our modeling efforts assume the latter definition of geometric spreading and seek to characterize the contribution of scattering to observed $P_n$ amplitudes in a purely elastic medium. Yang et al. (2007) developed frequency-dependent geometric spreading relations for $P_n$ and $S_n$ for simple reference velocity structures with constant velocities in the uppermost mantle lid (the high-velocity layer underlying the Moho and above any upper-mantle low-velocity zone) that differ significantly from commonly used frequency-independent power-law
geometric spreading corrections (e.g., Walter and Taylor, 2001).

\(P_n\) and \(S_n\) are generated by crustal sources and have paths that refract through the uppermost mantle with some secondary energy multiply reflecting from the underside of the Moho discontinuity as long as the radial velocity gradient is above the critical negative gradient (e.g., Yang et al., 2007). The distances at which \(P_n\) and \(S_n\) are observed as first P-wave and S-wave arrivals ranges from their crossover with \(P_g\) and \(L_g\) near 200 km to where P-wave and S-wave energy that has dived down to the vicinity of the transition zone overtakes them near 1500 km. The direct ray paths for \(P_n\) and \(S_n\) as far out as 1500 km distance bottom in the upper 50 km of the mantle lid for typical seismic velocity models, and the multiple underside Moho reflections bottom even shallower in the mantle lid. These shallow trajectories make \(P_n\) and \(S_n\) phases particularly sensitive to Earth’s sphericity and to details of the velocity structure of the lower crust and upper mantle through which they propagate nearly horizontally. Theoretical calculations of \(P_n\) propagation through a spherical body (Buldyrev and Lanin, 1965; Hill, 1973), as well as one-dimensional (1D) reflectivity simulations of \(P_n\) propagation (Yang et al., 2007) through laterally homogeneous layered models, provide ample documentation of \(P_n\) sensitivity to the Earth’s sphericity.

An analysis of variance (ANOVA) study conducted by Fisk et al. (2008) shows that \(P_n\) amplitude observations have the highest total path variance of the regional phases (\(P_n, S_n, P_g, L_g\)) typically used for regional source discriminants. This reflects the pronounced sensitivity of \(P_n\) to heterogeneous lithospheric structure and the fact that the \(P_n\) phase is not comprised of a suite of arrivals sampling a wide range of ray parameters that intrinsically average out the path effects, as is the case for the other phases (particularly \(P_g\) and \(L_g\)). The basic wave propagation characteristics of \(S_n\) are similar to those for \(P_n\), but the additional P-to-S conversions in the high frequency \(S_n\) signals appear to somewhat stabilize rms amplitude measures relative to those for \(P_n\), so we will focus on \(P_n\) in this study.

Geometric spreading corrections are typically defined for simple reference Earth models, motivated by seismic waveform modeling procedures; while it is clear that a 1D seismic model is not precisely correct for any path, our lack of knowledge of the actual 3D structure justifies using a simplified reference model about which there is expected to be a normally distributed variation of observations. For many phases this is reasonable; teleseismic body waves with turning points in the relatively homogeneous lower mantle appear to have minor scatter relative to 1D frequency-independent geometric spreading assumptions, while the behavior of complex multiple arrivals like \(L_g\) at regional distances can largely be accounted for by simple frequency-independent power-law amplitude decays by virtue of their intrinsic strong-wave interference (e.g., Yang, 2002). In this regard, \(P_n\) and \(S_n\) are important exceptions; even for a very simple spherical reference model involving a constant-velocity crustal layer over a constant-velocity upper-mantle layer, the computed geometric spreading is frequency-dependent, nonuniform with distance, and nonlinearly sensitive to perturbations of the structure (Yang et al., 2007). While observed \(P_n\) amplitude decay with distance may be well characterized by a simple power-law function, separating geometric spreading from attenuation effects that control the amplitude decay is very difficult when one cannot justify assuming a frequency-independent geometric spreading correction. Ignoring the frequency dependence of geometric spreading will inject artificial frequency dependence directly into attenuation estimates, undermining the latter’s use for path corrections or geophysical interpretations.

These considerations motivate using as realistic of a \(P_n\) geometric spreading relation as possible; however, the very sensitivity of \(P_n\) to spherical structure, which produces complexity in the geometric spreading, is also likely to yield sensitivity to small structural perturbations, with non-self-evident behavior. Full waveform calculations are made here to evaluate the sensitivity of \(P_n\) geometric spreading to deviations from a very simple reference model. We consider 1D gradients in the mantle lid velocity structure, 2D lateral volumetric heterogeneities in the mantle lid structure, and 2D topographic heterogeneity of the Moho, with the goals of assessing what practical choice of \(P_n\) geometric spreading is most reasonable and assessing the attendant uncertainties.

We find that modest, geologically reasonable structural heterogeneity produces strong \(P_n\) amplitude variability with distance, which ensures that considerable errors are associated with any \(P_n\) geometric-spreading parameterization. These errors will be manifested in, for example, attenuation models for \(P_n\). Adhering to the seismic modeling paradigm, our pragmatic recommendation is to use the complex frequency-dependent geometric spreading for a very simple basic Earth model (Yang et al., 2007) rather than an arbitrarily parameterized model such as a power-law for which there is no known associated 1D reference structure, allowing for large uncertainties on any derived attenuation model that accompanies it. The geometric spreading approximation for a laterally homogeneous (1D) Earth model from Yang et al. (2007) has the advantages of being consistent with theoretically predicted \(P_n\) propagation and corresponding to a specific parameterized velocity model. However, it does have multiple parameters, each nonlinearly dependent on changes in the velocity model, which makes applying this approximation slightly more complicated than selecting a single decay coefficient, as is the case for a power-law approximation. We argue the minor added complexity is necessary, as our models show that \(P_n\) does indeed have a nonlinear response to changes in mantle velocity gradient or small-scale structure. Empirically constraining the average mantle lid velocity gradient by first-arrival travel-time curvature can significantly improve the choice of reference model, but there will still be significant uncertainties associated with unresolved small-scale heterogeneities in the mantle lid structure.
Methods

Synthetic calculations in this study employ the 1D reflectivity method (Müller, 1977; Randall, 1994), as implemented by Yang et al. (2007), and the 2D P–SV finite-difference code of Xie and Lay (1994). We add linear radial mantle lid velocity gradients, randomly varying volumetric mantle lid P-wave velocity ($V_p$) heterogeneity, or Moho depth heterogeneity to the homogeneous basic Earth model (BEM) from Yang et al. (2007) and Sereno and Given (1990). This simple spherical reference model contains a 40-km-thick crust and a constant-velocity mantle (Fig. 1a). For both the reflectivity and finite-difference methods, we apply an Earth flattening transform (EFT) (Chapman, 1973) to the BEM, which yields 1D structures having positive velocity gradients with depth (Fig. 1b). All models are purely elastic (or asymptotically approximated as such for the reflectivity calculations as described by Yang et al., 2007), allowing us to isolate the effects of structural heterogeneity on $P_n$ geometric spreading. Perturbations to the BEM are herein discussed in detail for each class of heterogeneity that we investigate.

The source model is an isotropic point source located at a depth of 15 km. The source-time function for the reflectivity method is a delta function and is a Gaussian derivative source-time function with amplitude normalized to 1 for the finite-difference method. For the 1D velocity models with linear velocity gradients in the mantle lid, we compute both high-frequency reflectivity and lower-frequency finite-difference synthetics. The reflectivity synthetics span the range 0.75–12 Hz, with spectral estimation following the procedures described in Yang et al. (2007). The dominant frequency of our finite-difference synthetics is approximately 0.8–1 Hz. The 2D finite-difference modeling approach cannot achieve the very high frequencies of the 1D wavenumber integration method of Yang et al. (2007), and full exploration of frequency-dependent effects of the 2D models is beyond our computational limits. However, comparisons of the methods for simple 1D models indicate that the basic behavior of $P_n$ geometric spreading is accurately reproduced by the finite-difference computations for 1-Hz signals. We align the synthetic seismograms with respect to the mantle P-wave velocity (8.1 km/s) and use a fixed-window width of 9 s. The $P_n$ phase onset for the BEM case is approximately 3 s into the fixed window. This window effectively isolates the $P_n$ coda from contamination by the $P_g$ phase following it and allows for arrival time perturbations that result from heterogeneous path structure. The fixed window is only one of several windowing options common in studies of $P_n$, so we also calculate amplitudes of $P_n$ with a group velocity window (8.2–7.6 km/s) and with a very short fixed window based on $P_n$ onset that approximately captures the first period of the $P_n$ arrival. These three windowing methods span the range of measurement procedures typically used and allow us to consider the nature of geometric spreading for an isolated part of the wavefront versus a coda-dominated signal. Amplitudes are calculated as the rms amplitude of the windowed $P_n$ energy. Corrections are applied to the calculated rms amplitude to approximate a spherical wavefront ($A_r = A_o \times (1/\sqrt{x})$), where $A_r$ is the amplitude calculated on a spherical wavefront, $A_o$ is the initial amplitude, and $x$ is distance in kilometers) and to transform the amplitude ($A_r$) into that of a wave traveling within a spherical Earth (unflattening transform—see equation 3 in Yang et al., 2007). The $P_n$ amplitude calculations are made in the distance range from 350 to 1500 km to ensure the $P_g$ arrival does not bias the rms amplitude calculation, although the fixed window is still contaminated by $P_g$ for the closest distance.

$P_n$ Sensitivity to Radial Mantle Lid Velocity Gradients

The radial P-wave velocity structure of the uppermost mantle is seldom well-constrained by observations. The most direct constraint is usually provided by curvature of the $P_n$ travel-time branch as a function of distance (e.g., Morozova

Figure 1. (a) Crustal and mantle seismic velocity and density values, layer thickness, and conceptual $P_n$ path for the homogeneous spherical basic Earth model (BEM) used by Yang et al. (2007). (b) The associated 1D flat-Earth model, with slight density and seismic velocity gradients resulting from Earth flattening transform (EFT).
et al., 1999; Myers et al., 2010) if reliable measurements are available over a wide enough distance range. Even when there is apparent travel-time branch curvature, it is difficult to distinguish between a smooth gradient with depth versus a layered structure with a few or many small velocity increases. Receiver function methods and surface wave dispersion inversions sometimes indicate mantle lid structure, but resolution tends to be poor, especially for $P$ velocity, and layered structures are commonly assumed so that any radial gradients are only approximated. When lacking direct constraint, reference velocity structures often assume constant or near-constant velocity in the mantle lid by default, so the BEM model of Yang et al. (2007) and Sereno and Given (1990) is representative of many choices. However, there are regions in the Earth where the mantle lid-velocity does appear to increase with depth, with velocity gradients of up to 0.004 s\(^{-1}\) according to some studies (e.g., Zhao and Xie, 1993; Morozova et al., 1999), and areas of North Africa and Eurasia, for example, which evidence suggests have high lateral variations in mantle gradient (Myers et al., 2010). This raises the question of how useful the geometric spreading relations for the BEM are in such regions.

The effect of velocity gradient on $P_n$ geometric spreading is illustrated in Figure 2. Using the procedure described in Yang et al. (2007), we computed synthetic $P_n$ seismograms for the frequency band of 0.75–12 Hz, using the reflectivity method for models with positive velocity gradients in the mantle lid. The EFT further increases these gradients, causing $P_n$ ray paths to turn even more shallowly in the mantle lid than those for the BEM model. The amplitude–distance behavior is shown for multiple narrow frequency bands for the BEM (red curves) and for models with mantle velocity gradients of 0.002 s\(^{-1}\) (blue curves) and 0.004 s\(^{-1}\) (black curves). The BEM amplitude curves correspond directly to those for which the geometric spreading formulas of Yang et al. (2007) were derived. The systematic variation of $P_n$-amplitude frequency dependence, with increasing amplitudes and a decrease in the distance at which the minimum of the amplitude curves occur, is apparent. The overall shapes of amplitude curves for the different models are similar, with it being apparent that the effect of positive gradient is essentially to shift the curves toward higher frequencies relative to the BEM case. The 0.002 s\(^{-1}\) gradient model produces 0.75-Hz amplitudes that behave very similarly to the 9-Hz amplitudes for the BEM, and the 0.004 s\(^{-1}\) gradient model produces 0.75-Hz amplitudes very similar to the 12-Hz amplitudes for the BEM. This is problematic in that failure to constrain the lid-velocity gradient a priori can clearly lead to erroneous frequency dependence of any derived $P_n$ attenuation model in basically the same way that using a frequency-independent geometric spreading assumption does.

We made finite-difference calculations of ∼1-Hz $P_n$ amplitudes for models with radial $P$-velocity gradients in the mantle lid of 0.001 s\(^{-1}\), 0 s\(^{-1}\) (BEM), and −0.001 s\(^{-1}\). The rms amplitudes are shown in Figure 3. The BEM case

![Figure 2](image-url)  
Figure 2. $P_n$ amplitude versus distance curves for the BEM (red curves, for no physical gradient in the lid) and for two models with positive gradients in the mantle lid. The suite of curves for each case corresponds to frequencies ranging from 0.75 Hz (lowest curve) to 12 Hz (highest curve). The calculations were done with the reflectivity method, following the procedures of Yang et al. (2007).

![Figure 3](image-url)  
Figure 3. $P_n$ amplitude versus distance for the BEM (diamonds), a 0.001 s\(^{-1}\) positive mantle gradient (triangles), and a −0.001 s\(^{-1}\) negative mantle gradient (squares).
(diamonds) corresponds well with the 1-Hz BEM amplitudes in Figure 2, and the positive gradient case (triangles) gives the same shift of the amplitude curve as seen for the corresponding model 1-Hz reflectivity synthetics in Figure 2. The −0.001 s⁻¹ case almost cancels out the EFT effect (0.0013 s⁻¹), so this approaches the critical negative gradient that would yield a flat-Earth model with constant half-space velocity and true head-wave behavior of $P_n$, (see discussion in Yang et al., 2007). As expected, the amplitude decay approaches a linear trend in log-amplitude versus log-distance for that situation.

Figure 2 indicates that it may be possible to use the $P_n$ geometric spreading representation of Yang et al. (2007) for the BEM model and determine appropriate coefficients for any specific mantle lid velocity gradient that is larger than the critical negative gradient (−0.0013 s⁻¹), but this is only warranted if there are sufficiently tight a priori constraints on the specific model in the region for which the spreading is to be utilized. As this would need to be done on a case-by-case basis, we do not attempt to give generalized spreading corrections for varying mantle lid gradients here; the main result is recognizing the complex nature of the frequency-dependent shift that can result from the presence of small mantle lid velocity gradients comparable to those observed in various regions.

$P_n$ Sensitivity to Mantle $P$-Velocity Heterogeneity

Given the acute sensitivity of $P_n$ geometric spreading to a usually poorly constrained attribute of the reference velocity model (the radial velocity gradient in the uppermost mantle lid), the stability of the complex behavior of $P_n$ geometric spreading for the BEM in the presence of modest lateral heterogeneity is called into question. Most standard seismological practices assume rather simple reference models, known to be gross approximations of the real structure, under the assumption that errors in the reference model produce only mild biases in other estimated properties such as attenuation. This is simply not the case here, given the non-linear sensitivity of the $P_n$ whispering gallery arrival to small changes in 1D structures. Clearly, Earth’s actual mantle lid has multiscale lateral heterogeneities as well. Does their presence modify the acute sensitivity of $P_n$ spreading relative to the reference BEM structure?

We explore the effects of 2D lateral heterogeneity in the mantle lid velocity structure by adding random lateral velocity heterogeneity characterized by rms velocity fluctuations of a particular strength distribution with random length-scale fluctuations exponentially distributed about prescribed horizontal (Ax) and vertical (Ay) length scales. This allows us to consider 2D random heterogeneity models like that shown in Figure 4a, where the heterogeneities are superimposed on a constant half-space velocity model.

Figure 4. (a) Example realization of a velocity model with random $P$-wave velocity ($V_P$) heterogeneity in the mantle lid. This model has 0.5% rms random volumetric heterogeneities, exponentially distributed about a horizontal length scale of 40 km and a vertical length scale of 3 km. (b) Example realization of a velocity model with random Moho depth fluctuations. This model has 3% rms random Moho depth variations exponentially distributed about a horizontal length scale of 40 km.
Effects of 2D Random Velocity Heterogeneities in the Mantle Lid and Moho Topography

131

on the Earth-flattened BEM. Because our finite-difference simulations approximate a purely elastic system, random volumetric heterogeneities include rms perturbations of P-wave velocity ($V_P$), shear-wave velocity ($V_S$), and density. For each set of random model parameters, $V_S$ varies by the same percent rms as $V_P$, while density has half the percent rms fluctuation as $V_S$ and $V_P$. Analysis of the effects of random heterogeneity requires a statistical sampling of the effects associated with different realizations of the random velocity parameters. For each set of random parameters ($Ax$, $Ay$, $\%$ rms variability), we generate five velocity realizations, and calculate synthetic seismograms for individual realizations. The ensemble average from all five simulations is calculated to represent the result for a given set of random parameters. This modest number of intensive calculations appears to be sufficient to avoid significant bias in behavior associated with any single random configuration (Fig. 5). As is always possible with random model parameters, the possibility of outlier behavior and bias of the ensemble still exists, and we do observe significant fluctuation in model $P_n$ amplitudes, so our focus will be on gross attributes of the calculated $P_n$ behavior, not on individual measurements.

Figure 5 shows the synthetic $P_n$ amplitudes (∼1-Hz rms amplitude in each 9-s-long $P_n$ window) for five realizations of structures with statistical properties corresponding to those in Figure 4a, along with averages of the ensemble, and the finite-difference result for the BEM of Yang et al. (2007). An interesting behavior in Figure 5 is that the presence of volumetric velocity fluctuations with large horizontal aspect ratios systematically affects the basic shape of the geometric spreading, reducing the minimum and allowing better representation by a linear log-log (power-law) type behavior at this frequency. This suggests that the 1D geometric spreading behavior is rather delicate, with modest heterogeneity disrupting the specific interference that gives rise to the complex shape of the amplitude curve with distance. It would be prohibitively expensive (and numerically challenging) to explore the full frequency dependence of this behavior with current computers, but it is likely that at higher frequencies the delicate energy partitioning of the direct $P_n$ and the coupled whispering gallery (as discussed by Yang et al., 2007) would be at least comparably disrupted.

Figure 6 compares the effect on $P_n$ amplitude of variable percent rms $V_P$ variation in the mantle lid for each of four random heterogeneity configurations with aspect ratios ranging from isotropic (Fig. 6a: $Ax = 10$ km, $Ay = 10$ km) to large aspect ratio lenslike (Fig. 6d: $Ax = 40$ km, $Ay = 3$ km). As the aspect ratio increases, the deviations from the BEM solutions tend to increase for weaker heterogeneities. The overall amplitude of $P_n$ also increases with increased rms strength of $V_P$ variation. This is not surprising—heterogeneities with stronger velocity contrasts are expected to disrupt the wave field more effectively than more subtle velocity fluctuations, expanding the portion of the wavefront that can be scattered into the $P_n$ arrival window. This is illustrated in Figure 7 by seismic profiles of synthetic $P_n$ waveforms for the BEM, and for Earth models with large aspect ratio $V_P$ heterogeneity scale parameters corresponding to those in Figure 6d, with 1% and 2% rms $V_P$ fluctuations. Note the distortion of the $P_n$ waveform and the presence of significant early coda, indicating a high level of forward scattering of $P_n$ energy within the mantle lid. While the initial motion amplitudes are reduced relative to the homogeneous model, the overall rms values of the full window increase, indicating that signals with a wider range of ray parameters are being scattered into the window.

It is also interesting to note the difference in the shape of the amplitude curves in Figure 6 as the strength of heterogeneity increases. The same measurements are regrouped in Figure 8 by the windowing method, where the averaging lengths vary for models with common levels of 0.5% (Fig. 8a,d,g), 1% (Fig. 8b,e,h), and 2% rms (Fig. 8c,f,i) $V_P$ fluctuation. In the 0.5% case, the curve is concave-up for the

![Figure 5. $P_n$ amplitude at ∼1 Hz plotted with distance for five realizations of random mantle lid volumetric $V_P$ heterogeneity with the same averaging length scales and rms strength distribution shown in Figure 4a ($Ax = 40$ km, $Ay = 3$ km, RMS = 0.5%). The averages (circles) of these five random realizations are shown, as well as the $P_n$ amplitudes for the BEM (diamonds).]
more isotropic heterogeneities and approximately linear for the more lenslike heterogeneities. When the rms $V_P$ fluctuation is 1%, there is a stronger increase in $P_n$ amplitude as the averaging lengths increase in aspect ratio, with the appearance of a peak around 900 km distance. In the 2% rms $V_P$ variation cases, there is less variation with aspect ratio, and
the amplitudes at distances from approximately 400–900 km increase, effectively smoothing out the local amplitude peak at 900 km, as is apparent in the 1% cases. The calculations for 2% rms $V_p$ heterogeneity approach a linear (power-law) trend in log-amplitude versus log-distance, with the slope becoming more steeply negative with increasing aspect ratio of the mantle lid heterogeneity. It might be tempting to view these results as confirmation of the validity of the power-law geometric spreading approximation, but these modeling results correspond to a specific Earth model, as would any linear power-law fit to them. Furthermore, if a log-linear fit to any specific model case was specified, it would not work for most other cases—it would be specific to a single Earth structure or a narrow suite of structural configurations, the fine-scale of which is unconstrained in the actual Earth.

As Figure 6 and Figure 8 clearly demonstrate, $P_n$ amplitudes in most model cases increase in response to a heterogeneous mantle lid. The degree to which the amplitudes increase and approach a log-linear trend varies from case to case and by windowing method, as does the rate of amplitude decay. If $P_n$ scattering attenuation were to be included in an apparent attenuation model instead of being inadvertently removed by the geometric spreading approximation, it might be possible in some regions of strong scattering to produce models with negative apparent attenuation. However, Yang et al. (2007) applied their BEM geometric spreading correction to a large data set for Eurasia and obtained very reasonable (nonnegative) values, so it seems likely that this would only be the case for very extreme cases.

Each of the amplitude values shown in Figure 6 and Figure 8 are the average of five individual simulations with random configurations for a given set of parameters, thus the overall behavior is not the effect of any specific structural configuration. For example, the peak in $P_n$ amplitude near 900 km for $V_p$ fluctuations of 1% (Fig. 8b,e,h) is likely the result of a complex relationship between the strength of the $V_p$ heterogeneities and the angle at which $P_n$ energy penetrates into the uppermost mantle. Mantle lid $V_p$ heterogeneities of $\sim$1% are favorable for concentrating horizontally refracted energy, particularly when higher aspect ratio heterogeneities are present. Stronger heterogeneities do this effectively with little dependence on the shapes of the heterogeneities. Energy that penetrates into the lid with higher angles of incidence is unlikely to be trapped by these random heterogeneities unless very strong features are immediately encountered. The heterogeneous structures thus capture a limited range of ray parameters, and the increase in amplitude at 900 km represents a tuning of the refractions captured for a specific level of heterogeneity for 1-Hz waves. This interaction between angle of incidence and strength of velocity fluctuation likely underlies why only the strongest $V_p$ heterogeneities have strong effect on $P_n$ amplitudes at all ranges (Fig. 8c,f,i) and why the higher aspect ratio heterogeneities (Fig. 6c,d) have the highest $P_n$ amplitude levels for all distances. The case of $P_n$ amplitude calculated in the short fixed time window (Fig. 8g,h,i) is an exception to this pattern, as $P_n$ has the lowest amplitudes at distances greater than 1000 km for $V_p$ fluctuations of all aspect ratios, which become increasingly lowered with heterogeneities of increasing aspect ratio. This is the result of the short window excluding all of the $P_n$ coda following the initial $P_n$ arrival, and reflects the shift in energy distribution resulting from increased mantle heterogeneity. The longer time windows (group velocity and fixed window) show increased amplitude with higher aspect ratio because they are dominated at greater distances by the scattered $P_n$ energy that
Figure 8. Ensemble-averaged $P_n$ amplitudes as a function of distance for all configurations of random volumetric mantle heterogeneity (Fig. 6), grouped by percent rms fluctuation and windowing method. (a–c) Amplitudes calculated with a 9-s fixed time window; (d–f) amplitudes calculated with a group velocity window (8.2–7.6 km/s); and (g–i) amplitudes calculated with a short (3-s) fixed window positioned based the onset of $P_n$. $P_n$ amplitude for the BEM is shown (diamonds) for comparison to heterogeneity with horizontal and vertical averaging lengths of: $10 \times 10$ km (squares), $20 \times 10$ km (triangles), $20 \times 6$ km (×s), and $40 \times 3$ km (asterisks) for (a, d, g) 0.5% rms $V_p$ fluctuations; (b, e, h) 1% rms $V_p$ fluctuations; and (c, f, i) 2% rms $V_p$ fluctuations.

is captured and returned to the surface by the heterogeneous velocity structure, whereas the short window shows greatly decreased amplitude of the initial $P_n$ arrival at greater distances. Regardless of the windowing scheme used, the strongest heterogeneities with the highest aspect ratios have the most dramatic effects on $P_n$ amplitude. The specific geometric spreading behavior of the BEM or the 1D gradient models is clearly a result of rather delicate energy partitioning between diving energy and the suite of multiply reflected underside Moho reflections that is corrupted by the presence
Effects of 2D Random Velocity Heterogeneities in the Mantle Lid and Moho Topography

135

mantle lid propagation and whispering gallery development, near the source and the receiver, as well as throughout the mantle lid propagation and whispering gallery development, should yield $P_n$ sensitivity to the Moho roughness. Models with large-scale variations in crustal thickness are available, and one can make geometric spreading calculations for a specific deterministic model if the structure is known; however, relatively little constraint exists on the spectrum of small-scale Moho irregularities likely to affect any specific path. Once a model structure is known, one can make geometric spreading calculations for a specific deterministic model if the structure is known; however, relatively little constraint exists on the spectrum of small-scale Moho irregularities likely to affect any specific path. For example, model results for a 1-s dominant period. The main difference between the distributions is that the exponential model is much richer in small-scale structure and produces longer persisting coda than the smoother Gaussian model.

We created velocity models with Moho irregularity having four different averaging length scales, with random horizontal length scale exponentially distributed about distances ranging from 20 to 160 km. Topography of the Moho boundary is modeled with 1% ($\pm 0.4$ km), 3% ($\pm 1.2$ km), or 5% ($\pm 2.0$ km) rms fluctuations for each of the distance scales. There are few observational constraints on the appropriate statistics for Moho irregularities in general, but these values are representative of fluctuations in reflection seismology imaging. The average Moho depth always remains 40 km, as in the BEM. Results of these simulations are shown in Figure 9, grouped by horizontal averaging length, and in Figure 10, grouped by rms height of Moho topography and windowing scheme. These are again ensemble average values from five different model realizations for each set of random parameters. In Figure 9a, for 20-km averaging length, the 3% height variation actually reduces the average $P_n$ amplitudes to below those for the BEM. The strongest positive perturbations to $P_n$ amplitudes relative to the BEM result from Moho variations distributed around the 40-km averaging length scale (Fig. 9b). For this horizontal averaging scale, the amplitude scatter becomes significant for 5% rms heterogeneity in the Moho topography. There is only minor sensitivity to the choice of horizontal averaging function for values larger than 40 km.

Figure 10 displays the effects of percent topography fluctuation more clearly, with 1% rms variability having little effect on $P_n$ amplitudes for any of the horizontal length scales simulated and 5% height fluctuation clearly disrupting the $P_n$ geometric spreading pattern relative to the BEM. The disruption mainly involves increased amplitude scatter without a systematic shift in the pattern, as was found in the volumetric heterogeneity simulations. Moho topographic variability of 3%–5% is required for all horizontal averaging scales in order to produce significant scatter in $P_n$ amplitudes. We attribute most of this amplitude variability to the sensitivity of the ray paths for $P_n$ as it first encounters and refracts at the Moho, although progressive scattering along the horizontal path contributes overall. The consistency of the $P_n$ amplitude calculations (relative to the homogenous model case) across all windowing methods confirms that fluctuations in amplitude are mostly controlled by the first refraction of $P_n$ at the Moho, and the effect of forward-scattered wave energy along the Moho on $P_n$ amplitudes is relatively minor, as compared to the amplitudes calculated in the model cases of mantle heterogeneity, which are dominated by scattered $P_n$ energy. The amount of forward-scattered $P_n$ energy depends on the direction that the boundary is dipping below the source, as well as how steeply it is dipping, and even with five realizations being averaged, there is rapid fluctuation in amplitudes that emerge at various distances. When the synthetic waveforms are examined (Fig. 11), one can see that Moho variability strongly affects $P_m$ arrival time (mainly because the crustal thickness is locally thicker or thinner near the source and receiver, depending on the specific Moho topography) but also results in moderate scattering of $P_n$ energy, as evidenced by the enhanced $P_n$ coda following the first arrival. Given that all of the scattering in these models must originate at the Moho, the coda levels are influenced by both near-source and near-receiver scattering along with scattering of energy out of the whispering gallery of underside Moho reflections.

Discussion and Conclusions

The behavior of $P_n$ geometric spreading is subtle and complex, even for simple 1D structures (e.g., Menke and Richards, 1980; Sereno and Given, 1990; Yang et al., 2007). The calculations in this study provide a basic explanation for the high variance observed in $P_n$ rms amplitudes from the ANOVA analysis (Fisk et al., 2008) mentioned in the Introduction. Variable 1D and 2D structures readily produce nonlinear, frequency-dependent variations superimposed on the already complex behavior for the simple BEM. This variability is somewhat analogous to the higher variance found for $m_b$ measurements for teleseismic $P$ waves versus the less-variable $m_b(L_g)$ for complex, but heavily path-averaging regional $L_g$ phases; the energy in the $P_n$ window is comprised of a fairly narrow range of ray parameter arrivals that are very sensitive to the structure along any specific path. This study demonstrates that the effects of variable 1D mantle lid $P$ velocity gradients preserve the basic functional
form of the frequency-dependent complexity of the constant-velocity mantle lid model, with positive gradients causing a shift of amplitude curves toward higher frequencies for the zero-gradient BEM. The effects are so strong that specific frequency-dependent coefficients would need to be determined for the spreading model for any specific gradient. The nonlinear sensitivity raises questions about the choice of spreading model given typical situations where there is

Figure 9. Ensemble-averaged $P_n$ amplitudes plotted as a function of distance for all configurations of random Moho topography heterogeneity, grouped by horizontal averaging length scale, $A_x$. $P_n$ amplitude for the BEM (diamonds) are compared to amplitudes for simulations with 1% rms (squares), 3% rms (triangles), and 5% rms (×s) Moho depth variations for horizontal averaging length scales of (a) 20 km, (b) 40 km, (c) 80 km, and (d) 160 km.
little or no information on the local mantle lid velocity gradients. Of course, if the data for a given region are sufficient to constrain the mantle lid velocity gradient (as may be the case across the Russian platform; e.g., Morozova et al., 1999), specific spreading computations can provide a local reference behavior relative to which attenuation estimates can more reliably be determined. In practice, this is still challenging, given the degenerate nature of layered versus gradient model fits to first-arrival times.

Computations with lateral volumetric heterogeneities in the mantle lid and statistical irregularities in Moho topography add significant complexity to the general problem of

Figure 10. Ensemble-averaged $P_n$ amplitudes plotted as a function of distance for all configurations of random Moho topography heterogeneity, grouped by percent rms depth fluctuation and windowing method. (a–c) Amplitudes calculated with a 9-s fixed time window; (d–f) amplitudes calculated with a group velocity window (8.2–7.6 km/s); and (g–i) amplitudes calculated with a short (3-sec) fixed window positioned based the onset of $P_n$. $P_n$ amplitude for the BEM (diamonds) are compared to horizontal averaging length scales of 20 km (squares), 40 km (triangles), 80 km (×s), and 160 km (asterisks), for percent rms depth fluctuations of 1% for (a, d, g); 3% for (b, e, h); and 5% for (c, f, i).
approximating $P_n$ geometric spreading. Our 2D finite-difference simulations indicate that the precise shape of the geometric spreading behavior for 1D models can be disrupted by even moderate levels of lateral heterogeneity, and the behavior for 1-s period waves moves toward a power-law representation as the level of volumetric heterogeneity increases and as the horizontal-to-vertical aspect ratio of the heterogeneities increases. This behavior may provide a rationale for using a conventional power-law type geometric spreading correction, but how closely such a model approximates actual $P_n$ behavior depends critically on the level and type of heterogeneity present in the real Earth. For example, simulations with subtle mantle heterogeneity (0.5% rms $V_P$ fluctuations; Fig. 8a,d,g) indicate that $P_n$ geometric spreading can be more closely approximated by the 1D homogeneous BEM relations determined by Yang et al. (2007), while simulations with stronger mantle heterogeneity (Fig. 8c,f,i) can be better approximated by a power-law type correction, at least for 1 Hz, although a single linear decay rate would not fit all cases approaching a log-linear decay. The $P_n$ amplitudes of simulations through velocity models between these two end members cannot be particularly well fit by either type of geometric spreading correction.

For rough Moho topography heterogeneity, departures from the 1D $P_n$ geometric spreading pattern prove much less significant. Models with 3%–5% topographic fluctuations over scale lengths of approximately 80 km seem to have the greatest effect on $P_n$ amplitude near 1 Hz, but, for nearly all simulations, the BEM geometric spreading correction of Yang et al. (2007) would be an acceptable reference model.

These results present something of a conundrum: the 1D BEM structure considered by Yang et al. (2007) is very simple but produces complex, frequency-dependent, geometric spreading behavior. Our results show that the behavior is nonlinearly sensitive to the model parameters, and the 1D gradient models indicate that this extends to the frequency dependence. However, our 2D models with the strongest, but still geologically plausible, mantle heterogeneities produce relatively simple patterns of geometric spreading, at least for 1 Hz. It is likely that $P_n$ simulations over a broader frequency band and for 3D structures would also have high sensitivities to different scale lengths and velocity contrasts as a function of frequency. The effects of 2D or 3D $V_P$ heterogeneity on shorter period signals have not been quantitatively assessed for the long propagation distances considered here; doing so presents formidable computational challenges. We speculate that eventually simulations in heterogeneous 3D structures will enhance the tendencies manifested in our 2D calculations.

Prior studies of $P_n$ attenuation efforts have usually assumed simple frequency-independent power-law geometric spreading behavior for $P_n$ (e.g., Sereno et al., 1988; Pasyanos et al., 2009), which is not a realistic form of spreading for any known 1D (i.e., laterally homogeneous) velocity model. This assumption may actually yield reasonable results to the extent that the specific power-law corresponds to a fairly high degree of uppermost mantle velocity heterogeneity in the real 3D Earth, which may be evidenced in the data by the amount of coda following the initial $P_n$ arrival (Fig. 7 and Fig. 11). Of course, the precise spectrum and statistical distribution of real mantle lid heterogeneities is not known.

Figure 11. Finite-difference synthetic $P_n$ seismograms generated in (a) BEM; (b) a velocity model containing 3% rms random Moho depth variation with horizontal averaging scale length of 40 km; and (c) a velocity model with 5% rms random Moho depth variation along horizontal averaging scale length of 40 km. Each trace is normalized on the largest amplitude.
for any specific path, just as the precise best 1D lid velocity gradient is not reliably known, so it would be difficult to estimate a particular small-scale structural configuration with a power-law geometric spreading approximation that has an appropriate decay rate. That situation is unlikely to change in general. Conventionally, seismic reference models used for computing geometric spreading of seismic signals with a limited ray parameter range are selected to be very simple structures rather than such complex (and unspecified) structures that the wave field is scattered to the point of allowing an energy flux power-law assumption to be made (as is quite reasonably done for the multiray parameter phases like $L_g$). It should be recognized that if one assumes (arbitrarily) a power-law form of geometric spreading for $P_n$, the underlying reference Earth model is explicitly not a conventional 1D constant-velocity mantle model or even a constant lid-gradient model, but is intrinsically a statistically heterogeneous model with unspecified properties. It is rather uncomfortable to invoke such a model. In addition, we recognize that even though scattering due to velocity heterogeneities is an elastic effect, the single-wavefront definition of geometric spreading is often assumed, and elastic scattering effects are considered part of the apparent attenuation.

Our recommendation is that, in the absence of specific regional constraints on the mantle lid velocity gradient and on small-scale heterogeneity in the lid and in Moho topography, the best choice for $P_n$ geometric spreading is the frequency-dependent version found for the BEM by Yang et al. (2007). This has the primary merit of having a known, simple 1D Earth model associated with the geometric spreading behavior and a parameterization that can accommodate the nonlinear response of $P_n$ to large-scale changes in the Earth velocity model (e.g., mantle gradient), which is consistent with standard seismological modeling practices. If this model is used in determining an apparent attenuation structure, the attenuation model will very likely have explicit frequency dependence. As is always the case, deficiencies in the spreading corrections will likely project into the attenuation model, but, as long as the resulting attenuation model is viewed as a collective parameterization of intrinsic $P$ wave attenuation, correction for scattering attenuation (which could involve negative $Q$s), and frequency-dependent errors in the geometric spreading relations (which, based on our modeling results, could approach a factor of 10 for any relation chosen), at least the attenuation model is defined relative to a known elastic Earth model. With most applications involving a convolution of the geometric spreading and attenuation models, the trade-offs will not bias source strength estimates; but one must exercise care in specific interpretation of attenuation parameters in terms of Earth properties. Decoupling geometric spreading and attenuation can only occur with a priori constraint on the structure (e.g., using travel-time curves to constrain the upper-mantle velocity gradient), but small-scale structure of the type modeled here is largely unconstrained. Investigators should be aware that by adopting a specific geometric spreading approximation, they are also adopting assumptions about Earth structure. It is therefore imperative that investigators document the assumptions inherent in any geometric spreading approximation chosen in attenuation modeling studies to prevent assumptions specific to a given study region from being erroneously applied to other regions. It could be argued that, with amplitude variations between a heterogeneous Earth model and a homogeneous Earth model as large as they are, an approximation of $P_n$ geometric spreading should in some way account for scattering due to small-scale heterogeneity. This is problematic, as the small-scale heterogeneity is not known. The data can serve as somewhat of a guide, with observed $P_n-P_s$ coda variations indicating a strong or weak scattering domain, but quantitatively linking the data to a specific heterogeneous model remains beyond our scope as a community. By continuing efforts to constrain the mantle lid velocity gradient empirically, or to use independent knowledge of the Moho topography, calculations of geometric spreading for corresponding models can be performed to enable better resolution of true attenuation effects.

It is clear from these simulations that either the 1D BEM geometric spreading corrections of Yang et al. (2007) or any power-law correction will have significant uncertainty in approximating the actual geometric spreading of $P_n$ for specific data sets; neither is ideal for all cases. The extent of this uncertainty is difficult to quantify, given the lack of constraints on the small-scale seismic velocity heterogeneity spectrum within the real Earth. Thus, propagating realistic uncertainties in the geometric spreading model into formal uncertainties in attenuation model estimates remains very difficult. We seek to perform future calculations for 3D structures with greater bandwidth, with the goal of providing useful quantification of the uncertainties in attenuation modeling due to geometric spreading approximation.

Data and Resources

All data used in this paper came from published sources listed in the references.

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