Rupture speed dependence on initial stress profiles: Insights from glacier and laboratory stick-slip

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Highlights:

• Slow slip events similar to natural ones on the Whillans Ice Plain can be created from a simple, homogenous laboratory model of plastic sliding blocks.

• Both the laboratory and natural events have low rupture velocities, which increase with increase of applied stress.

• The slip-predictability requires variable system strength that can be explained by a dependence on the loading conditions.
Abstract:

Slow slip events are now well-established in fault and glacier systems, though the processes controlling slow rupture remain poorly understood. The Whillans Ice Plain provides a window into these processes through bi-daily stick-slip seismic events that displace an ice mass over 100 km long with a variety of rupture speeds observed at a single location. We compare the glacier events with laboratory experiments that have analogous loading conditions. Both systems exhibit average rupture velocities that increase systematically with the pre-rupture stresses, with local rupture velocities exhibiting large variability that correlates well with local interfacial stresses. The slip events in both cases are not time-predictable, but clearly slip-predictable. Local pre-stress may control rupture behavior in a range of frictional failure events, including earthquakes.

1. Introduction

The physical controls on rupture and slip velocities are a persistent and perplexing issue in rupture mechanics. Once a crack exceeds a critical length, it generally accelerates to the dynamically limited Rayleigh wave velocity, which is between 90-95% of the shear wave velocity. Most natural slip events, like earthquakes, follow this expectation and rupture at a rate approaching, and occasionally surpassing the shear wave speed of the rocks through which they propagate (Kanamori and Brodsky, 2004). However, since the discovery of slow slip events over a decade ago, we now also understand that faults can fail with rupture velocities significantly below shear wave velocity and slip velocities that only exceed the long-term plate motion by an order of magnitude (Dragert et al., 2001; Peng and Gomberg, 2010; Beroza and Ide, 2011). Glaciers can also slip in transient events that can have rupture velocities ~10% of the local shear
wave velocity and slip velocities an order of magnitude above the long-term slip velocities (Wiens et al., 2008; Winberry et al., 2009). For this new class of slow-slip events, the question remains what controls the rupture and slip velocities.

The case of Whillans Ice Plain, Antarctica is particularly important in the study of slow rupture as this natural system produces frequent, large, and well-documented slip events. Each day, as many as two slip events occur beneath the Whillans Ice Plain over a region ~100 km long (Fig. 1) (Bindschadler et al., 2003; Wiens et al., 2008; Winberry et al., 2009; 2011; Walter et al., 2011). During periods between fast slip events, the ice flows downhill at a steady rate of less than 0.001 m/min, then suddenly increases its speed by more than an order of magnitude at tidal periods, slipping up to ~0.5 m in ~30 min. Individual rupture fronts propagate with average velocities on the order of ~100 m/s (Wiens et al., 2008) from two distinct nucleation regions that vary with the tide (Winberry et al., 2009; Pratt et al., 2014).

Early work attributed the anomalously low rupture velocity of the Whillans events to the low local shear wave velocity of till (Bindschadler et al., 2003). Surprisingly, more recent studies have established that the rupture velocity varies systematically with loading over the tidal cycle and therefore cannot be controlled solely by the material properties (Wiens et al., 2008). Total slip, recurrence time and nucleation location also covary with rupture velocity and the tidal cycle (Winberry et al., 2009; Walter et al., 2011; Winberry et al., 2011).

The systematics of the Whillans Ice Plain system suggests a major role for the variations in tidal loading in controlling the final rupture. In this study, we refine the fundamental observational constraints on the propagation of slow slip at Whillans Ice Plain with new field observations and proceed to construct a laboratory analog model of the slip events that reproduces the fundamental dynamics.
2. Rupture Propagation

2.1. Whillans Ice Plain stick-slip

To study the rupture process in detail, we installed an array of 11 broadband seismometers on the Whillans Ice Plain for a period of 1 month during the 2010 field season (Fig. 1). We use this data to locate the initiation points of slip events and measure the mean rupture velocity in a slightly different way than was possible with previous studies. The new local seismic data allows us to both track the rupture across the network and to use beamforming to improve the location of the end of the rupture, which is critical to improving the rupture velocity estimates.

We estimate the mean rupture velocity by measuring the travel time between the nucleation point and the generator of a distinct barrier phase. The approximate azimuth of the barrier phase relative to the seismic network is determined by finding the direction that focuses the observed radiated energy, i.e., beamforming (Rost and Thomas, 2002). Estimates of azimuth indicate a source in the direction of Phase B on Fig. 1, with a wave speed at approximately the surface wave speed for the region. The estimate of the barrier phase location provides a fixed point for rupture propagation and we use this estimate to calculate the mean rupture velocity, similar to previous work (Walter et al., 2011) (See supplementary material).

Loading of the Whillans Ice Plain basal interface is both temporally and spatially variable. It varies temporally, as we find that the location of the initiation point varies with tidal loading, consistent with previous work (Bindschadler et al., 2003; Winberry et al., 2011). Those events that begin at low tide (first arrival at station S03, blue circle in Fig. 1) nucleate closer to the grounding line than those that begin at high tide (first arrival at station S08, red circle in Fig.
The tidal height in the Ross Sea is calculated using the model of Padman et al. (2003), where the sign of the vertical tide indicates whether an event initiates at low (negative) or high (positive) tide. The tides act as a small-amplitude perturbation (Bindschadler et al., 2003) to the gravitationally-driven flow of the Whillans Ice Plain as it flows into the Ross Ice Shelf.

Loading also varies spatially, as some previous studies suggest that basal interface conditions may play a role in the location of the nucleation (Winberry et al., 2009; Pratt et al., 2014). In order to highlight regions of the Whillans Ice Plain where basal coupling may be high, we assimilate GPS datasets from two separate GPS campaigns collected during an experiment in 2004-2005 (Winberry et al., 2009) and a project investigating subglacial lake activity spanning the years 2007-2012. For each experiment, we choose a single 14-day period when the most stations are operating, and compute the seismic coupling. We define the local seismic coupling at a station as the total displacement that occurs during the slip events over a 14-day period divided by the total displacement that occurs at each GPS station over that time period. This is analogous to the seismic coupling defined for subduction zone faults (Ruff and Kanamori, 1983). For the earlier data (Winberry et al., 2009), we compute the seismic coupling spanning days 332-346 in 2004 and for another experiment, we compute the seismic coupling spanning days 339-353 in 2008. We assume that the temporal variation in coupling is negligible over the 4 years between deployments and that combining 14 days of data effectively averages the coupling ratio over multiple slip events. Seismic coupling on the Whillans Ice Plain is shown in Fig. 2A.

Once ruptures nucleate, the events on the Whillans Ice Plain include four major features. First, the mean rupture velocities, $V_{\text{mean}}$, of Fig. 3A are significantly slower (100-240 m/s) than both the shear wave velocity in ice (1900 m/s) (Blankenship et al., 1987) and plausible values of
the wave velocity for the underlying sedimentary basins at a scale relevant to rupture propagation (~800-2800 m/s) (Trey et al., 1999).

Secondly, $V_{\text{mean}}$ co-varies with the time since the previous event (Fig. 3A) and slip measured during each event (Fig. 3B). Previous analysis utilized a combination of distant seismic arrivals and/or GPS data to suggest such a relationship (Walter et al., 2011; Winberry et al., 2011). Here we provide an assessment using local seismic data (Fig. 3A). The correlation implies that the $V_{\text{mean}}$ co-varies with the mean basal shear stress accumulated in the interseismic period up to the time of slip nucleation (Bindschadler et al., 2003; Winberry et al., 2009).

Thirdly, the rupture velocity varies significantly along the rupture path (Fig. 2). This is most easily seen by examining the high and low tide events separately. Figs. 2B and 2C indicate that both groups of events rupture at high velocities near the nucleation site. Furthermore, low tide events, which rupture over the same region, rupture at different velocities than high tide events. Fig. 4A shows apparent rupture speed (the rupture speed is apparent because it is calculated along one dimension, between observation points) from either S03 or S08 (depending on low or high tide event) to S11 for a series of events. Slip events that begin at low tide, propagate at much faster speeds towards S11 than high tide events. Within the 2010 seismic network, the low tide events have significantly higher initial rupture speeds, yet slow down quickly to average velocities that are lower than those for high tide events (Fig. 3A).

Fourth, rupture events nucleate in the strongly locked, and therefore highly loaded, regions of the Whillans Ice Plain (Winberry et al., 2009). The spatially interpolated pattern of seismic coupling (Fig. 2A) corresponds directly to the rupture pattern as illustrated by the average arrival times (Figs. 2B, 2C). The seismic coupling maps the inter-event strain field,
indicating that areas of high coupling are relatively more loaded at the beginning of the slip event. Slip nucleates in the area with highest loading.

2.2. Laboratory experiments

In the laboratory analog model, we slide one poly(methyl-methacrylate) (PMMA) block past another PMMA block that is held fixed (Fig. 1C). An applied force, $F_S$, displaces the bottom of the lower block and is gradually increased until a stick-slip event occurs along the 200 × 6 mm interface. Strain gauges at 7-10 points adjacent to the interface locally measure all 2D stress components. In addition, we actively monitor ruptures along the interface by measuring the entire real contact area every 1.7-20 $\mu$sec. This allows us to track the instantaneous velocity of a typical rupture front as a function of its length together with the temporal evolution of the real contact area as the rupture progresses.

The two transparent poly(methyl-methacrylate) (PMMA) blocks (shear wave speed, $c_S=1370$ m/s) are pressed together with a uniform normal forces $F_N$ that varies between 3000-6000 N. The dimensions of the top (bottom) blocks were $x=200, y=6, z=100$ mm (300 x 60 x 30mm), where $x, y$, and $z$ are, respectively, the propagation, thickness, and normal loading directions. The contacting faces of the two blocks are first machined to be flat to within 5 $\mu$m and then roughened to an RMS roughness of approximately 1 $\mu$m, thereby forming a rough but uniform contacting surface. The real area of contact is measured with a laser light sheet (see supplementary information).

The bottom block is mounted on a low-friction translational stage constrained to move along the $x$ axis. Once $F_N$ is applied, the shear force, $F_S$, is applied to the bottom block in the negative $x$ direction via a load cell of stiffness $10^7$ N/m at a continuous loading rate of 10–20
N/s. The upper face of the top block is rigidly held in place, so translation of the bottom block imposes elastic stresses on both blocks until rupture takes place along the frictional interface. A rigid stopper is optionally pressed against the leading edge of the upper block, creating a loading effect that locally increases the shear stress in the vicinity of the contact point.

We use the laboratory stick-slip sequences to recreate the four major rupture propagation observations from the field. We perform two sets of experiments (Fig. 1C): uniform application of $F_S$ to the edge of the bottom plate (Fig. 3C), which produces an approximately uniform shear stress along the interface, and preferential application of $F_S$ to the leading edge of the top plate (Rubinstein et al., 2007; Rubenstein et al., 2011) (Fig. 1C with stopper), which produces a high stress concentration near the plate edge (Fig. 4C).

We compare the results of these experiments to the field observations. Firstly, anomalously low $V_{mean}$ occur in the lab (Fig. 3C). Average rupture velocities of as low as 100 m/s are observed. Since the shear wave velocity is 1370 m/s, these correspond to $V/c_s<0.1$, as observed in nature. An important distinction between the lab and nature is that laboratory slip events that are loaded sufficiently to first slowly elongate to a transition length and, afterwards, rapidly accelerate to velocities that can either approach or exceed the Rayleigh wave speed. The Whillans Ice Plain slip events appear never to have sufficient loading to reach this transition length and thus remain at relatively low rupture velocities.

Secondly, the experiments (Fig. 3C) show a systematic increase of $V_{mean}$ with the inter-event duration like the Whillans Ice Plain (Fig. 3A). Since applied $F_S$ is increased at a constant rate in the experiments, inter-event duration has a one-to-one relationship with force increase before each event. In Fig. 3 we also find that displacement measurements (slip) at the trailing edge of the shear device (Fig. 3D) systematically increase with the $V_{mean}$. 
Thirdly, when applying the non-uniform stress configuration (application of rigid stopper shown in Fig. 1C), we see that the experiments are able to reproduce the observations of fast initial rupture, followed by slow rupture. As shown in Fig. 4B, once nucleation barriers are overcome, ruptures nucleate with high initial velocities that are followed by slow rupture propagation.

Fourthly, the laboratory events with non-uniform stress configuration nucleate at the edge adjacent to the rigid stopper, which is an area of highest shear loading locally. Specifically, the ratio of shear stress to normal stress is highest near the nucleation point (Fig. 4C). This is consistent with previous similar experiments (Ben-David et al., 2010). As discussed above, the rupture slows as it propagates away from this high stress region.

The relatively simple frictional laboratory experiments presented in Figs. 3 and 4 capture the major features of the Whillans Ice Plain observations, including a rupture velocity that co-varies with the loading conditions and is significantly lower than the shear wave velocity.

3. Discussion

3.1. Role of loading in determining failure conditions and rupture dynamics

We have established that the model system reproduces the essential features of the natural system: low mean rupture velocity, co-variance between rupture velocity and loading time, decreasing rupture velocity during propagation and nucleation at the site of highest loading.

The most important conclusion from this work is that slow slip events can be created due to frictional failure with no further complicating factors. Homogeneous, solid sliding plastic blocks reproduce the essential features of the Whillans Ice Plain. No more complex model is necessarily required.
We speculate that the success of the simple model for the Whillans Ice Plain stems in part from the relative simplicity of the isolated stick-slip system. Tectonic earthquakes seldom occur in isolation and are constantly influenced by the motion of neighboring faults. In the absence of these complications, a laboratory model closely related to a classical slider-block does an excellent job of reproducing behavior, even for slow-slip events.

A secondary conclusion of this study is that the observed systematic variations in rupture velocities are governed by the applied stresses. In the framework of fracture mechanics, this is an expected result. It is well-established that a crack can propagate at applied stresses, $\Delta \tau$, that are much lower than material yield strengths (e.g., Bonamy, 2009; Lengline et al., 2011). Here, we define $\Delta \tau$ as the difference between the externally applied stress at the time of nucleation and the dynamic stress during failure (e.g. Rice, 1980). All else being equal, the rupture velocity increases with the amount of released elastic energy, which is proportional to $\Delta \tau^2$. Rupture and slip velocities for theoretical cracks do not, necessarily, reach the limiting rupture velocity. What determines the applied stress, $\Delta \tau$, for the onset of friction instabilities? The answer lies in the mechanism for rupture nucleation, which is not presently understood (e.g. Ben-David et al., 2010). If the nucleation condition were governed by a material-characteristic static friction coefficient, $\Delta \tau$ would be independent of the tidal loading conditions. The results presented in Fig. 3 suggest that this is not the case and that the applied stress at initiation, $\Delta \tau$, is varying among the events.

In earthquake physics, events where the amount of slip is deterministically controlled by the loading since the last event are termed slip-predictable (Fig. 5; Shimazaki and Nakata, 1980). As shown in the hypothetical illustration (Fig. 5A), the shear stress at failure is variable for slip-predictable earthquakes. The Whillans Ice Plain behavior over multiple stick-slip cycles
is consistent with a slip-predictable model (Fig. 5C). Therefore, the shear stress at failure is variable. The question remains: what controls the failure stress and nucleation timing?

Winberry et al. (2009) suggested that the peculiar condition of ice near its melting point could produce a sufficiently rapid (nonlinear) healing process to generate variability in the yield strength of the bed to explain the observations. However, we obtain the same type of rupture variability in the laboratory with a simple PMMA system. Therefore, interevent healing is not likely the answer.

An alternative possibility is that material heterogeneity on the base allows different regions, with differing frictional thresholds, to be activated as the ice sheet flexes through the tidal cycle (Fig. 2A). Material heterogeneity is an attractive hypothesis as a control on event nucleation (Pratt et al., 2014), however, the laboratory experiments show similar behavior even in the absence of spatial variability. The continuous variation of rupture speed with force before the event in Fig 3C demonstrates that material heterogeneity is not required. The variability of the high tide events in Fig. 3A (inter-event durations ~0.55-1.05 d for similar tide level) also suggests that there is more variability in the natural rupture velocities than can be explained by invoking two different frictional thresholds at the two nucleation sites.

A third possibility is that averaging the stress for failure over the entire interface misses an important aspect of the loading geometry. As the ice sheet (or plastic sheet) flexes, the bed experiences a spatially complex field of concentrations of normal and shear stresses. Recent experiments (Ben-David and Fineberg, 2011; Passelègue et al., 2013) have shown that a constant value for the ratio of applied shear to normal stress on the entire block is a poor predictor of failure. The same result is shown in the current experiments. The coefficient of friction for the entire sample $\mu_s$ is defined as the ratio of applied normal force, $F_n$, to shear force, $F_s$, as defined
in Fig. 1. Fig. 3E shows a variation in $\mu_S$ that is much smaller than the nearly 100% variation observed under different loading conditions (Ben-David and Fineberg, 2011; Passelègue et al., 2013), but still resolvable.

Events with a higher average value of $\mu_S$ at failure have a higher value of $\Delta\tau$ and therefore are expected to have a higher mean rupture velocity $V_{\text{mean}}$. This expectation is met in the laboratory. The recorded $V_{\text{mean}}$ increases with $\mu_S$ (a measure of the globally applied stresses) and the data from experiments with different normal stresses collapses to a single trend (Compare Fig. 3E to Fig. 3C).

The correspondence between the initially applied stresses and rupture velocity also exists at local scales. At the Whillans Ice Plain grounding line, flexure may result in localized basal stress concentrations. Local GPS data collected across the grounding line 80 km from the study site during a few days in January 2011 shows that when the ice shelf level goes below the grounding line at low tides, its motion causes an upward flexing of the ice shelf within 5-10 km of the grounding line (Fig. 6). Unfortunately, the deployment is not adjacent to the nucleation points, but does capture the ice flexure across a grounding zone with similar ice thickness to the low tide nucleation area. Upward movement at the forebulge may indicate a normal stress reduction (e.g. Walker et al., 2013), though the normal effective stress is reduced only if the subglacial tills behave at least partially as a drained aquifer.

This effect and the estimation that local shear stress at low tide in this region has been estimated to be over 3 times higher than at high tide (Winberry et al., 2011), leads to the expectation that the local shear to normal stress ratio ($\tau/\sigma$) in the vicinity of the nucleation point should be significantly larger during low tides than at high tides and may explain the fast initial rupture for the low tide events (Fig. 4A). High tide events are not likely influenced by elastic
flexure, as they nucleate nearly 30 km from the grounding line. Nucleation during high tides closer to S08 may also be influenced by high hydropotential where less liquid water would be available for lubrication (Winberry et al., 2011), bedrock, or other higher competence material that generate basal asperities, which act to localize strain.

A flexural geometry on the Whillans Ice Plain that results in localized high local shear to normal stress ratios ($\tau/\sigma$) where nucleation occurs during low tides, is in analogy with the laboratory measurements summarized in Fig. 4C. Here, the laboratory events nucleate at the edge of the apparatus, adjacent to the rigid stopper, and the rupture velocity during the event directly corresponds to local value of $\tau/\sigma$. Thus in both cases, events nucleate in regions where the basal interfacial stresses are higher (local $\tau/\sigma$), relative to other areas on the frictional surface.

4. Conclusions

The success of the highly idealized system in generating the major features of Whillans Ice Plain slip events suggests a relatively simple path forward toward understanding the nature of slow slip. A laboratory experiment with no material heterogeneity, complex rheology, or phase changes has recreated the basic observations of slow rupture velocities that depend on loading and local stresses.

The slip-predictable events have a failure condition that varies from event to event depending on the loading conditions. Nucleation of Whillans Ice Plain seismic events occurs during both high and low tides at significantly different stress levels (Bindschadler et al., 2003; Winberry et al., 2009; Walter et al., 2011). This observation complements recent laboratory experiments demonstrating a large range of system-averaged stress ratios over which a frictional system can either be stable or unstable to rupture (Ben-David et al., 2010; Ben-David and
Once nucleation takes place, the rupture speed of stick-slip sliding is controlled by the local stress conditions at the sliding interface, in both the nucleation zone and as the rupture event propagates through other areas.

Frictional processes in fault zones may occur under similar physical conditions, as natural faults have large spatial variations in pre-rupture stress distributions (Lay and Kanamori, 1981). These results imply that loading geometry can have significant effects on resulting rupture dynamics in the absence of any complicating factors.

5. References


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provided by the PASSCAL facility of the Incorporated Research Institutions for Seismology (IRIS) through the PASSCAL Instrument Center at New Mexico Tech. Seismic data collected during this experiment is available through the IRIS Data Management Center. The facilities of the IRIS Consortium are supported by the National Science Foundation under Cooperative Agreement EAR-0552316 and by the Department of Energy National Nuclear Security Administration.
Fig. 1. Whillans Ice Plain location and illustrations of loading conditions. (A) Locations of stations deployed during the 2010 field season and the major geographic features: the grounding line, Ice Raft A and Subglacial Lake Whillans. The rupture front is initially observed at station S03 for low tide events. The rupture front is initially observed at station S08 for high tide events. Solid and dashed lines are used to indicate distances over which rupture speeds are calculated. Grounding line is the Antarctica Surface Accumulation and Ice Discharge (ASAID) grounding
line (Bindschadler et al., 2011). Loading configuration prior to slip and propagation of rupture fronts during slip events for the two systems: (B) Whillans Ice Plain bidaily stick-slip events and (C) laboratory measurements of stick-slip at an interface between two poly(methyl-methacrylate) (PMMA) blocks. The optional stopper is used to create spatially nonuniform stresses along the interface (cf. Fig. 5B-C) that are peaked near the stopper. The oblique gray shading indicates light that illuminates the interface for rupture front detection by a high-speed camera.
Fig. 2. Inter-event loading and slip event nucleation geometry. Black triangles indicate locations of broadband seismometers. (A) Seismic coupling inferred from the continuous GPS stations (white circles). The local seismic coupling is the total displacement that occurs during the sum of all slip events over a 14-day time period divided by the total displacement that occurs at each GPS station over the same 14-day period (See Supplementary Materials for further
(B) Low-tide event average arrival times and (C) high-tide event average arrival times based on the 2010 field season seismic network (black triangles). Time 0 in both (B) and (C) is the arrival time at the closest station. The contours are interpolated slip front arrivals at the discrete seismic stations. The x- and y-axes of (A-C) represent the South Polar Stereographic grid (km).
Fig. 3. Average rupture speed estimates over the glacier (A-B) and laboratory (C-E) scales.

(A) Co-variation of Whillans Ice Plain average rupture speed with the inter-event duration (recurrence interval). Note that the colors correspond to the tide level at the time of nucleation.

(B) Co-variation of Whillans Ice Plain average rupture speed with event displacement, as measured at a GPS station co-located with S10 near the initiation point. (C) PMMA plastic
block laboratory average rupture speed co-variation with applied shear force before the event.

(D) Co-variation of average rupture speed with laboratory event displacement, measured at the trailing edge of the shear device (E) Average rupture speed as a function of measured ratio

\[ \mu_S = \frac{F_S}{F_N} \] for each event. This normalization by the non-constant effective friction coefficient reduces the scatter. Colors/symbols in Fig. 3C-E are as noted in Fig. 3C and the results in Fig. 3C-E are obtained by the loading described in Fig. 1C without the optional stopper.
Fig. 4. Initial apparent rupture speed and average rupture speed measurements for each slip event. (A) For each Whillans Ice Plain event, average rupture speed (black) from Fig. 3A is connected by a vertical red line to apparent rupture speed near the nucleation point (red), with tide height at time of slip event initiation on the x-axis. (B) Laboratory measurements of average rupture speed (black) and initial rupture speed (red) when $F_S$ is preferentially applied to the sample edge with the optional stopper. The high initial rupture speeds correspond to regions of high local shear/normal stress ratios (C) A detailed comparison of the rupture velocity to the spatially dependent shear to normal stress ratio ($\tau/\sigma$) for the second event within the stick-slip sequence described in (B). (Inset) The real contact area as a function of time and space from which the rupture velocity measurements are derived. Each horizontal line depicts the change in the real contact area along the entire interface in x, after averaging in the thickness (y) dimension. Sequential lines were photographed at 1.7 $\mu$s intervals, and all measurements are normalized by the contact area at t=0. For each time t>0, a sharp drop in contact area (color) occurs at the tip of the rupture front.
Fig. 5. Schematic illustration of slip-predictable earthquake behavior (adapted from Shimazaki and Nakata, 1980) and observed horizontal displacement for Whillans Ice Plain over multiple slip events. (A) In the slip-predictable model for earthquake occurrence, an event does not occur at a constant threshold shear stress ($\tau_1$). Rather, after each event, the shear stress reduces to a minimum fault stress ($\tau_2$). If shear stress accumulates at a constant rate in between events, then the eventual magnitude of subsequent events can be predicted at any point in time. (B) Illustration of displacement through time shows a slip history that directly reflects the stress
drops in (A). (C) Horizontal displacement from GPS on the Whillans Ice Plain for multiple slip events suggests slip-predictable behavior.
Fig. 6. Flexure along the grounding line on the Whillans Ice Plain. (A) Schematic of flexure cross-section under a line load, including features of the flexure profile. (B) The elevation at the...
GPS station located at 24 km along the profile. (C) Elevations (with mean removed) of a set of five stations with lines connecting the stations at the same point in time, which demonstrates the flexure geometry across the Whillans Ice Plain grounding line. Vertical errorbars are shown for the first measurement as one standard deviation of vertical position signal, indicating a noisy signal. The average vertical position has been subtracted from the vertical position time series so that all stations plot horizontally on the same line; though there exists a permanent slope across the grounding line (Horgan and Anandakrishnan, 2006). (D) Map outline showing the seismic station (triangles) and the 5 GPS stations (inverted triangles) temporarily deployed across the grounding line that were used to measure (C).
Supplementary material

Laboratory methods

During each experiment, both $F_N$ and $F_S$ are continuously measured at 100 Hz to better than 1% accuracy. The local strains are acquired by the use of miniature strain gages mounted in a rosette configuration (Vishay 015RJ), mounted on one face of the top block, 2-3 mm above the interface. Each strain gage is wired to its own Wheatstone bridge, acquiring data at a rate of 1 Hz with an RMS noise level in the range of 0.04-0.08MPa. Acquisition of both the applied forces and contact area is triggered by an acoustic sensor mounted to the $x=0$ face of the top block.

The real area of contact, $A$, is measured by a sheet of light illuminating the entire contact surface, where the incident angle of the light sheet is chosen to be well beyond the critical angle for total internal reflection. When non-contacting locations are encountered at the interface, the light is reflected out of the system. Only when the light encounters a contact point, does it traverse the interface. As a result, the instantaneous transmitted light intensity is proportional to $A(x,t)$ over the entire ($x \cdot y$) 200-6mm interface. The transmitted light is imaged using either a VDS CMC-1300/485N or Phantom V710 fast camera. Integration of the acquired images over $y$ provides simultaneous measurements of $A(x,t)$ at 1280 spatial points in $x$ (0.16 mm/point), with a temporal resolution between 1.5-20 $\mu$sec. The rapid acquisition of $A(x,t)$ is triggered by means of an acoustic transducer mounted on the trailing edge ($x=0$) of the top block. An example of the contact area measurements together with spatial measurements of the consequent local rupture velocity and shear to normal stress ratio ($\tau/\sigma$) prior to rupture event 2 in Fig. 4B is presented within the inset of Fig. 4C. The shear stress, $\tau(x)$, was calculated from the local 2D strain measurements assuming plane stress boundary conditions according to: $\tau(x) = \alpha_{xy} + h \cdot \Delta \alpha_{xx}/\Delta x$
where $\sigma_{ij}$ is the stress tensor (using plane stress conditions) and the last term is the result of a Taylor expansion of $\sigma_{xy}$ to account for the fact that we perform our measurements at a finite height, $h=2$-3 mm, above the interface.

Mean rupture speed estimate on the Whillans Ice Plain

The inferred location in the direction of that beam is shown in Fig. 1. The location of the barrier phase differs from the assumed location of the barrier phase in a previous experiment (Walter et al., 2011), which was assumed to be adjacent to the area where Whillans Ice Stream flows into the Whillans Ice Plain. This discrepancy is likely due to a lack of stations and limited station aperture in the previous study.

Using the distance from the nucleation region to the barrier phase as the rupture length, the rupture velocity, $V_{\text{mean}}$, is

$$V_{\text{mean}} = \frac{L_{\text{rupture}}}{t_{S11} - \frac{d_{\text{PhaseB-S11}}}{V_{\text{Rayleigh}}}}$$

where $V_{\text{Rayleigh}}$ is the Rayleigh wave speed in ice inferred from beamforming (1.8 km/s), $d_{\text{PhaseB-S11}}$ is the distance to station S11 observing the barrier phase (75 km), $L_{\text{rupture}}$ is the length of rupture from station S03 or S08 to the barrier phase (90 and 93 km, respectively), and $t_{S11}$ is the difference in time between the origin time and arrival of Phase B at station S11. We use arrival of the barrier phase at an on-ice station rather than relying on the sporadic arrivals at station VNDA in the Dry Valleys, Antarctica. We infer the barrier location by projecting the backazimuth of the barrier phase to the edge of the ice (Fig. 1). We determine from Eq. S1 the average rupture speed for each event (Fig. 3A). $V_{\text{mean}}$ scales with inter-event period, an
observation consistent with data recorded during the 2008 field season (Walter et al., 2011). This measurement is in addition to the apparent rupture velocity measured more directly in Fig. 4.

**Beamforming to locate barrier phase**

We utilize delay-and-sum beamforming to determine the azimuth and speed of the barrier phase seismic wave, which allows for the calculation of average rupture speed on the ice. Beamforming is useful when the source location and/or propagation velocity are unknown or poorly constrained (Rost and Thomas, 2002). We construct a beam for each of 360 degrees, \( j \), of source locations surrounding our 2010 network. We loop through apparent wave velocities, \( v \), from 1200 to 2600 m/s at 100 m/s intervals and compute the slowness, \( u \), for each azimuth by

\[
\mathbf{u} = (u_x, u_y) = \left( \frac{\sin \theta}{v}, \frac{\cos \theta}{v} \right)
\]

(S2)

The waveforms from all 11 stations, \( y_{i} \), are time-shifted according to the travel-time for a particular plane wave apparent velocity relative to a point at the center of the array. We stack the seismograms at each of these iterations in \( j \), to produce the beam, \( B_j(t) \), at that azimuth, where

\[
B_j(t) = \sum_{i=1}^{N} y_i(t + r_i \cdot \mathbf{u})
\]

(S3)

We take the maximum amplitude at each azimuth, \( j \), for iterations of \( v \).

In order to test the performance of the beamforming algorithm, we prescribe an origin location, produce synthetic “seismograms,” then beamform the synthetic data. For the seismograms, we produce a cosine impulse function with duration of 20 s and add sine functions with random periods (using MATLAB’s random number generator) for each of the 11 stations.

Using the geometry of the 2010 network and the prescribed location (Fig. S1A), we compute the
expected time delay for a wave speed of 1800 m/s and shift the synthetics accordingly (Fig. S1B). We beamform the data, as described previously, and produce the polar plot in Fig. S1C. The recovered backazimuth is 346°, which is roughly consistent with the prescribed origin of the synthetic source, whose azimuth from the center of the array is 348°.

The maximum beams of barrier phase B are shown on the polar plot in Fig. S2A. The highest amplitude stack for this event (Fig. S2B) is at an azimuth of 189° with an apparent velocity of 1,800 m/s. Shear-wave speed in ice is approximately 1,900 m/s (Blankenship et al., 1987), and so our inference of a barrier phase wave speed of 1,800 m/s is consistent with the velocity of a surface wave through ice. The mean azimuth for 4 of 44 events with sufficiently high amplitude is 196 ± 11°, on the South Polar Stereographic grid. The restricted number of analyzed events is due to the low amplitude barrier phase for the on-ice stations. Only stacked beams with signal-to-noise ratio (SNR) greater than 10 are considered sufficiently high amplitude to be included in the analysis.

**Supplementary material references**


Fig. S1: Example of beamforming method on synthetic data. (A) Location of synthetic origin relative to station geometry, (B) example seismograms for the 11 stations with random noise, and (C) polar plot showing the synthetic beamform. Colorbar indicates relative amplitude of stacked signals.
Fig. S2: Example of beamforming method for a single event. (A) Polar plot showing the beamform for a single event and (B) example seismograms for raw seismograms and time-shifted seismograms corresponding to the maximum beam amplitude. Heavy dark lines indicate linear stack of seismograms, divided by the number of stations.