The Spatial Density of Foreshocks

Emily E. Brodsky, Dept. of Earth & Planetary Sci., University of California, Santa Cruz

Abstract

Aftershocks follow a well-defined spatial decay pattern in the intermediate field. Here I investigate the same pattern for foreshocks. Foreshock linear density decays as $r^{-1.5+/-0.1}$ over distances $r$ of 0.1-30 km for 15 minutes before magnitude 3-4 mainshocks. This trend is the same as that of the aftershocks within the error of the measurement. This consistency of spatial decay can be explained by the clustering inherent in earthquake interactions. No additional preparatory process beyond earthquake triggering is necessary to explain the spatial decay.

1. Introduction

Earthquake triggering is a possible route into studying earthquake initiation and, by extension, predictability. Earthquake interactions are generally dissected by looking at relationships between earthquakes that are close in either time or space. For instance, Felzer and Brodsky [2006] examined the spatial pattern of smaller earthquakes following small magnitude mainshocks within 5 minutes. The study found that the linear density $\rho$ of aftershocks followed a well-defined pattern of $\rho \propto r^{-\gamma}$ where $r$ is the distance from the mainshock. The persistence of this spatial trend to large distances was interpreted as indicative of a consistent triggering process across the entire range of distances. Since seismic waves are thought to be significant triggerers at great distances, the inference was that seismic waves are an important part of triggering at all distances [Hill et al., 1993; West et al., 2005; Hill and Prejean, 2007; Van der Elst and Brodsky, 2010].

Foreshocks are the best-known predictor of earthquakes. Early thinking attributed foreshocks to a stress accumulation process that ultimately culminated in a large earthquake [e.g., Jones and ...]
Molnar, 1979]. In this scenario, the spatial distribution of foreshocks is controlled by the stress
distribution and fatigue on the fault plane. This line of thinking has most recently been supported
by a connection between fluid flow and foreshocks [Lucente et al., 2010, Terakawa et al., 2010].
Other work has entertained alternative possibilities such as the foreshocks being symptomatic of an
earthquake cascade. Earthquakes are constantly triggering each other. When seismicity is tightly
clustered, the likelihood of a large earthquake increases and, after the fact, the seismicity cluster is
interpreted as foreshocks [Helmstetter, et al., 2003; Felzer et al., 2004]. This type of clustering
results in distinct patterns, like the Inverse Omori’s Law [Helmstetter et al., 2003]. Still other work
has suggested that both kinds of foreshocks exist in earthquake catalogs [McGuire et al., 2005].
This paper starts by establishing that the foreshock spatial trend is similar to the aftershock one. At
first, this observation is disconcerting. If the spatial decay of $r^{-\gamma}$ is generated by aftershocks, then
why is it present before the mainshock? I will follow up the observation by showing that a
statistical seismicity model in which aftershocks follow the spatial decay law of $r^{-\gamma}$ automatically
generates foreshocks with the same spatial decay. The trend can be a natural consequence of the
clustering provided by the mutual triggering of the earthquake cascade.

2. Observation

I begin by using the well-located Lin-Shearer-Hauksson (LSH) catalog of Southern California
earthquakes 1981-2005 to compare the spatial distribution of aftershocks and foreshocks [Lin et al.,
2007]. Only magnitudes >2 are included to ensure completeness. For the purpose of comparison, I
use a similar windowing criteria as Felzer and Brodsky [2006] to isolate sequences. Mainshocks are
defined as earthquakes that are at least 4 days after and 0.5 day before any larger earthquake at any
distance. Aftershocks are defined to be earthquakes that follow a larger identified mainshock within
the specified time period $\Delta t$. Foreshocks are earthquakes that are followed by a larger identified
mainshock within the specified time period \( \Delta t \). When comparing foreshocks and aftershocks for a particular sequence, the same value of \( \Delta t \) is used to define both types of seismicity.

These windowing criteria are not meant to imply a physical limit to interaction. The insensitivity of the aftershock spatial decay to the windowing details was established in previous work [Felzer and Brodsky, 2006]. The point of the present study is simply to compare foreshock decays to aftershock decays given the same conditioning on the data.

To measure density, the mainshocks (and their accompanying sequences) for the entire catalog are combined to improve the statistical sampling. A single, ordered vector of aftershock-mainshock distances is created, \( \bar{r} \). The linear density of aftershocks (and foreshocks) is estimated at the midpoints between each element of the combined aftershock distance vector \( \bar{r} \) by

\[
\rho \left( \frac{r_i + r_{i+1}}{2} \right) = \frac{1}{r_{i+1} - r_i}
\]

For mainshocks with magnitude <4, the fault rupture length is less than the spatial precision of the data (~0.1 km) and therefore the point approximation embedded in eq. 1 is appropriate. Larger magnitude mainshocks require a better geometrical model for earthquake density estimation.

The aftershocks within 15 minutes of magnitude 3-4 mainshocks yield a fit of \( \gamma = 1.5 +/- 0.08 \) for data from distances of 0.1-30 km (Fig. 1). (Error ranges throughout the paper are standard deviations of 1000 bootstrap trials). The value of \( \gamma \) differs slightly from Felzer and Brodsky [2006] because it is based on epicenters, not hypocenters. This study uses epicenters to facilitate comparison with the 2-D ETAS model below. For \( \Delta t = 5 \) minutes, a similar trend appears, but fewer quakes are recorded (Fig. 1b).
For close distances, the largest mainshocks in this range can no longer be treated as point sources and so the distance $r$ is likely inaccurate. For large distances, the number of triggered events is sufficiently small that unrelated seismicity overwhelms the signal. The background level can be seen by shuffling the event times randomly for the same earthquake locations and re-measuring the density. The thin light lines in Fig. 1 show 100 such reshufflings and the thick light lines show the mode values of the time-randomized set. As an unusually closely spaced pair of events can easily dominate the median or mean of the density, the mode is the best representative of the ordinary background level of seismicity. This background level demonstrates that indeed the background interferes with recording the aftershock signal at large distances, but an aftershock signal is visible up to at least 30 km distance from the mainshock.

Interestingly, the foreshocks follow almost the same trend (Fig. 1). For 15 minutes before, the decay of the composite foreshock sequence from the future mainshock site follows $r^{-1.5+/-.1}$. For the 5 minutes before the mainshocks, the foreshock linear density is $r^{-1.6+/-.02}$. This observation is similar to Richards-Dinger et al. [2010]. The foreshock decay is also truncated by the background seismicity at large distances.

The specific cases of $\Delta t= 5$ and 15 minutes provide insight into the behavior at short times when unrelated background earthquakes are rare. The recovered value of $\gamma$ for a full suite of values of $\Delta t$ ranging from 2- 40 minutes shows that these results are representative of short times when there are enough earthquakes for the measurement to be made (Fig. 2). At extremely short times (<2 minutes), too few events are cataloged to measure $\gamma$. For $\Delta t$ up to 15 minutes, the foreshock and aftershock values are indistinguishable. Beyond 15 minutes, the foreshock sequences have a slightly smaller value of $\gamma$. 
3. Clustering Model

In order to recreate the observed foreshock trend, I will employ an Epidemic Triggering Aftershock Sequence (ETAS) model that combines empirical statistical laws of seismicity to generate self-consistent synthetic earthquake catalogs [Ogata, 1999]. An earthquake triggers successive earthquakes with a probability determined by the magnitude of the mainshock. Omori’s Law and a spatial decay relationship determine the time and distance separating the aftershock from the mainshock. Successive earthquake magnitudes are then determined by Gutenberg-Richter statistics and then each earthquake generates its own aftershocks.

The ETAS formulation has been explored extensively in the literature [Helmstetter et al., 2003, 2005; Felzer et al., 2004; McGuire et al., 2005; Zhuang et al., 2008]. Windowing and selection criteria often result in behaviors that are challenging to evaluate analytically. I therefore use a numerical model directly for this work. The parameters used here are meant to illustrate the relationship between the foreshock and aftershock spatial decay. They are certainly not exhaustive, but are realistic and suffice to elucidate the role of clustering in generating a foreshock spatial pattern. Published versions of the basic ETAS model utilize variants of the observational laws. As the permutations of the model are important and the practicalities of implementation are sometimes difficult, I will go into some detail on the exact forms used in the numerical calculations done here.

3.1 ETAS Implementation

Background seismicity is imposed as a Poissonian process with rate $\lambda$ over the duration of the observed catalog (24 years) and the magnitude of each earthquake determined by Gutenberg-Richter. The background rate is chosen so that the total number of synthesized events (including aftershocks) is approximately equal to the observed number. Each background earthquake generates
aftershocks, which in turn spawn their own sequences through four empirical equations cast in terms of probability distributions:

I) Gutenberg-Richter. The number of aftershocks with magnitudes greater than or equal to \( M \) is

\[
\log N(M) = -bM + a 
\]

(2)

where \( a \) and \( b \) are constants. To obtain a cumulative density function (CDF), \( N(M) \) is normalized by the total number of earthquakes in the catalog, i.e., \( N(M_{\text{min}}) \) where \( M_{\text{min}} \) the minimum magnitude of the simulated catalog. A random number \( p_1 \) is then selected between 0 and 1 for each aftershock and inverted using the CDF into a magnitude, i.e.,

\[
M = -\log (p_1(10^{-bM_{\text{min}}} - 10^{-bM_{\text{max}}}))/b 
\]

(3)

where \( p_1 \) is a random variable between 0 and 1 and \( M_{\text{max}} \) is the maximum magnitude size to be generated. The extra term in brackets is a normalization that accounts for discarding values of \( p_1 \) that result in \( M > M_{\text{max}} \) [Sornette and Werner, 2005].

II) Omori’s Law. The rate of seismicity as a function of time \( t \) from a mainshock is

\[
dN_A/dt = K(t + C_{\text{omori}})^{-\beta} 
\]

(4)

where \( \beta \) and \( C_{\text{omori}} \) are constants and \( K \) is a function of mainshock magnitude to be discussed below. The constant \( C_{\text{omori}} \) is difficult to measure as it is often biased by completeness problems at short times after an earthquake. Recent work has suggested that \( C_{\text{omori}} \) is significantly shorter than previously imagined [Peng et al., 2007]. In probabilistic terms,

\[
t = [(t_{\text{max}} + C_{\text{omori}})^{1/\beta} p_2 + (1-p_2) C_{\text{omori}}(1-\beta)]^{1/(1-\beta)} - C_{\text{omori}} 
\]

(5)

where \( p_2 \) is a random variable between 0 and 1.

III) Aftershock Productivity. The numerator of Omori’s Law determines the number of aftershocks from a mainshock of magnitude \( M \).
where $C$ and $\alpha$ are constants. Studies agree that $\alpha = 1$ for Southern California seismicity [Felzer et al., 2004; Helmstetter et al., 2005]. The parameter $C$ is defined for the instantaneous rate in Omori’s Law. In order to use eq. 6, the total number of aftershocks must be computed for a sequence and so in practice what is required is the integral of eq. 4 over the finite time, $t_{\text{max}}$, that is the duration of the simulated direct aftershock sequence. The practical equation is therefore,

$$N_{AS} = C' \, 10^{\alpha(M-M_{\text{min}})}$$

where $C' = C[(t_{\text{max}} + C_{\text{Omori}})^{1-\gamma} - C_{\text{Omori}}^{1-\beta}]/(1-\beta)$

Eq. 7 is implemented deterministically based on aftershock magnitudes. For non-integer values of $N_{AS}$, the fractional part is interpreted as a probability. For each sequence, a uniform random variable $p$ is generated between 0 and 1 and if the value is less than the non-integer fraction, an extra aftershock is added. Table 1 reports a value of $C'$ that fits the observed productivity in the LSH data set with the aftershock identification criteria used here (Supp. Fig. 1). Aftershock productivity probably varies in time and space, but the variations of $C'$ are not sufficiently well mapped out to justify a more sophisticated implementation at this time.

IV) Distance Decay. The distance fall-off is implemented by normalizing the distribution to form a CDF assuming that the aftershocks are distributed over distances greater than $d_{\text{min}}$ from the mainshock. Since $\gamma > 1$, no upper bound to the distance fall-off is necessary. The probabilistic form for $r$ for each aftershock is

$$r = d_{\text{min}} (1-p_3)^{1/(1-\gamma)}$$

where $p_3$ is a random variable between 0 and 1. As there are no observational constraints on the angular distribution, $\theta$ is selected with uniform probability from 0 to $2\pi$. 
In the implementation of ETAS, it is helpful to note the order of calculation. First the total number of aftershocks must be calculated for each extent earthquake using eq. 7, then each aftershock is assigned a time, magnitude and location using eqs. 3, 5 and 8. Then the aftershocks generate their own aftershocks by the same procedure until the cascade comes to an end.

3.2 Simulation Results

The ETAS catalog was made to mimic the observed catalog using the parameters of Table 1. The procedures used to isolate mainshocks and identify 15 and 5-minute aftershock and foreshocks are identical to those used on the observational data in Fig. 1.

The overall trends of the Fig. 1 are reproduced in the simulated catalog (Fig. 3). Both the aftershock and foreshock densities have a best-fit spatial decay consistent with the imposed exponent. This consistency in foreshock and aftershock behavior exists even though there was no specific foreshock preparatory process in the ETAS model. The utility of the statistical simulation is that it shows that non-intuitive behavior, like that of Fig. 1, emerges naturally from the earthquake sequences. Like the Inverse Omori’s Law, the spatial decay of the foreshocks is symptomatic of the type of increased seismicity that is likely to trigger a large earthquake.

The behavior of the best-fit values of $\gamma$ over a range of time windows reproduces the major trends of the observational data (Fig. 4). Like in the real catalog, $\gamma$ is unmeasureable at extremely short times ($\Delta t < C_{Omori}$). The decay of the seismicity becomes more gradual (best-fit $\gamma$ decreases) for both aftershock and foreshock sequences as time increases due to the random walk of secondary sequences [Helmstetter et al., 2003] combined with interference from unrelated background events. (Superposition of sequences with offset origins result in a composite fit with reduced values of $\gamma$).

In the simulations, both the decrease in $\gamma$ and the variability of realizations is more pronounced for
the foreshock sequences because of the relatively small number of events. Multiple simulations are required to identify the expected behavior. The mean of the best-fit values of $\gamma$ for foreshocks over 50 simulations by more than 1 standard deviation from that of the aftershocks for time windows $\geq 14.5$ minutes (Fig. 4b). These simulation results are consistent with the real data showing significant departures between the aftershock and foreshock decay for $\Delta t>15$ minutes (Figure 2).

One conspicuous difference between the simulations and the real catalog is that the ratio of identified aftershocks to foreshocks is much higher in the simulation than the observation. In the actual catalog, the ratio is 2.2 for $\Delta t=5$ min and 2.8 for $\Delta t=15$ min, while the simulations result in ratios between 7 and 8 for the parameters used here. Aftershock/foreshock ratios have been analyzed extensively elsewhere [e.g., McGuire et al., 2005] and are not the focus of this study. Nonetheless, some assessment of the source of the discrepancy is in order.

One possibility is that the observed catalog artificially depresses the aftershock/foreshock ratio because of incompleteness. It is more difficult to detect aftershocks than foreshocks because the mainshock and temporally high seismicity rate can obscure individual events. Therefore, the magnitude of catalog completeness for aftershocks can be significantly higher than normal [e.g., Peng et al., 2007].

To initially evaluate the role of catalog incompleteness, I restrict the data to magnitudes $>2.5$ (Supp. Fig. 2). The observed aftershock to foreshock ratio for $\Delta t=15$ min increases slightly to 2.9. The same restriction on the simulated data decreases the aftershock/foreshock ratio because the ratio depends on the identified mainshock magnitude range [McGuire et al., 2005]. In a suite of 100 ETAS simulations with the values in Table 1 and $M_{th}=2.5$, for $\Delta t=15$ min the mean ratio is 4 with a standard deviation of 0.8. This improved consistency between simulation and observation is
expected based on previous work on foreshock rates in the region [Felzer et al., 2004] and indicates that the lack of aftershock detection is an important issue in the observations. However, the simulated ratios still do not exactly match the observations. It is possible that the completeness threshold needs to be raised still further to ensure aftershock detection, but further restriction limits the dataset to an unreasonably small size.

As an additional probe of time-variable completeness, I exclude a short window immediately before and following the mainshock in measuring the aftershock and foreshock densities (Supp. Fig. 3). This strategy has the advantage of preserving the number of events at larger times when the completeness threshold drops to the overall catalog value. Excluding a window of 1 minute on either side of the mainshock increases the aftershock/foreshock ratio to 3.3 for $\Delta t = 15$ min. This ratio is still smaller than the simulated ratio, but it does again indicate that early-time aftershock detection is likely a problem in the observations.

The aftershock-foreshock ratio can be adjusted in the simulations by reducing the minimum magnitude of simulation $M_{\text{min}}$ or increasing the productivity constant $C'$, both of which have the effect of increasing the average number $n$ of aftershocks per mainshock [Sornette and Werner, 2005]. As $n$ increases, the system approaches a critical state and the foreshock rate should increase relative to the aftershock rate [McGuire et al., 2005]. The calculations also become prohibitively expensive. Thus far, the observed foreshock to aftershock ratio has not been simulated. One possible explanation is that there is a genuine difference in the ETAS process from the observations. If so, the aftershock/foreshock ratio is the only evidence of such a process. Another explanation is that aftershock completeness is still the overwhelming observational problem.

More importantly for this study, the spatial decay of the foreshock density in both restricted datasets remains consistent with the aftershock decay. This consistency of Supp. Fig. 3 is particularly useful
in light of the commentary of Richards-Dinger et al. [2010] that the observed aftershock density
decay may be controlled by events that occurred before the arrival of the seismic waves. No such
events are included in the fits of Supp. Fig. 3 as P waves travelling at 6 km/s pass through 360 km
in the first minute.

At very large distances, the simulated catalog has clusters due to unisolated ongoing sequences. The
isolation distance of 100 km is insufficient for the simulation because of the more homogeneous
distribution of epicenters, i.e., there are no simulated faults.

4. Conclusions
Earthquakes are preceded by a halo of earthquakes that decays with distance much like aftershocks.
These foreshocks are an expected consequence of earthquake interaction and clustering. Their
existence and location near the eventual rupture does not in itself indicate a preparatory strain
accumulation process beyond ordinary earthquake triggering.

A foreshock preparatory process could exist in the Earth beyond the clustering process, but it is not
required by the observations. The only potential evidence seen here for such a preparatory process
is in the ratio of total number of aftershocks to foreshocks. However, the data suggest that catalog
completeness is a serious problem that could artificially depress the aftershock/foreshock ratio and
therefore this ratio is not the most reliable indicator of the underlying physics. The spatial decay of
both the aftershocks and foreshocks is well explained by the clustering model.

Acknowledgements. Many thanks to Karen Felzer, Agnes Helmstetter, Keith Richards-Dinger,
Peter Shearer, and Ross Stein for helpful conversations and ideas. This work was funded in part by
the Southern California Earthquake Center. SCEC is funded by NSF Cooperative Agreement EAR-
0106924 and USGS Cooperative Agreement 02HQAG0008. The SCEC contribution number for
this paper is 1481.
References


West, M., J. J. Sanchez, and S. McNutt (2005), Periodically Triggered Seismicity at Mount

Differences between spontaneous and triggered earthquakes: Their influences on foreshock

**Figure Captions**

**Figure 1.** Aftershock and foreshock decay for mainshock magnitudes 3-4 and sequences separated
from mainshocks by (a) $\Delta t = 15$ minutes and (b) $\Delta t = 5$ minutes. For $\Delta t = 15$ minutes, a least-squares
fit over 0.1-30 km yields $\gamma = 1.5 +/- 0.08$ for aftershocks and $1.5 +/- 0.1$ for foreshocks. For $\Delta t = 5$
minutes, $\gamma = 1.5 +/- 0.1$ and $1.6 +/- 0.2$ for aftershocks and foreshocks, respectively. Background
density is represented by time-randomized catalogs. The thin, light lines connect the measured
density (Eq. 1) for each of 100 time-randomizations with the same aftershock (blue) and foreshock
(red) criteria as used on the original data. The thick lines are mode background values for
logarithmic bins of the 100 realizations.

**Figure 2.** The best-fit value of $\gamma$ for aftershocks and foreshocks for a range of time windows. As in
Figure 1, mainshocks are magnitude 3-4 and the linear density decay is fit over 0.1-30 km. Shaded
regions show the error on the fit.

**Figure 3.** Simulated aftershock and foreshock decay of magnitude 3-4 mainshocks with (a) $\Delta t = 15$
minutes and (b) $\Delta t = 5$ minutes. The best-fit decay exponent for $\Delta t = 15$ minutes is $\gamma = 1.5 +/- 0.04$ for
the aftershocks and $1.4 +/- 0.09$ for the foreshocks. For $\Delta t = 5$ minutes, $\gamma = 1.5 +/- 0.05$ for the
aftershocks and $1.5 +/- 0.1$ for the foreshocks. ETAS parameters are in Table 1.

**Figure 4.** The best-fit value of $\gamma$ for simulated aftershocks and foreshocks for a range of time
windows. ETAS parameters are as in Figure 1. (a) Best-fit values from a single simulation with
error ranges on fit shown by the shaded region as in Figure 2. (b) Mean of the best-fit values of $\gamma$

for 50 simulations with the standard deviations over the suite of simulations shown by the error

bars. As in Figure 3, mainshocks are magnitude 3-4 and the linear density decay is fit over 0.1-30

km. Vertical dashed line shows the value of $C_{Omori}$ in the simulations.

Table 1. ETAS Parameters for Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completeness threshold $M_{th}$</td>
<td>2</td>
<td>Catalog completeness</td>
</tr>
<tr>
<td>$t_{max}$</td>
<td>24 years</td>
<td>Catalog duration</td>
</tr>
<tr>
<td>Minimum simulation magnitude $M_{min}$</td>
<td>0</td>
<td>Computationally limited</td>
</tr>
<tr>
<td>Spatial decay exponent $\gamma$</td>
<td>1.5</td>
<td>Aftershock density spatial fit (Fig. 1)</td>
</tr>
<tr>
<td>Productivity constant $C'$</td>
<td>0.03 Earthquakes</td>
<td>Aftershock productivity fit (Supp. Fig. 1)</td>
</tr>
<tr>
<td>Omori Exponent $\beta$</td>
<td>1.34</td>
<td>Hardebeck et al. [2008]</td>
</tr>
<tr>
<td>Background rate $\lambda$ for $M&gt;M_{min}$</td>
<td>300 Earthquakes/day</td>
<td>Matches observed rate</td>
</tr>
<tr>
<td>Minimum aftershock distance $d_{min}$</td>
<td>100 m</td>
<td>Minimum location accuracy</td>
</tr>
<tr>
<td>Omori delay constant $C_{Omori}$</td>
<td>130 s</td>
<td>Peng et al. [2007]</td>
</tr>
<tr>
<td>$b$</td>
<td>1</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>Felzer et al. [2004]; Helmstetter et al. [2005]</td>
</tr>
</tbody>
</table>
Figure 1. Aftershock and foreshock decay for mainshock magnitudes 3-4 and sequences separated from mainshocks by (a) Δt = 15 minutes and (b) Δt = 5 minutes. For Δt = 15 minutes, a least-squares fit over 0.1-30 km yields γ = 1.5 ± 0.08 for aftershocks and 1.5 ± 0.1 for foreshocks. For Δt = 5 minutes, γ = 1.5 ± 0.1 and 1.6 ± 0.2 for aftershocks and foreshocks, respectively. Background density is represented by time-randomized catalogs. The thin, light lines connect the measured density (Eq. 1) for each of 100 time-randomizations with the same aftershock (blue) and foreshock (red) criteria as used on the original data. The thick lines are mode background values for logarithmic bins of the 100 realizations.
Figure 2. The best-fit value of $\gamma$ for aftershocks and foreshocks for a range of time windows. As in Figure 1, mainshocks are magnitude 3-4 and the linear density decay is fit over 0.1-30 km. Shaded regions show the error on the fit.
Figure 3. Simulated aftershock and foreshock decay of magnitude 3-4 mainshocks with (a) $\Delta t = 15$ minutes and (b) $\Delta t = 5$ minutes. The best-fit decay exponent for $\Delta t = 15$ minutes is $\gamma = 1.5 \pm 0.04$ for the aftershocks and $1.4 \pm 0.09$ for the foreshocks. For $\Delta t = 5$ minutes, $\gamma = 1.5 \pm 0.05$ for the aftershocks and $1.5 \pm 0.1$ for the foreshocks. ETAS parameters are in Table 1.
Figure 4. The best-fit value of $\gamma$ for simulated aftershocks and foreshocks for a range of time windows. ETAS parameters are as in Figure 1. (a) Best-fit values from a single simulation with error ranges on fit shown by the shaded region as in Figure 2. (b) Mean of the best-fit values of $\gamma$ for 50 simulations with the standard deviations over the suite of simulations shown by the error bars. As in Figure 3, mainshocks are magnitude 3-4 and the linear density decay is fit over 0.1-30 km. Vertical dashed line shows the value of $C_{Omori}$ in the simulations.
Supplementary Figure 1. Simulated and observed identified aftershocks as a function of magnitude using the parameters in Table 1. The fit between the simulation and observations for the magnitudes in the study range (M 3-4) is used to calibrate the simulation aftershock productivity $C'$. Solid lines indicate best fit trends to the simulations and observations for M 3-4 mainshocks. The apparent depletion of aftershocks for mainshocks with magnitude> 4 is likely a completeness issue as documented elsewhere [Felzer et al., 2004; Helmstetter et al., 2005].
Supplementary Figure 2. Observed and simulated aftershock and foreshock sequences with a completeness magnitude of $M_{th}=2.5$. All other simulation parameters are the same as Table 1. (a) Observed earthquake densities for $\Delta t=15$ minutes and (b) $\Delta t=5$ minutes. (c) Simulated earthquake densities for $\Delta t=15$ minutes and (d) $\Delta t=5$ minutes.
a)

b)
Supplementary Figure 3. Observed and simulated aftershock and foreshock sequences excluding a 1 minute interval both before and after the mainshock. All other simulation parameters are the same as Table 1. (a) Observed earthquake densities for Δt=15 minutes and (b) Δt=5 minutes. (c) Simulated earthquake densities for Δt=15 minutes and (d) Δt=5 minutes.