Roughness of fault surfaces over nine decades of length scales

Thibault Candela, François Renard, Yann Klinger, Karen Mair, Jean Schmittbuhl, and Emily E. Brodsky

1ISTerre, University of Grenoble I, CNRS, OSUG, BP 53, 38041 Grenoble, France.

E-mail: thibault.candela@ujf-grenoble.fr; francois.renard@ujf-grenoble.fr

2Physics of Geological Processes, University of Oslo, Oslo, Norway. E-mail: karen.mair@fys.uio.no

3Institut de Physique du Globe de Paris, Sorbonne Paris cité, Univ. Paris Diderot, UMR 7154 CNRS, 1 rue Jussieu, F-75005 Paris, France. E-mail: klinger@ipgp.fr

4UMR 7516, Institut de Physique du Globe de Strasbourg, Strasbourg, France. E-mail: jean.schmittbuhl@eost.unistra.fr

5Department of Earth and Planetary Sciences, University of California–Santa Cruz, Santa Cruz, California 95064, USA. E-mail: brodsky@es.usc.edu
Abstract

We report on the roughness measurements of five exhumed faults and eight surface ruptures over a large range of scales: from 50 micrometers to 50 km. We used three scanner devices (LiDAR, laser profilometer, white light interferometer), spanning complementary scale ranges from 50 micrometers to 10 m, to measure the 3-D topography of the same objects, i.e. five exhumed slip surfaces (Vuache-Sillingy, Bolu, Corona Heights, Dixie Valley, Magnola). A consistent geometrical property emerges as the morphology of the slip surfaces shows a straight line covering five decades of length-scales in a log-log plot where axes are fault roughness and spatial length scale. The observed fault roughness is scale dependent, with a common self-affine behavior described by four parameters: two power-law exponents $H$, constant among all the faults studied and slightly anisotropic ($H_\parallel = 0.58 \pm 0.07$ in the slip direction and $H_\perp = 0.81 \pm 0.04$ perpendicular to it), and two pre-factors showing a quite large variability. For larger scales between 200 m and 50 km, we have analyzed the 2-D roughness of the surface rupture of eight major continental earthquakes. These ruptures also show self-affine behavior ($H_\parallel = 0.8 \pm 0.1$), which is consistent with the slip-perpendicular behavior of the smaller-scale measurements. We show that small degrees of non-alignment between the slip orientation and the exposed trace result in sampling the slip-perpendicular geometry. Although a data gap exists between the scanned fault scarps and rupture traces, the measurements are consistent within the error bars with a single self-affine scaling exponent in the slip-perpendicular direction, i.e. consistent dimensionality, over nine decades of length scales.
1. Introduction

Faults appear most commonly as more or less continuous linear breaks at the surface of the Earth, but their traces show wavy irregularities at all scales (Brown and Scholz, 1985). These irregularities, which we call roughness, control the geometry (Power et al., 1987), mechanics, and transport properties of fault zones (Power and Durham, 1997) and contribute to their 3D architecture (Bistacchi et al., 2010; Faulkner et al., 2010). Fault roughness may control several faulting processes and parameters such as the total resistance to slip, the aseismic versus seismic behavior, the alteration of shear resistance during sliding, the magnitude of stress concentration and heterogeneity in the fault zone (Chester and Fletcher, 1997; Chester and Chester, 2000), and the deformation and damage of the rock on either side of the fault (Arvid and Fletcher, 1994; Dieterich and Smith, 2008; Griffith et al., 2010).

The morphology of fault planes controls the dynamics of faulting and slip. Based on the results of kinematic source inversion models that reconstitute the spatio-temporal evolution of slip during an earthquake, several studies show that both coseismic slip and stress appear to be very heterogeneous along the fault plane (Bouchon, 1997; Mai and Beroza, 2002). A possible explanation is that the fault plane is rough and asperities concentrate stress and slip heterogeneities at various spatial scales (Schmittbuhl et al., 2006; Candela et al., 2011a, 2011b). Earthquake activity and aseismic creep can create and destroy this roughness that causes the heterogeneity of the stress field in the fault zones.

Other studies have also shown the importance of non-planar structures in the rupture propagation (e.g. Aochi and Madariaga, 2003) and the close relationship between the rupture geometry and its propagation velocity (Vallée et al., 2008; Bouchon et al., 2010). Numerical models of earthquake rupture and strong motion need accurate 3-D morphological models of fault surfaces to improve simulations of rupture scenarios.

As direct observations are not possible at the depths of earthquake nucleation, data of exhumed fault scarps (Power et al., 1987; Renard et al., 2006; Sagy et al., 2007; Candela et al., 2009; Brodsky et al., 2011 and references therein) or earthquake surface ruptures (Wesnousky, 2006, 2008; Klinger, 2010) provide the means to characterize fault plane morphology over a wide range of spatial scales.
Pioneering fault roughness measurements (Brown and Scholz, 1985; Power et al., 1987; Power and Tullis, 1991; Schmittbuhl et al., 1993) performed on exhumed scarps found that their roughness cannot be described by a single number such as the standard deviation of the roughness amplitude. Rather, fault surface topography was measured as non-stationary and more particularly self-affine fractal, the amplitude of the topography increasing with the wavelength under consideration.

A 2-D rough profile (Figure 1) is self-affine if it remains statistically invariant under the scaling transformation \( \delta x \rightarrow \lambda \delta x, \delta z \rightarrow \lambda^H \delta z \) (Feder, 1988; Meakin, 1998), where \( \delta x \) is the coordinate along the 2-D profile, \( \delta z \) the roughness amplitude and \( H \) the Hurst exponent (or roughness exponent). If the power law scaling exponent lies in the range \( 0 \leq H < 1 \), different magnification factors will be needed in the directions parallel and perpendicular to the profile for a small portion of the profile to appear statistically similar to the entire profile (Figure 1). As a consequence, in this case, the slope at large scales along a self-affine profile scales as \( s = \delta z / \delta x \propto \delta z^{H^{-1}} \), and tends to flatten for long wavelengths, suggesting a significant role of the small spatial scales (Schmittbuhl et al., 1995a). Since the exponent \( H \) does not contain any information on the amplitude of the signal, and is related only to the progression in space, a second parameter is needed to describe fully the power law and the amplitude of the scaling behavior. In this study, we call this parameter the pre-factor of the power law.

Pioneering studies measured 2-D profiles on exhumed fault planes using mechanical profilometers (Brown and Scholz, 1985; Power et al., 1987; Power and Tullis, 1991). Recently, with the development of a new generation of 3-D laser scanners, fast and accurate acquisitions of topographic data are now available, allowing more fault surfaces to be characterized (Renard et al., 2006; Sagy et al., 2007; Candela et al., 2009; Brodsky et al., 2011). These first results confirmed the roughness of several faults over a wide range of spatial scales and their morphologically anisotropic property with a Hurst exponent smaller in the slip direction than perpendicular to it.

In the present study, we investigate roughness properties of five fault surfaces in different geological contexts (with varying lithology, accumulated displacement, tectonic regime) using three independent scanner devices (a Light Detection And Ranging apparatus – also called LiDAR, a laser
profilometer, and a white light interferometer), spanning a range of scales from $5 \times 10^{-5}$ m to 10 m. In addition, we analyzed the geometry of eight map-scale high resolution rupture traces of large continental earthquakes, giving access to a range of scales from 200 m to 50 kilometers.

The availability of LiDAR has greatly facilitated the measurement of topography data from natural fault surfaces. However, the level of noise inherent in the measurements of these instruments, which is estimated by scanning flat reference surfaces (see Section 3), requires caution when interpreting the results of LiDAR topography measurements. In contrast to the noise level of the LiDAR instruments, the noise level inherent in the laboratory laser profilometer and the white light interferometer (WLI) profilometer is much lower, well below the magnitude of the surface topography. For this reason the laser profilometer and WLI data can be considered to be essentially noise free for the measurements presented here.

We characterize the self-affine scaling of both data sets (scanned fault surfaces and rupture traces) using the same statistical tool, i.e. the Fourier power spectrum analysis. This is a robust technique well-suited for characterizing self-affine roughness of fault zones (Candela et al., 2009). A self-affine model implies that the Fourier spectrum as a function of either the spatial frequency or wavelength plots as a linear trend in log-log coordinates. Two parameters describe such a self-affine model in the spectral domain: the slope of the power spectrum and its pre-factor (i.e. the intercept at a given length scale) on a log-log plot of the Fourier spectrum. The slope (directly proportional to $H$) describes how the roughness changes with scale, while the pre-factor determines the magnitude of the surface roughness at a given scale (Mandelbrot, 1983, p. 350, Power and Tullis, 1991). Both parameters are necessary and sufficient to describe a self-affine geometry. We compute these two parameters for the fault surfaces and rupture traces in order to decipher if a global tendency emerges and/or to characterize the fluctuations.

In Section 2, the characteristics of both the scanned fault surfaces and rupture traces are described. Section 3 is devoted to the presentation and the application of the roughness analysis method. In Section 4, we present results that indicate an anisotropic self-affine model with two different Hurst exponents in perpendicular directions and varying pre-factors, could fit each fault scarp data set,
despite different geological contexts. Secondly, we highlight a similar self-affine regime that fits the eight earthquake rupture traces. In Section 5 we discuss the possibility that a unique anisotropic self-affine geometrical model is maintained from the micrometric scale to the map-scale earthquake surface rupture. The variability of the pre-factors for each data set (fault surfaces and ruptures traces) is discussed with respect to the different geological contexts sampled and more particularly the accumulated slip. Physical processes at work in the generation of the observed roughness are also debated.

2. Fault roughness data

2.1. Exhumed fault scarps

We have analyzed five natural fault surfaces which were selected because of their particularly well preserved slip surfaces, large exposures and few pits or weathering patterns. Existing data sets on the Vuache-Sillingy (Renard et al., 2006; Candela et al., 2009, Angheluta et al., 2011) and Magnola (Candela et al., 2009) faults have been updated and extended with three new faults (Corona Heights, Dixie Valley, and Bolu). A complete list of our fault data, including GPS locations, nature of the rock, direction of motion, and estimated finite geological offset, is given in Table 1. The finite geological offset is always the most difficult of these parameters to estimate. In fact, if the total offset of a fault zone can be estimated from the displacement of geological markers, the total displacement could have occurred on several parallel slip surfaces, making it difficult to estimate the total slip on a given slip surface. For each of our scanned surfaces, we will consider a range of slips bracketed by the extreme values: the total slip of the fault zone and a minimum slip we could estimate from field observations.

The Corona Heights and Dixie Valley faults cut through silicate rocks. The Dixie Valley (Basin and Range province in Nevada) fault has a mainly normal slip component and cross-cuts through rhyolite. Chemical changes during faulting at depth have altered the mineralogy and chemical composition of the rock in the fault zone (Power et al., 1987; Power and Tullis, 1989, 1992). The material that forms the fault consists almost entirely of secondary quartz, a mineral which is extremely resistant to weathering and allows extremely good preservation of the slip surfaces. Using
gravity studies combined with reflection seismology studies (Okaya and Thompson, 1985), Power and Tullis (1989) estimate that the total normal slip is probably between 3 and 6 km. Geological and mineralogical constraints indicate that the slickenside surface formed at depths of less than 2 km and temperatures less than 270 °C (Power and Tullis, 1989). Major historical earthquakes and microearthquakes occurred on the region of the studied fault scarp (Wallace and Whitney, 1984; Dozer, 1986). Additionally, Power and Tullis (1989) have argued that seismic faulting played a role in the development of the slickenside surfaces, based on textural features they described in the fault surface materials.

The Corona Heights strike-slip fault (Figure 2), located in the Castro district of San Francisco, cross-cuts brown Franciscan cherts and was exposed by post-1906 earthquake anthropogenic quarrying. The relatively recent exposure of the fault and the high resistance of cherts to weathering allows for excellent preservation of the slip surface (Figure 2). In most places on the outcrop, an anastomosing set of slip surfaces is present (Figure 2). Individual patches of the fault surfaces may have been activated at different times and different depths. Although the total slip for the fault zone as a whole could be large (> 1000 m), individual surfaces have recorded smaller (~1 m) displacements which are difficult to precisely estimate due to the absence of well-defined structural markers.

The other three faults offset limestone rocks (Vuache-Sillingy in the French Alps, Magnola in the Appenines, and Bolu in Turkey). The Vuache-Sillingy fault is an active strike-slip fault system in the western part of the French Alps and has accumulated a total displacement in the kilometer range (Thouvenot, 1998). The fault surface we analyzed (Renard et al., 2006; Candela et al., 2009) lies on a short segment of this fault system, where the accumulated slip was small, in the range of 10 – 30 meters, as estimated on aerial photographs. The fault plane was exhumed in the 1990s by quarrying and, as a consequence, the LiDAR measurements were performed on fresh, vegetation free surfaces, where weathering is minimal. The Magnola fault (Candela et al., 2009), in the Fuccineo area, is part of the extensive fault system in central Apennines, Italy. This 15 km long normal fault shows microseismic activity and presents an average vertical displacement larger than 500 meters. The site we study has been recently exhumed (Palumbo et al., 2004; Carcaillet et al., 2008) with less alteration
by weathering than older exhumed portions of the fault. The Bolu fault is part of the North Anatolian
strike-slip fault system. The study area (Figure 3) is a part of the section that ruptured during the 1944
earthquake (Kondo et al., 2005; Kondo et al., 2010; Barka, 1996). The small vertical component of
the motion (~1 m), compared to the dominant horizontal strike-slip motion (~3.5 m), was responsible
for the partial exhumation of the fault plane (Figure 3) during the 1944 earthquake (Barka, 1996).
More recently, anthropogenic activity (excavation for a garbage dump) also contributed to the
exhumation of the outcrop. The total geological offset of the North Anatolian fault can be of about
85 ± 25 km (Hubert-Ferrari et al., 2002), but it is not easy to define the slip accommodated
specifically on each individual sub-parallel slip surface constituting the fault zone of the Bolu
segment. Paleo-seismological investigations on the Bolu segment give a lower bound of
approximately 20 m (Kondo et al., 2005; Kondo et al., 2010).

The five faults studied show slip activity during the Quaternary. The Bolu fault records both the
propagation and termination of the 1944 earthquake. The Dixie Valley outcrop lies north of a segment
that broke in 1954; it is the same outcrop studied by Power and Tullis (1987). For three of these
faults, the slip surfaces were exhumed recently by anthropogenic activity during the 20th century
(Vuache-Sillingy, Bolu, and Corona Heights). These three surfaces were therefore exposed to
atmospheric alteration for only a short period of time and therefore their roughness reflects only
faulting processes. The Dixie Valley fault was exhumed by normal faulting activity combined with
local quarrying. In this region, desert weather conditions and the silicate rocks result in slow chemical
alteration. For this fault, we have chosen surfaces free of mechanical erosion, where the mirror-like
polishing due to the latest slips activity was still present. Finally, the Magnola slip surfaces were those
for which the alteration was the most important. For this fault, we selected slip surfaces that were
exhumed by the quaternary vertical activity of this normal fault, and for which the erosion was
minimal (i.e. surface away from a stream that could have increased the erosion rate); however we
cannot discount that weathering has altered the roughness properties of these surfaces. We will show
later that the roughness properties of the Magnola fault do not deviate significantly from those of the
other faults and interpret this observation as evidence that weathering processes did not influence our roughness analyses at the spatial scales we considered.

Even if it is difficult to accurately determine under which conditions at depth (confining pressure, temperature, strain rate and chemical environment) fault surfaces were built, the five fault surfaces studied here sample a range of different control parameters (total fault zone displacement (10’s of meters to ~10’s of kilometers), lithology (rhyolite, chert and limestone), tectonic regime (strike-slip, oblique and normal) which possibly could have controlled fault surface roughness.

2.2. Scanner devices and digital elevation models of fault roughness

Fault surface topography was scanned in the field using five different 3-D portable LiDAR laser scanners that use the time of flight of a light beam to accurately measure distances. The laser scanner records the topography of each exposed fault surface by collecting a cloud of points whose three dimensional coordinates correspond to points on the fault surface (Renard et al., 2006; Sagy et al., 2007; Candela et al., 2009; Resor and Meer, 2009; Zhanyu et al., 2010).

The actual point spacing depends on the distance between the target and the scanner and a chosen angular spacing. For each fault outcrops, fresh sub-surfaces were selected and scanned for higher resolution acquisition (Figure 2, 3).

Our data sets cover surface scales from 1 m² to 800 m² at a spatial length scale resolution $\Delta x$ from 1 mm to 30 mm. This spatial length scale resolution $\Delta x$ corresponds to the point spacing after the data processing (see Section 3), and is systematically taken to be twice as large as the average irregular spacing during the acquisitions, that is from 0.5 mm to 15 mm. The actual precision in the spatial positioning is estimated to be at most half the original average spacing, that is ±0.25 mm to ±7.5 mm. The height precision achievable depends on scanning conditions and is closely related to the spatial length scale resolution and to the roughness amplitude of the surface. In Table 1, $\delta$ represents the estimated amplitude of the instrumental noise.
The scans were combined with digital photographs to distinguish clear slip surfaces from eroded areas. This manual cleaning of the extremely large datasets (several tens of millions of points) was completed using 3-D Reshaper software, a point cloud editor and visualization tool. Once all non-fault features such as trees, grass, or anthropogenic structures were removed, the whole fault scarp or selected smaller patches were analyzed (Figure 2, 3). Typically, in our data sets, less than 5% of spurious points were removed from the raw scanner data. As pointed out by Candela et al. (2009), the estimation of the fault surface properties was not significantly biased by the presence of randomly distributed holes and missing data in the cloud of points (see Appendix A.1 for a quantitative analysis of the bias inherent to data acquisition in the estimation of the geometrical properties of fault surfaces).

In the laboratory, we used a home-made laser profilometer (Méheust, 2002), to measure samples of the fault surfaces (between 200 mm\(^2\) and 1000 mm\(^2\), Figure 2 and 3), set on a 2-axis moving table. Each surface is scanned by geometric triangulation, measuring the distance between the sample and a laser head (Schmittbuhl et al., 2008; Candela et al., 2009). One difference between the LiDAR and the profilometers is that with the laser profilometer, the data are regularly spaced. The spatial length scale resolution \(\delta_x\) is equal here to the horizontal step, \(i.e.\) 20 microns. The actual precision in the spatial positioning is \(\pm 1\) micron and the vertical resolution (\(\delta_z\)) is better than 1 micron. The reliability and accuracy of the cloud of points obtained with this laser profilometer required that only few spurious points were removed (less than 0.01%).

At the millimeter scale, the topography of several slip surfaces (between 0.5 mm\(^2\) and 40 mm\(^2\), Figure 2 and 3) was measured using white light interferometry microphotography (Dysthe et al., 2002). This is done with a microscope that uses a broad-band white light source and that is coupled to a Michelson interferometer. A reference arm creates interference fringes with maximum intensity at equal optical path lengths of the imaging beam and reference beam. By vertical movement of the sample and simultaneous image capturing, the interference, intensity envelope, and thereby the relative height of the imaged surface at each pixel is determined with a resolution of \(\delta_z = 3\) nm. The
horizontal resolution depends on the lens used; the highest magnification it is at the diffraction limit of white light, of about 0.5 micron. In the present study, we have selected horizontal steps (\(\delta x\)) between 1 and 2 microns (Table 1). The actual precision in the spatial positioning is estimated to be \(\pm 0.025\) micron. As for data acquired with the laser profilometer, the cloud of points obtained are regularly spaced and only some spurious points have been manually removed. The whole suite of characteristics of the scanners devices and digital elevation models (spatial precision, resolution \(\delta x\), noise on the data \(\delta z\)) used in this study are shown in Table 1.

As examples, two fault surfaces (Corona Heights fault, Figure 2 and Bolu fault, Figure 3) have been selected to illustrate the topographic data acquired at all scales with the three techniques presented. These fault surfaces are composed of many discrete slip surfaces delimiting bumpy lenses elongated in the direction of slip (Figure 2). These multi-scale bumpy lenses give the wavy aspect of the fault surfaces that are overprinted by fine linear polished striations and coarser corrugations generated by abrasions (Figures 2 and 3).

### 2.3. Earthquake surface rupture data

In contrast to the exhumed fault scarps presented in Section 2.1, where the 3-D roughness is characterized, surface rupture data at larger scales provide only 2-D measurements (Figure 4), since full fault surfaces are not accessible for direct measurement. Fault trace roughness has been previously analyzed (Scholz and Aviles, 1986; Wechsler et al., 2010). In the present study, high resolution geological maps of large continental strike-slip earthquake surface ruptures, in various geological settings, were analyzed using the dataset of Klinger (2010); see for an example Figure 5, in Klinger (2010). For each event, the surface ruptures length, earthquake magnitude, and total geological offset are provided in Table 2.

Of these three parameters, we note that the total slip of a fault is the least constrained one because of the difficulty in finding markers of the displacement over long distances on strike-slip continental faults. For Owens Valley fault (Beanland and Clark, 1994), the total offset was estimated in the range \(\leq 20-30\) km. For the Haiyuan fault (Zhang et al., 1987) it is in the range from 10-15 km
(Burchfiel et al. 1991) to 95±15 km (Gaudemer et al. 1995). For the Gobi-Altay fault, the total displacement lies in the range of 2-20 km (Kurushin et al., 1997). For the Superstition Hills fault, the total slip is still debated and, by comparison with the closeby San Jacinto Fault, the total displacement is estimated to be less than 24 km (Sharp et al. 1967). For the Luzon earthquake, the cumulated displacement of the fault is in the range of 50-200 km (Karig, 1983; Mitchell et al. 1986), and Rigenbach et al. (1993) noticed that because of the activity of this fault since the beginning of the Pleistocene, the total offset should be smaller than 200 km. The Hector Mine and Landers earthquakes occurred on faults that are part of the East California Shear Zone that has a total geological offset close to 65 km (Jachens et al., 2002). The individual geological total slip for Landers (3.1 to 4.6 km) and Hector Mine (3.4 km) faults were estimated from the offset of magnetic anomalies (Jachens et al., 2002). Finally, Korizan (1979) and Zirkuh (1997) earthquakes occurred on the Abiz fault that accommodates 60 km of right lateral motion along the Sistan suture between Iran and Afghanistan (Berberian et al., 1999).

These rupture traces have been acquired using field cartography that allows mapping of the geomorphologic traces of the rupture, combined with slip distributions and high resolution satellite images (Klinger et al., 2005, 2006; Klinger, 2010). The actual point spacing is irregular and its average is between 200 m and 1300 m. However, the precision of each point based on the combined field observations and high resolution satellite images is close to the meter-scale (Klinger et al., 2005). This last detail is crucial for interpreting a multi-meter fine description of the roughness of the rupture trace.

To avoid any bias due to local wiggles of the digitized rupture trace, the data set is re-sampled to ensure consistent spatial sampling, independent of the length of each rupture (Klinger, 2010). This re-sampling procedure does not affect the Fourier transform computation and makes it possible to keep the scaling information of the rupture traces (see Appendix A.2).

Geometric discontinuities that are commonly observed on the eight high resolution maps of large continental strike-slip earthquake surface ruptures are fault azimuth changes or bends, and relay zones, which are also referred to as jogs. These discontinuities reflect the multi-scale en-echelon
pattern of the fault system, and range from a few hundred meters to several kilometers in size. Due to
the presence of abrupt steps associated with relay zones, which influence the Fourier transform and
therefore bias the roughness analysis, the ruptures traces are divided into individual segments (Figure
4). A reconstruction of the entire rupture trace can still be made when removing steps but the
information contained in the first order large scale segmentation has been removed. Note that for
some earthquake surface traces (Luzon, Superstition Hills, Hector Mine), no abrupt steps were
detected and the whole rupture trace can be directly analyzed.

3. Analysis of scaling properties of roughness data

In this section we detail the procedure to characterize the scaling properties of the scanned fault
surface topography. The same approach is followed for the digitized earthquake surface ruptures since
they are considered as rough profiles extracted along one direction of a fault surface.

First the original cloud of points (Figure 3) with irregularly spaced points was rotated, so that the
mean rupture surface was horizontal and parallel to major axis \((X, Y)\). The 3-D set of points was
transformed into a 2-D \((X, Y)\) + 1-D \((Z)\) data set where \(Z\) is the vertical direction, perpendicular to
the mean fault plane (Renard et al., 2006; Sagy et al., 2007; Candela et al., 2009; Zhanyu et al.,
2010). A set of parallel cuts was taken through the cloud of points to obtain a series of thin bands of
points striking at an angle \(\theta\) from the \(X\) axis. Then, each band of points was projected to obtain a
series of profiles with irregularly spaced points. The thickness of the bands of points is closely related
to the average spacing \((X, Y)\) of the original raw data. Linear interpolation on a regular spacing is
performed independently on all profiles to yield a set of heights \(h(X_i)\), function of the coordinate
\(X_i\) along the cut. The regular spacing taken for the linear interpolation has been systematically
chosen to be twice as large as the average irregular spacing of the original profiles.

To describe the scaling properties of these rough profiles (Figure 1), we search for possible spatial
correlations of the height fluctuations. Along each profile, we computed auto-correlation functions. If
the auto-correlation function of a rough profile is a power law and scales as
<h(x), h(x + \delta x) \propto \delta x^{2H}, then the rough profile is self-affine with H the Hurst exponent, if multi-affinity is excluded. One way to estimate the Hurst exponent is to compute the Fourier transform. This method is well-suited and robust for recognizing and characterizing self-affine roughness, as demonstrated by Candela et al. (2009). The Hurst exponent H can be estimated from the Fourier power spectrum, which has a power law form for a 2-D self-affine profile (Barabási and Stanley, 1995; Meakin, 1998). The steps in the procedure to compute the Fourier power spectrum of each profile are as follows: 1) First, removing of the residual drift in the signal is performed in order to avoid any ramp artifact for the Fourier analysis (see Schmittbuhl et al., 1995b for a quantitative analysis of this trend artifact). Indeed, adjusting the reference mean fault plane is always hard to determine exactly a priori. In addition, even if corrections are possible a posteriori, trend suppression is non-trivial and complex. All length scales are involved in the faulting process, even very large ones, and therefore suppressing macroscopic information may influence the scale invariance analysis at large scales. For the following analyses, and as suggested by Schmittbuhl et al. (1995a), the trend has been defined simply as the line fit through the first and last point. 2) In order to ensure that there are no step functions at the end of the finite window, we apply a 3% cosine taper. Note that a taper function of 5 and 10% has been tested and the results appear robust. 3) The Fourier power spectrum \( P(k) \), i.e. the square of the modulus of the Fourier transform, is calculated as a function of wavenumber \( k \). 4) The Fourier power spectrum is normalized by dividing the power at each wavenumber by the length of the profile. 5) Each cloud of points is computed as a whole by stacking and averaging all 2-D Fourier transforms to reduce the noise associated with individual profiles. In other words, for each fault patch, power spectral estimates with regularly spaced wavenumbers is obtained by averaging the power spectra of the individual profiles in a geometric progression (Figure 5).

As an example, the computed Fourier power spectra along the slip direction and perpendicular to it from patches of the Corona Heights fault surface acquired with each device (LiDAR, laser profilometer, WLI) are displayed in Figure 6. The uncertainty in the average spectral values obtained for each fault patch can be estimated following the method of Bendat and Piersol (1986). A one sigma
confidence interval for the spectral power is given by: 

\[ \frac{\sqrt{n_y} \hat{P}(k)}{\sqrt{n_y} + 1} \leq P(k) \leq \frac{\sqrt{n_y} \hat{P}(k)}{\sqrt{n_y} - 1} \]

with 

\[ n_y(k) = n_y \Delta y k + 1, \]

where \( P(k) \) and \( \hat{P}(k) \) are the actual and calculated spectral power, respectively; \( n_y \) and \( n_s \) are the total number of profiles spaced a distance \( \Delta y \) apart perpendicularly to the profile direction, and the number of independent profiles used to calculate \( \hat{P}(k) \), respectively. Note that \( n_s \) depends on scale. For the largest wavenumbers, there are many more independent estimates of the total spectral power, and hence the error estimate is smaller than at the smallest wavenumbers (see Figure 6).

Note that for wavelengths below 50 mm, the fault surfaces we scanned with the LiDAR are so smooth that at this small scale, their spectral power falls within the range of those of the flat plate we use as a planar reference surface (Figure 6). That means that even if our LiDAR surface measurements were acquired at a spatial length scale resolution of \( 1 \text{ mm} \) to \( 10 \text{ m} \) (cf. Section 2.2), total power estimates of the surfaces are accurate only between 0.05-10 m scales. In contrast, the laser profilometer data and WLI data can be considered to be essentially noise free since their inherent noise level falls well below the magnitude of the fault surface topography (Figure 6).

When plotting the average power spectrum as a function of wavenumber in a log-log space, a self-affine function reveals a linear slope, which is itself a function of \( H \) through 

\[ P(k) = C k^{-1-2H} \]

(with \( C \) the pre-factor). Taking into account the possible uncertainties in the spectral power as previously described for our entire data set of fault surface patches, the upper limit of the error bar on the Hurst exponent estimated using a least square method is equal to \( \pm 0.05 \) and does not vary significantly with the wavenumber. Due to the fact that only one rough profile for each rupture trace is analyzed (Figure 4), the noise in the spectrum is higher compared to the fault surface patch (constituted of an average of a multitude of profiles). For a single power spectral estimate, the standard deviation is equal to the mean (Press et al., 2007). So that the upper limit of the error bar in the estimated Hurst exponent of the eight rupture traces analyzed is equal to \( \pm 0.1 \).
4. Fault roughness results

4.1. Two universal Hurst exponents

In order to extract the Hurst exponents characterizing the scaling behavior of the faults roughness, we have fit the linear part of each averaged spectrum obtained in the slip direction and perpendicular to it for each individual fault patch (Figure 6). Because we are ultimately interested in the variability of pre-factors, as discussed in the next section, we have worked separately on each fault patch with each type of instrument instead of calculating average spectra for the full surface. We have focused our analysis of fault surface roughness between the largest scale accessible with the LiDAR, i.e. approximately 10 m, and to the scale of 0.05 mm accessible by the WLI. However, the WLI gives access to scales down to 1 micron.

Figure 7 compiles all Hurst exponents derived from each individual fault patch (41 sub-surfaces in total) in the direction of slip and perpendicular to it. The five faults scanned can be characterized by two different global Hurst exponents between 0.05 mm and 10 m (see Table 1): $H_\parallel = 0.58 \pm 0.07$ in the direction of slip and $H_\perp = 0.81 \pm 0.04$ perpendicular to the slip. We will refer to $H_\parallel = 0.6$ and $H_\perp = 0.8$ afterwards. This result highlights the fact that, for one direction (parallel or perpendicular to slip), the relative amplitude of short and large wavelengths remains identical for the whole data set. Fault surfaces with a Hurst exponent smaller or larger than one standard deviation of their distribution ($\pm 0.07$ in slip direction and $\pm 0.04$ normal to slip) exist, but all Hurst exponents fall within two standard deviations (Figure 7). Consequently, our global Hurst exponents appear to appropriately characterize the scaling behavior of the five faults studied when taking into account the fluctuations indicated in Table 1.

A systematic bending of the Fourier power spectra (along both the parallel and the perpendicular slip directions) at the length scale 0.05 mm is observed on the WLI data (Figure 6, 9). This change of regime occurs at larger length scales relative to the expected WLI resolution, and we suspect that it could be the hallmark of the transition between two physical processes appearing at the grain scale.
Chen and Spetzler (1993) suggest that a characteristic length scale appearing at the grain scale is due to a change in the dominant mode of deformation from small scale intergranular cracking to intragranular cracking at large scale. The same observation and interpretation was made by Meheust (2002) on tensile cracks. Sagy et al. (2007) also suggested a change in behavior at the sub-centimeter scale for large-slip faults. However, it is beyond the scope of this work to quantitatively characterize this possible characteristic length scale and we limit our scaling analysis to scales larger than 0.05 mm.

4.2. Pre-factor variability

In Section 4.1, we focused the analysis on the slope of the power-law, which appears to be constant along slip and perpendicular to it for each fault surface. In addition, for each scanned fault patch, the along slip direction is smoother than perpendicular to it. This is seen by the lower position of power spectrum amplitudes at each scale (Figure 6). More precisely, the power-law relationships between $P(k)$ and $k$ have both lower power-law exponents and pre-factors. However, our results point out that the pre-factor derived from each fault patch is variable along the fault scarps. Here, we illustrate this variability on the data acquired with the LiDAR on the Corona Heights fault, along the slip direction. A significant vertical shift is observed on the Fourier power spectra corresponding to the six different patches of the Corona Heights fault (Table 1, Figure 8). The slopes, however, are similar (Table 1, Figure 7). We emphasize that, even if for some fault patches it is not the case, for other patches this spatial variability on the pre-factor is clearly larger than the size of the error bars in the average spectral values (see Figure 8, for example between Corona-A and Corona-E).

As proposed by Power et al. (1987) and Power and Tullis, (1989), who have observed a similar vertical shift in their Fourier power spectra, a tempting explanation of the variability of the pre-factor within a same fault surface is that the finite displacement accumulated by rougher fault patches (or segments with larger pre-factors) is smaller than for the smoother fault patches. In other words, it is possible that the spatial heterogeneity on the pre-factor illustrated in Figure 8 highlights variable accumulated displacement (probably much less important than the total offset recorded by the fault
zone) on each individual segment (or fault patch) constituting the whole fault zone. However, as emphasized by Power and Tullis (1989), this possibility is difficult to quantify because the total displacement for sub-parallel individual surfaces (which represent a part of the total offset of the fault zone) cannot be observed with certainty in the field. Alternative explanations for the difference in surface characteristics would include formation of the individual fault patches at different conditions (depth, confining pressure, temperature, strain rate) or the presence of different initial heterogeneities in the rock before deformation. It is also possible that the variability of roughness reflects the natural variability of an underlying stationary pre-factor distribution.

In order to highlight the specific trend of each fault in the spectral domain, we have averaged the similar spectra (that means spectra with an approximately identical slope but with a slightly different vertical position) obtained from each scanner device. Each curve on Figure 9 therefore represents an average of similar spectra obtained for multiple fault patches. In this way, this technique gives a smoother spectrum that represents the global self-affine character of the entire fault surface at the specific scales accessible by each device, while preserving good wavelength resolution. However, as a direct consequence of the previously described spatial variability of the pre-factor along the fault surface (Figure 8), the global spectra obtained for each device are vertically shifted in Figure 9. Consequently, even if each global spectrum presents a similar slope, in most cases it is difficult to connect them all together with a unique linear trend (Figure 9). It is worthwhile to point out here that when we compute the scaling from the LiDAR measurements, we average all the 2-D power spectra of all individual rough profiles extracted from different fault patches (Figure 5, 8). Each of these profiles has a different pre-factor but the average gives one specific pre-factor characterizing the global 3-D geometrical property of the fault surface at the LiDAR scale. At smaller wavelengths, with another device (i.e. laser profilometer or WLI), we select several sub-regions with one given average pre-factor among the whole population that we explored at larger scale (i.e. with the LiDAR) but not necessarily equal to the average pre-factor of the large scale measurements (see Figure 8, 9). In other words, with the laser profilometer or the WLI, we expect to sample rougher or and smoother sub-regions of the fault (i.e. with a pre-factor magnitude that is, respectively larger or smaller) than the
average behavior recorded by the LiDAR by combining data over the full surface. As a consequence, it is clear that for the same fault scarp, the specific global spectra calculated for each device by stacking similar spectra obtained for several fault patches, might be vertically shifted in some cases as observed in Figure 9. Finally, this variability in the pre-factor: (i) explains why the Hurst exponents have to be necessarily calculated separately for each device (cf. Section 4.1), and (ii) implies that it is not necessary to invoke a change in slope to connect the vertically shifted global spectra in Figure 9.

To summarize, our analysis highlights that the roughness of the five fault scarps studied can be characterized over more than 5 decades of length scales (between 0.05 mm and 10 m) by two universal Hurst exponents in the direction of slip and perpendicular to it even if it is difficult to point out a single pre-factor in both directions. This description of the fault surfaces is independent of the geological context, i.e. lithology (rhyolite, chert and limestone) and tectonic regime (strike-slip, oblique and normal). Moreover, no clear relationship is observed between the range of pre-factor magnitude extracted from the global spectra shown in Figure 9 for each fault (see Table 3 for a complete list of the maximum and minimum pre-factors of each faults), and the slip estimated for the fault zone (see Figure 10).

4.3. Roughness of large continental earthquakes surface ruptures

In Figure 11, a compilation of the roughness results is provided for the eight surface ruptures. Results of the Fourier power spectrum analysis are shown for the individual segments (bounded by abrupt steps), and the whole rupture trace. For each surface rupture trace, each individual segment has a roughly identical self-affine exponent (see also Table 2) at large scales (i.e. above the regime controlled by data re-sampling – see Section 2.3). Moreover, the profiles obtained for the entire “reconstructed” rupture traces represent an average of the roughness over all the individual segments and keep an approximately identical self-affine exponent (Figure 11).

The variability in the power spectra amplitude (i.e. the pre-factor) of the profiles extracted from the exhumed fault scarps (see Section 4.2) is also observed between each individual segment of the rupture traces. More precisely, along the rupture traces, individual segments display variability in the
roughness amplitude but keep the same self-affine scaling properties of amplitude versus length scale (Figure 11). Since the 2-D roughness scaling properties of the reconstructed rupture traces correspond to a sum of those of the individual segments, it is to be expected that their spectra fall between the smoothest and the roughest short segments (Figure 11). In this way, the range of pre-factors inferred by the individual segments can be interpreted as a typical fluctuation or error in the estimated pre-factor of the “reconstructed” rupture traces.

On the same log-log graph (Figure 12a), a stack of all the spectra calculated on the whole rupture traces complements Figure 11 and emphasizes that all of the surface rupture data can be described by a unique self-affine exponent of $0.8 \pm 0.1$. Another interesting result highlighted in Figure 12a is that, even though the eight rupture traces analyzed clearly sample variable geological settings, the same self-affine exponent fits all of the data best. In particular, even though the surface rupture traces are related to fault zones which have accumulated a large range of finite geological offsets (see Table 2), no trend is revealed between this parameter and the 2-D roughness scaling of the rupture traces. Indeed, taken into account both (i) the average pre-factor of each whole rupture traces and (ii) their respective typical fluctuations extracted from the corresponding individual segments, no correlation appears with the finite geological offset (Figure 12b), admitting however that the standard deviation in the data is quite large and could hide the physical correlation, if any.

5. Discussion

5.1. Can surface measurements of faults be correlated with earthquake processes at depth?

Firstly we discuss how our measurements, performed on surface outcrops, could be relevant to faulting processes that occur at crustal depths up to 15 km. On the five faults we have studied, two of them (Magnola and Dixie Valley) have a mainly normal component and their roughness has therefore recorded processes at work in the first kilometers of the crust. For the three other strike-slip faults, the roughness has recorded only shallow depth faulting processes. Dixie Valley and Bolu outcrops have recorded the surface termination of major earthquakes in the last century. For all faults, we find roughness exponents in the range $0.6 - 0.8$. This can be compared to a recent study of the
roughness of exhumed faults in the Sierra Nevada and Italian Alps (Griffith et al. 2010, Bistacchi et al. 2011). In these studies, the roughness of fault traces was characterized on outcrops for which the deep origin (c.a. 10 km depth) of the faulting process could be identified because of the presence of pseudotachylites and specific mineral assemblages. Griffith et al. (2010) measured Hurst exponents with a quite large variability, in the range 0.4 – 1. Interestingly, Bistacchi et al. (2011) report Hurst exponents in the range 0.6 – 0.8 for spatial scales in the range 0.5 mm – 500 m, similar to those observed in our data and suggesting that the roughness property of faults is also maintained at depth.

5.2. Implications of the self-affine scaling of the fault roughness on the machinery of earthquakes

One of the main contributions of the present study is to provide a robust and realistic fault geometry model, which is still currently missing. Here, we demonstrate that one way to describe fault surfaces with few parameters is to consider them as self-affine. Using our realistic geometry of rough fault surfaces in seismic rupture dynamic models will provide new keys to move forward towards a better understanding of the fundamentals of earthquake behavior. In this sense, based on self-affine geometrical model of fault surfaces as highlighted in our present work, recent numerical and theoretical studies have shown results in agreement with seismological observations. Indeed, quantification of roughness has been used to infer the scaling of the stress and slip variations along the fault plane after an earthquake (Schmittbuhl et al., 2006; Candela et al., 2011a, 2011b). Fault roughness is also useful for calculating the expected seismic radiation (Dunham, 2011).

5.3. Reconciling the Hurst exponent of the scanned fault surfaces and rupture traces

The rupture trace geometry of strike-slip earthquakes should correspond to the extrapolation of the fault surface roughness sampled on fault scarps along the slip direction and the Hurst exponent should be close to 0.6. However, our results highlight a small difference in the Hurst exponents of the eight rupture traces compared to the average trend in slip direction of the five fault surfaces scanned. The spatial correlations of the rupture trace irregularities are characterized by $H_R = 0.8 \pm 0.1$ and
pre-factors magnitudes, close to those in the direction perpendicular to slip, collected on the five fault surfaces that were scanned. We propose that the roughness increase at the rupture trace scale is due to the fact that the slip direction is not strictly sampled as it is the case for fault surfaces scanned. Slip distributions of strike-slip earthquakes show that at some locations a vertical component could be significant (Florensov and Solenko, 1965; Kurushin et al., 1997). Therefore, at the surface rupture scale, a slight vertical slip component along the whole rupture trace of the strike-slip earthquake would explain how we could sample a roughness oblique to the slip direction.

This interpretation is investigated in Figure 13. The Hurst exponents and the pre-factors of a synthetic anisotropic self-affine surface (see Appendix A.1), computed using a Fourier based method (Candela et al., 2009), and with a roughness root-mean-square standard deviation that scales as $RMS = 0.005L^{0.6}$ in slip direction and $RMS = 0.015L^{0.8}$ normal to it, were calculated on series of 2-D profiles extracted at an angle $\theta$ from the slip direction. The same procedure was performed on a scanned patch of the Corona Heights fault (Corona-A in Table 1). A clear similarity is observed in the angular variability of the roughness exponent computed for the synthetic surface and the natural fault patch (considered as representative of our set of fault surfaces). Both surfaces expose the same non-linear dependence of the Hurst exponent with the azimuthal direction of profile extraction ($\theta$), as pointed by Renard et al. (2006) and Candela et al. (2009) using different statistical tools. When departing a few degrees from the direction of the smallest exponent, i.e. the slip direction, the Hurst exponent sampled is already very close to the largest exponent, i.e. the direction normal to slip (Figure 13). This effect could explain the slight difference in the roughness exponents observed between the surfaces ruptures and fault surface scanned along slip. In the same way, this non-alignment of the ruptures traces with the slip direction, could also explain that their pre-factors magnitudes are close to the maximum sampled for the fault surface scanned.

Moreover, the traces of the ruptures show a structural complexity underlined by relay zones (compressive and extensive jogs), and bends, which could locally accommodate a significant vertical component at many locations along the fault (Klinger et al., 2006). In this case, the rupture trace morphology corresponds to a combination of topography along slip direction and normal to it, which
might partially explain the increase of the measured Hurst exponent and pre-factor compared to those of the along-slip fault surfaces that were scanned.

Taking into account the previous arguments, we propose that the self-affine regime observed at the outcrop scale is also present at the map-scale of earthquake rupture traces. In other words, even though there is a gap of data between the scanned fault surfaces and the rupture traces that remains to be investigated and the traces do not directly constrain the slip-parallel geometry, a unique self-affine geometrical model could fit our measurements from the WLI scale to the map-scale of rupture traces. Indeed, considering the variability of the pre-factor observed both on the scanned fault scarps (see Section 4.2) and the rupture traces (Section 4.3), it seems reasonable to directly connect both data sets by a unique line covering 9 decades of length-scales (Figure 14). As already mentioned, analyzing the roughness of fault traces in Italian Alps, Bistacchi et al. (2011) have recently reported Hurst exponents in the range 0.6 – 0.8 for spatial scales in the range 0.5 mm – 500m and therefore bridging our gap of data.

5.4. Pre-factor variability independent of displacement

In Section 4.2, by combining the measurements acquired with various scanner devices and covering complementary scales, we have shown that no clear relationship exists between the range of roughness amplitude (i.e. the range of pre-factor magnitude) of each exhumed fault surface and the slip estimated for the scanned fault surface (Figure 10). Note here that, following Sagy et al., (2007), even if we focus on the smoothest sub-surfaces of each fault (i.e. the lower bound of the range of pre-factors), no trend is revealed with the estimated offset accommodated (see Figure 10). Other recent studies have noted changes in fault roughness and damage parameters as a function of slip for faults spanning a range of offsets shifted from the dataset here (Sagy et al., 2007; Mitchell and Faulkner, 2009; Savage and Brodsky, 2011; Brodsky et al., 2011). Sagy et al. (2007) noticed a difference for faults that have slip more than 10 m versus those that slip less than 1 m. The fault roughness data presented in the present study only spans offsets that are within the large-slip faults population by these criteria. The lack of a discernible evolution signal is consistent with the suite of data of Brodsky
et al. (2011) that also shows that smoothing is a very weak process once >10 m of offset is accumulated.

Another important detail of our roughness scaling analysis is that no correlation was observed between the finite displacement by the fault zone hosting the rupture trace and its roughness (see Section 4.3 and Figure 12b). This observation is consistent with the study of Klinger (2010), who showed that the correlation between the characteristic fault segment length and the thickness of the seismogenic crust is maintained, independently of the slip accumulated. In addition, we suggest in this study that a consistent spatial organization persists over the entire range of length scales accessible (i.e. from several hundred meters to ~50 km), independent of the total geological offset. Note here that in following the reasoning of Klinger (2010), a specific length scale should appear at approximately 20 km, i.e. the thickness of the seismogenic crust. However, because of the lack of sufficient frequency content between 20 km and 50 km, this probable characteristic length scale is not clearly revealed by our analysis.

Smoothing of fault geometry has been suggested by other studies (Wesnousky, 1988; Manighetti et al., 2007), and is thought to result from surface roughness of faults being inversely related to their total displacement. Such smoothing of a fault trace has primarily been studied by examining large scale geometrical asperities, such as step-overs of several kilometers. In our study, given that we remove these first-order geometric discontinuities to perform our Fourier transform analysis, we lose these hallmarks of the fault maturity.

The notion of "geometric regularization" or “maturity” refers to the intuitive idea that the fault zone geometry becomes more and more flat during the successive slips by abandoning or smoothing the complexity of the initial structures (segments). Ben-Zion and Sammis (2003) recommended that it is necessary to separate the abandoned structural units from those which actively participate in the accommodation of the slip to reveal a possible regularization of the geometry of the fault zone with slip. It is therefore important to emphasize that both the fault scarps scanned on the field and the rupture traces are markers of the morphology of the active structures of fault zones. Our results thus demonstrate that the active portions of faults from the spatial scale of micrometer up to at least the
thickness of the seismogenic crust preserve a complex geometry during successive displacements. In other words, the complexity of an active fault surface is spatially organized following a self-affine geometrical model, independently of the lithology and the tectonic regime. Our observations, as those of Brodsky et al. (2011) and Klinger (2010), suggest that a re-roughening mechanism is active in the fault zone to maintain the geometrical complexity during successive slips. For example, as proposed by Klinger (2010), rupture might naturally branch on preexisting secondary faults as a result of the dynamic stress build-up ahead of the rupture (Poliakov et al., 2002; Bhat et al., 2004). This effect could explain the persistence of some level of complexity and prevent the complete smoothing of the fault geometry.

5.5. Scale-free process at the origin of fault roughness

Previous studies on natural fault surfaces have shown that the slope of the Fourier spectrum could change from large to small scales (e.g., Lee and Bruhn, 1996), implying that different processes may be involved during the generation of surface fault textures at different spatial scales. In fact, any inflexion of the spectrum would correspond to a characteristic scale. Such a crossover scale could be interpreted as the transition between physical processes, as is the case for dissolution surfaces such as stylolites (Renard et al., 2004). In the present study, five fault surfaces in different geological settings have been scanned with three different scanner instruments spanning complementary length scales. Considering the variability of the pre-factor (see Section 4.2), an anisotropic mono-affine geometric model characterized by two different global Hurst exponents $H_\parallel = 0.6$ and $H_\perp = 0.8$ best fits the entire data set between 10 m and 0.05 mm. In our data set, because the slopes follow a consistent trend, and considering the level of noise, no variation of the Hurst exponent could be detected between 10 m and 0.05 mm. This observation could be evidence that a unique scale-free process could be at the origin of the scanned fault scarps.

In contrast to mode I fracture roughness for which a lot has been done in physics and mechanics communities, few studies have focused on the processes at work in the generation of the shear surface roughness. Since the 1990s, increasing experimental evidence showed that the roughness exponent for
fractures surfaces had a universal value of about 0.80 (Bouchaud et al., 1990; Bouchaud, 1997). Simultaneously with these experiments, theoretical and numerical works have been produced at a steady rate with the aim of modeling the fracture propagation in order to identify the origin of the self-affine scaling of fracture roughness (Bouchaud, 1997, Alava et al., 2006; Hansen & Schmittbuhl, 2003; Bonamy and Bouchaud, 2011). The common denominator of all this work is to model fracture propagation as a network of elastic beams, bonds, or electrical fuses with random failure thresholds and subject to an increasing external load. In other words, in order to reproduce the self-affine roughness exponent experimentally observed, the fracture propagation is assimilated as a damage coalescence process in a heterogeneous material. The damage becomes localized by long-range elastic interaction between multi-scale cracks. In this sense, we propose that elastic interactions related to linkage of many discrete slip surfaces, controlling the generation of the multi-scale bumpy lenses observed on the scanned fault outcrops, could be the scale-free process at the origin of fault roughness. Indeed, even if shear cracks involve significantly higher energy release rates than tensile cracks (Atkinson, 1991), some similarities in the elastic influences on the stress field at the crack tip can be obtained theoretically (Gao and Rice, 1986; Gao et al., 1991; Atkinson, 1991; Schmittbuhl et al., 2002).

Even if a unique self-affine regime is maintained up to the map-scale of the rupture traces, it remains difficult to support the fact that their roughness is also controlled by the process at work for the scanned fault scarps. However, it is noteworthy that the rupture traces seem to be also formed by multi-scale segments (see for example the Figure 5 in Klinger, 2010) whose growth and coalescence could be also controlled by long range elastic interactions.

6. Conclusion

The roughness properties of fault surfaces is summarized in Figure 14, where the Fourier power spectra of the Corona Heights fault surfaces perpendicular to the slip direction and the spectra obtained for the continental earthquake surface ruptures are plotted together. First, even if it is difficult to point out a unique power-law fit due to the pre-factor variability, a single anisotropic self-
affine model ($H_\parallel = 0.6$ and $H_\perp = 0.8$) best fit our 3-D scanned fault measurements from 0.05 mm scale to 10 m. Secondly, an identical self-affine behavior is consistent with the map-scale of the 2-D rupture traces. In order to connect both data sets while considering the variability of the pre-factor, we propose that a unique self-affine geometrical model is maintained over nine decade of length scales (between 0.05 mm and at least the thickness of the seismogenic crust, i.e. ~20 km). Even if the lack of data between the scanned fault surfaces and the rupture traces remains to be investigated, recent works of Bistacchi et al. (2011) bridging the length scale gap appear to support our interpretation.

In addition, even if in both cases, for the scanned fault surfaces and the ruptures traces, we have focused our analysis on the active portion of the fault zone, it appears that once a small amount of offset has been achieved, the geometric complexity is maintained regardless of the amount of further slip accommodated. Consequently, we propose that processes that create this roughness and processes that destroy it must reach a dynamic equilibrium.

Acknowledgments: This study was supported by the Agence Nationale pour la Recherche grant ANR-JCJC-0011-01, ANR ECCO TRIGGERLAND and ANR RiskNat SUPNAF. Field work was supported in part by NSF Grant EAR-0711575. The authors are very grateful to Michel Bouchon and Fabrice Cotton for encouragements and scientific stimulations. William L. Power is really thanked for his detailed and constructive comments on an early version of this work. The first author gratefully acknowledges Andrew P. Rathbun and Megan Avants for their constructive comments that helped to improve the content and clarity of the manuscript, and James D. Kirkpatrick for fruitful discussions concerning various aspects of this paper.
References


Figure captions

Figure 1: Two-dimensional roughness profiles from the Corona Heights fault surface parallel and perpendicular to slip. A: Profiles parallel to the slip direction and, B: perpendicular to the slip direction. For both directions, a magnified portion of the profiles has a statistically similar appearance to the entire profiles when using the scaling transformation  $\delta x \rightarrow \lambda \delta x$ $\delta y \rightarrow \lambda \delta y$ $\delta z$ $\rightarrow \lambda \delta z$. For each scale, profiles have been shifted vertically for clarity.

Figure 2: Corona Heights fault. Multiple bumpy discrete slip surfaces constituting lenses and striations can be detected at all scales, from the measurement resolution of each scanner device to the size of the entire exposure. A: Whole outcrop view. The inset corresponds to the surface shown on Figure 2C. B: Zoom on the fault showing different segments constituting the surface. C&D: Map of fault surfaces scanned using LiDAR topography. The inset in C corresponds to the patch shown on Figure 2D. E and F: Maps of fault surfaces scanned with the laser profilometer. G and H: Zoom on the Figure 2E-F scanned with the white light interferometer.

Figure 3: Bolu Fault. A: Photograph of the fault zone in the Bolu limestone. Dashed red contour corresponds to the limits of the cloud of points shown in B. C: Photograph of a well-preserved slip surface constituting the fault zone. D: LiDAR data of surface in C. E and F: Zoom on the fault surface and the corresponding topographic map acquired with the laser profilometer, which still includes anisotropic roughness features in the slip direction.

Figure 4: Digitized surface rupture trace of the three largest segments of the Landers earthquake. The data correspond to Figure 5G in Klinger (2010). The inset zooms on one of the steps removed for the Fourier transform analysis. Each segment is individualized by different colors and the steps are represented in pink. The reconstructed trace by removing steps is displayed in gray.
**Figure 5:** Example of the 3-D average power spectrum of one LiDAR fault patch (Corona-B) obtained by averaging in a geometric progression several thousands 2-D power spectra of individual profiles (we have changed the color of the spectra every 20 profiles successively extracted from the surface).

**Figure 6:** Typical average power spectra in the slip direction and perpendicular to it for fault patches scanned with the three devices used in our study. Errors bars with 68% confidence interval (one sigma), and power law fits performed in linear portions of each average spectra, are shown. At the LiDAR scale, the black arrow indicates the lower limit used for the fits. This lower limit underlines the length scale at which spectra flatten out when they intersect the noise spectrum calculated by scanning smooth, planar reference surfaces. For the laser profilometer and the WLI, the spectral power levels of our natural fault roughness data fall at a vertical higher position than the noise spectra. However, at the WLI scale, the vertical dashed blue line at the bending of spectra indicates the lower limit of our fits. Even if this systematic change of slope at the length scale 0.05 mm seems to be related to intrinsic physical fault roughness properties, we arbitrarily limit our scaling analysis to scales larger than 0.05 mm.

**Figure 7:** Plot of the Hurst exponents (see Table 1) in slip direction (B) and normal to it (A) of the 41 scanned fault patches. The average Hurst exponent is equal to 0.58 along the direction of slip and 0.81 perpendicular to it. The shaded area and dashed lines indicate 1σ and 2σ confidence intervals, respectively.

**Figure 8:** Illustration of the pre-factor variability along the Corona Heights fault. Log-log graph gathering the global laser profilometer spectrum with the 6 averaged Fourier power spectra obtained at the LiDAR scale. Note the intercepts range of all 2-D LiDAR spectra, performed from each individual profiles extracted from the six fault patches, highlighted by the two dashed dark power
laws. The global laser profilometer spectrum falls in this range of intercepts sampled by the whole population of the individual profiles that we explore at the LiDAR scale.

**Figure 9:** Global Fourier power spectra from the five faults analyzed along the slip direction (left) and perpendicular to it (right). Each curve at each scale (LiDAR, laser profilometer, white light interferometer) includes together the average spectra of several sub-surfaces (or fault patches in Table 1). Power-law fits (thick gray lines) with a roughness exponent of $H_{∥} = 0.6$ and $H_{⊥} = 0.8$, connecting the field and laboratory data in both directions, are shown on plot for eye guidance. The black arrow and the vertical dashed blue line indicate, respectively: the level of noise of the LiDAR and the lower limit for the fit performed at the WLI scale. Dotted black lines indicate the range of pre-factors, $C_{∥,⊥} \left( P(k) = C_{∥,⊥} k^{-2H_{∥,⊥}} \right)$, extracted from the power-law fits at $k = 1 \text{ m}$ (see all the values in Table 3), and used for the Figure 10. One way to more easily interpret this log-log graph and to compare our results with previous studies like Sagy *et al.* (2007), is to convert the power spectrum module in term of root-mean-square ($RMS$) roughness amplitude using Parseval’s Theorem (Brodsy *et al.*, 2011). Indeed if $0 < H_{∥,⊥} < 1$, for a profile of length $L$, the integration of $P(k) = C_{∥,⊥} k^{-2H_{∥,⊥}}$ over the wavelength $\lambda$ (with $\lambda = 1/k$) yields that the $RMS$ roughness correlates as $RMS = \left( \frac{C_{∥,⊥}}{2H_{∥,⊥}} \right)^{0.5} L^{H_{∥,⊥}}$. In this sense, red lines represent power-law fits for three self-similar rough surfaces (i.e. $H = 1$) with various pre-factors, that are: $RMS = 0.1 L$, $RMS = 0.01 L$, $RMS = 0.001 L$.

**Figure 10:** Log-log plots of the range of global pre-factor magnitude of the five faults extracted from Figure 9 (see also Table 3) versus the estimated slip (Table 1).

**Figure 11:** Compilation of the surface rupture roughness results: Digitized rupture traces (top) and corresponding Fourier power spectrum (bottom). Because of the abrupt steps biasing the Fourier
transform computation, we have performed this roughness analysis on each individual segment between two steps of the whole profile. The same color code is respected between the individual segments and the corresponding spectra. In addition, the gray rupture profile and the corresponding gray Fourier power spectrum represent a reconstruction of the entire profile. Power-law fits and the inferred Hurst exponents on the linear part of each curves at large scale (above the cross-over length scale indicated by the gray vertical bar and marking the beginning of the regime controlled by data re-sampling) are represented on each graph. Note here that for each rupture trace, the curves corresponding respectively to the selected segments and to the reconstructed profiles are characterized by a similar scaling exponent but variable pre-factors.

**Figure 12:** Variability of the pre-factor and slip accumulated for the eight studied rupture traces. A: Stack of all the Fourier power spectra of the whole surface ruptures stacked, underlining the global trend of the self-affine behavior at large scale. Power-law fit giving an average Hurst exponent of $0.8 \pm 0.1$ is indicated. B: Log-log plot of the average pre-factors (corresponding to those of the spectra of the whole rupture traces and indicated in Table 2) with their respective typical fluctuations (i.e. the range of pre-factors extracted from the spectra of the individual segments constituting the whole rupture traces) of each rupture traces in function of the finite geological offsets (see Table 2).

**Figure 13:** Angular dependence of the Hurst exponent (left) and the pre-factor (right) computed on a synthetic anisotropic self-affine surface with two input exponents in perpendicular directions ($H_{//} = 0.6$ and $H_{\perp} = 0.8$), and for a patch (Corona-A, see Table 1) scanned with the LiDAR on the Corona Heights fault surface. The Hurst exponents and the pre-factors were calculated on series of 2-D profiles extracted at an angle $\theta$ between the slip direction ($\theta = 0$) and the perpendicular direction ($\theta = 90$).

**Figure 14:** Comparison of the roughness of the earthquakes surface ruptures with the Corona Heights fault topography. The Fourier power spectra normal to the slip direction of the Corona Heights fault
surface are plotted on a log-log graph together with those obtained for the eight continental strike-slip
earthquakes surface rupture traces. The Corona Heights fault data are identical to those plotted on
Figure 9, and those of the surface ruptures correspond to that of Figure 12. Both data sets are
connected with a unique power-law fit (grey line) with a Hurst exponent of 0.8.
Table 1. Laser scanner characteristics and fault roughness results.

<table>
<thead>
<tr>
<th>Fault Name</th>
<th>Lithology &amp; Slip/*</th>
<th>Sense</th>
<th>Fault Patches</th>
<th>Scanner</th>
<th>Spatial precision</th>
<th>$\delta_x$</th>
<th>$\delta_z$</th>
<th>$H_{xy}$ (±0.05)</th>
<th>$H_{xz}$ (±0.05)</th>
<th>Average $H_{xy}$</th>
<th>Average $H_{xz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault Name</td>
<td>Lithology &amp; Slip*</td>
<td>Sense</td>
<td>Fault Patches</td>
<td>Scanner</td>
<td>$\delta x$</td>
<td>Spatial precision</td>
<td>$\delta z$</td>
<td>$H_{\parallel}$ (±0.05)</td>
<td>$H_{\perp}$ (±0.05)</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------</td>
<td>-------</td>
<td>---------------</td>
<td>-----------</td>
<td>-----------</td>
<td>------------------</td>
<td>-----------</td>
<td>--------------------------</td>
<td>------------------------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Dixie Valley</td>
<td>Rhyolites</td>
<td>Normal</td>
<td>Dixie-1</td>
<td>HDS 3000</td>
<td>5 mm</td>
<td>± 1.25mm</td>
<td>2 mm</td>
<td>0.66</td>
<td>0.79</td>
<td>0.59 ± 0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>several m to 3-6km</td>
<td></td>
<td>Dixie-2</td>
<td>Leica</td>
<td></td>
<td></td>
<td></td>
<td>0.63</td>
<td>0.80</td>
<td>0.82 ± 0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.61</td>
<td>0.84</td>
<td>0.84 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
<td>0.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Map-1</td>
<td>Lab. profilometer</td>
<td>20 μm</td>
<td>± 1μm</td>
<td>&lt; 1μm</td>
<td>0.57</td>
<td>0.82</td>
<td>0.59 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Map-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.61</td>
<td>0.81</td>
<td>0.81 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-D</td>
<td>WLI</td>
<td></td>
<td>2μm</td>
<td>0.025 μm</td>
<td>0.56</td>
<td>0.88</td>
<td>0.56 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-H</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.50</td>
<td>0.87</td>
<td>0.88 ± 0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.61</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dixie-C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.59</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M2</td>
<td>Lab. profilometer</td>
<td>20 μm</td>
<td>± 1μm</td>
<td>&lt; 1μm</td>
<td>0.60</td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Except for the Vuache-Sillingy fault surface, a lower and upper bound of the displacement is given.

Although total geological cumulated slip for the fault zone as a whole can be kilometric (*i.e.* the upper bound), scanned individual surfaces within the fault zone may have experienced considerably less slip (*i.e.* the lower bound).

$^5$ Spatial length scale resolution ($\delta x$).

$^*$ Vertical resolution ($\delta z$).
Table 2. Characteristics and roughness results of earthquake rupture maps used in this study. (references for each earthquake are given in the text)

<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
<th>Magnitude</th>
<th>Rupture length (km)</th>
<th>Total geological offset (km)</th>
<th>$H_R$</th>
<th>Pre-factor ($m^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owens Valley (USA)</td>
<td>1872</td>
<td>M$_w$ 7.5-7.8</td>
<td>81</td>
<td>≤ 20-30</td>
<td>0.6 ± 0.1</td>
<td>4 x 10$^{-2}$</td>
</tr>
<tr>
<td>Haiyuan (China)</td>
<td>1920</td>
<td>M$_s$ 8 to 8.7</td>
<td>200</td>
<td>15 to 95</td>
<td>0.8 ± 0.1</td>
<td>7 x 10$^{-3}$</td>
</tr>
<tr>
<td>Gobi-Altay (Mongolia)</td>
<td>1957</td>
<td>M 8.3</td>
<td>235</td>
<td>2 to 20</td>
<td>0.7 ± 0.1</td>
<td>2 x 10$^{-2}$</td>
</tr>
<tr>
<td>Superstition Hills (USA)</td>
<td>1987</td>
<td>M 6.6</td>
<td>18</td>
<td>≤ 24</td>
<td>0.7 ± 0.1</td>
<td>2 x 10$^{-3}$</td>
</tr>
<tr>
<td>Luzon (Philippine)</td>
<td>1990</td>
<td>M$_w$ 7.8</td>
<td>107</td>
<td>50 to 200</td>
<td>0.8 ± 0.1</td>
<td>1 x 10$^{-2}$</td>
</tr>
<tr>
<td>Landers (USA)</td>
<td>1992</td>
<td>M$_w$ 7.2</td>
<td>65</td>
<td>3.1 to 4.6</td>
<td>0.8 ± 0.1</td>
<td>1 x 10$^{-3}$</td>
</tr>
<tr>
<td>Zirkuh (Iran)</td>
<td>1997</td>
<td>M$_w$ 7.2</td>
<td>104</td>
<td>60</td>
<td>0.9 ± 0.1</td>
<td>3 x 10$^{-4}$</td>
</tr>
<tr>
<td>Hector mine (USA)</td>
<td>1999</td>
<td>M$_w$ 7.1</td>
<td>39</td>
<td>3.4</td>
<td>0.7 ± 0.1</td>
<td>7 x 10$^{-3}$</td>
</tr>
</tbody>
</table>
Table 3. Pre-factor (exhumed slip surfaces)

<table>
<thead>
<tr>
<th>Location</th>
<th>Pre-factor (m³)</th>
<th>Parallel to slip</th>
<th>Perpendicular to slip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vuache-Sillingy</td>
<td>min</td>
<td>8 x 10⁻⁸</td>
<td>2 x 10⁻⁹</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>2 x 10⁻⁵</td>
<td>1 x 10⁻⁴</td>
</tr>
<tr>
<td>Corona Heights</td>
<td>min</td>
<td>4 x 10⁻⁷</td>
<td>1 x 10⁻⁷</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>5 x 10⁻⁵</td>
<td>2 x 10⁻⁴</td>
</tr>
<tr>
<td>Bolu</td>
<td>min</td>
<td>2 x 10⁻⁸</td>
<td>3 x 10⁻⁷</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>5 x 10⁻⁵</td>
<td>2 x 10⁻⁴</td>
</tr>
<tr>
<td>Dixie Valley</td>
<td>min</td>
<td>1 x 10⁻⁴</td>
<td>2 x 10⁻⁵</td>
</tr>
<tr>
<td>Magnola</td>
<td>min</td>
<td>1 x 10⁻⁴</td>
<td>5 x 10⁻⁶</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>5 x 10⁻⁴</td>
<td>8 x 10⁻⁴</td>
</tr>
</tbody>
</table>
Figure 1

(A) Along slip

(B) Normal to slip
Figure 2

A

B

C

D

LiDAR

E

F

Laser profilometer

G

H

White Light Interferometer
Figure 4

![Graph showing the distance in km over distance in km for Landers.](image)
Figure 6

**LiDAR**

Corona-B

$H_{\perp} = 0.81 \pm 0.05$

$H_{\parallel} = 0.65 \pm 0.05$

**Laser profilometer**

Co-AGU

$H_{\perp} = 0.85 \pm 0.05$

$H_{\parallel} = 0.60 \pm 0.05$

**WLI**

Co-A-4

$H_{\perp} = 0.82 \pm 0.05$

$H_{\parallel} = 0.62 \pm 0.05$

- Along slip
- Perpendicular to slip
- Planar reference surfaces
Figure 7

Normal to slip

Fault patches

Parallel to slip

Fault patches

- LiDAR
- Laser profilometer
- WLI
- Blue: Vuache-Sillingy
- Magenta: Corona-Heights
- Yellow: Bolu
- Green: Dixie Valley
- Black: Magnolia
Figure 8

![Graph showing the relationship between $P(k)$ and $k$ with different datasets labeled as Corona-F, Corona-E, Corona-D, Corona-C, Corona-B, Corona-A, and a line labeled Laser profilometer.](image)

$P(k) \propto k^{-2.2}$
Figure 9

Along slip

Normal to slip

Corona Heights Fault, California

Bolu Fault, Turkey

Dixie Fault, Nevada

\[ P(k) \propto k^{-\alpha} \]

\[ k \propto \frac{1}{H_s} \]

- \( P(k) \) (m³)
- \( k \) (m⁻¹)

- LiDAR
- Laser profilometer
- White Light Interferometer
**Figure 10**

- **Along slip**
  - Magnolia fault
  - Vaucluse-Sillingy fault
  - Corona Heights fault
  - Bolu fault
  - Dixie valley fault

- **Normal to slip**
Figure 11

[Graphs showing data for different locations: Owens Valley, Laidon, Haiyun, and Landers. Each graph plots distance against a physical quantity, with a focus on the relationship between wave number (k) and power (P(k)).]
Figure 12

A

\[ H_R = 0.8 \pm 0.1 \]

\[ P(k) \text{ (m}^3\text{)} \]

\[ k \text{ (m}^{-1}\text{)} \]

- Luzon
- Haiyuan
- Zirkuh
- Landers
- Superstition Hills
- Owens Valley
- Gobi-Altay
- Hector Mine

B

\[ \text{Pre-factor, } C \text{ (m}^3\text{)} \]

\[ \text{Displacement (km)} \]

- 1 km
- 10 km
- 100 km
Figure 14

![Diagram showing the distribution of earthquake surface ruptures and fault heights. The graph plots the probability mass function (P(k)) against the fault length scale (k). The data points are color-coded to represent different sources, such as Earthquakes, surface ruptures, and fault heights from LiDAR, Laser profilometer, and WLI measurements. The graph includes a line indicating the scaling of the data with a slope of 0.8.](image-url)
Appendix A: Potential bias in roughness data

A.1. Reliability of the roughness results at the LiDAR scale: effect of the noise in the acquisition system

Before analyzing the main biases inherent to LiDAR data acquisition, we emphasize that raw scanner data acquired with the home-made laser profilometer and the WLI are considered as quasi-ideal since the level of the intrinsic white noise of these instruments is well below that recovered from the fault surfaces (see Figure 6). A detailed description of the conditions in which measurements with this scanner device departs from the reality is given by Méheust (2002). In addition, the slope of the spectra computed on WLI scans at spatial scales larger than 0.05 mm are consistent with those of laser profilometry for the same range of length scales (see Figure 9). Given that LiDAR data include the largest bias, we focus our noise analysis on this instrument. Consequently, the results provided by the following analyses can be considered as end-members of the noise effect estimation for the three instruments used in our study.

In the spirit of the work of Schmittbuhl et al. (1995) on the reliability of a self-affine measurement on 2-D rough profile, Candela et al. (2009) have reviewed different statistical methodologies which allow the assessment and characterization of the anisotropic self-affine behavior of fault topography. This work was mainly devoted to precisely define the intrinsic error of the statistical methods (such as the Fourier power spectrum) to estimate the scaling properties of fault surface roughness.

Here, a new test is performed by taking into account the error encountered in the spatial position \((X,Y,Z)\) of each points measured by the 3-D laser scanner. We use a synthetic anisotropic self-affine surface (Candela et al., 2009) of \(5 \times 5 \text{ m}\) with an original regular point spacing of \(5 \text{ mm}\) (Figure A1), and with two different Hurst exponents in perpendicular direction (\(H_\parallel = 0.6\) and \(H_\perp = 0.8\)). In order to simulate the error inherent to LiDAR data acquisition on the spatial position of each points \((X,Y,Z)\), we add Gaussian white noise with a distribution of \([0, 2.5 \text{ mm}]\) on the original position of each point, to obtain the perturbed grid \((X^1,Y^1,Z)\). Then the height \(Z^1\) of each
point at these new positions \((X^1, Y^1)\) is computed by interpolating (bilinear interpolation) the four nearest points of each of these new positions on the original ideal model (Figure A1). In a final step, Gaussian white noise, with a standard deviation equal to 5 mm, is randomly added on the interpolated heights \(Z^1\), to yield the error of the LiDAR data in the vertical position.

After generating this noisy cloud of points \((X^1, Y^1, Z^1)\), we extract profiles oriented along slip direction and perpendicular to it as is done for the measurements, and estimate both Hurst exponents. The results are then averaged over 100 realizations of synthetic surfaces. Due to the noise, the Fourier power spectra flatten at short length scales (as for examples in the slip direction shown in Figure A1), which results in a slight underestimation of the Hurst exponents. For both directions, along slip and perpendicular to it, we find that the median estimates of \(H//\) and \(H\perp\) of the biased synthetic surfaces are \(0.59 \pm 0.06\) and \(0.74 \pm 0.11\) respectively, compared to the noise-free data where the Hurst exponents were \(0.6 \pm 0.07\) and \(0.78 \pm 0.05\), respectively. Note that the error bar of the estimated Hurst exponent of the biased synthetic surfaces is approximately twice larger in the direction perpendicular to slip relative to that in the slip direction. In both directions, even if the Hurst exponent is slightly underestimated for the noisy data, its value is still included in the range given by the standard deviation of the noise-free data. Therefore, the noise in the LiDAR data could be estimated as well as the reliability of Hurst exponent values.

A.2. Reliability of the roughness results at surface rupture scale: effect of re-sampling

For each earthquake, once the surface rupture map has been digitized, the data set is re-sampled to a regular spacing to ensure consistent spatial sampling, independent of the length of each rupture. This re-sampling is performed to avoid bias due to local wiggles of the rupture trace (Klinger, 2010). We verify here how this re-sampling process affects the spectra of the Fourier transform. The original digitized rupture trace of the Hector mine earthquake, taken as an example, is re-sampled with various constant values of \(dx\) in the range 60-620 m (Figure A2). In the Fourier power spectra of this set of digitized rupture traces, two regimes can be observed. At small scales, i.e. large wave numbers,
(between approximately 120 m and 1200 m) the behavior can be attributed to data re-sampling. At large scales, i.e. small wave numbers (above 1200 m), a power law giving a Hurst exponent \( H_R = 0.74 \) represents the best fit. The cross-over length scale between the two regimes corresponds to the maximum spacing between two points in the original data. Whatever the value of \( \delta x \) taken for re-sampling the data, the cross-over length scale remains at the same position. We propose that the regime at large scales, characterizing roughness properties of the digitized ruptures traces is therefore not affected by data re-sampling; both the slope and the pre-factor of each spectrum are identical (Figure A2).

The same re-sampling procedure has been performed on ideal synthetic self-affine profiles in order to precisely define if the scaling property could be modified (Figure A3). An original self-affine profile (\( RMS = 0.01 L^{0.6} \)) with regular spacing of 500 m and a total length of 100 km (extracted from a synthetic surface as previously presented), is modified by adding to the \( X \) coordinates a Gaussian white noise perturbation with a distribution of [0,250 m] (Figure A3). This altered profile is re-sampled with different values of \( \delta x \) in the range [40-350] m. The Fourier spectra (Figure A3) indicate that for the different values of \( \delta x \), the large scale regime (above 750 m) characterizing the input self-affine behavior with a Hurst exponent of 0.6 is not biased. This test validates that the re-sampling procedure makes it possible to keep the scaling information of the rupture traces at spatial scales above 1200 m.
Figure A1: Effect of noise inherent to LiDAR data acquisition. A: Ideal synthetic $5 \times 5 \, m$ self-affine surface with an original regular point spacing of $5 \, mm$. B: Illustration of the addition of a noise in the regularly spaced original grid $(X, Y, Z)$. C: Comparison of the Fourier power spectra in direction of slip obtained for the ideal synthetic surface and the noisy synthetic surface. Both vertical dashed grey lines indicate limits taken for fitting the Hurst exponent. D: Distribution of measured Hurst exponents, on 100 simulations, for the direction of slip and perpendicular to it. Black bars show the ideal simulated fault surface models, and the gray bars correspond to the noisy simulated fault surface models. The solid lines (black for the ideal case and gray for the natural case) represent the fits of the measurements to a normal distribution with mean $\mu$ and standard deviation $\sigma$ given at the top left for the noisy natural case and the top right for the noise-free case.

Figure A2: Effect of re-sampling effect on earthquake surface rupture roughness: example of Hector Mine earthquake. A: Digitized surface rupture traces of the Hector Mine earthquake. The original rupture map with an irregular point spacing (pink profile) is re-sampled in order to ensure consistent spatial sampling with a regular spacing $(\delta x)$. The inset indicates the position of the zoom located on the right, showing the irregular point spacing on the top and the gradual increase of the re-sampling (or decreasing of $\delta x$) with a regular point spacing of profiles downwards. B: Fourier power spectra of the digitized rupture traces shown in A. Spectra colors are the same than in A. On the right: the spectra have been shifted vertically to improve the visibility. The blue vertical bar on both graphs highlights the cross-over length scale, at approximately $1200 \, m$, between both regimes: the lower regime is attributed to data re-sampling, the upper regime characterizes roughness properties of the digitized profiles with a Hurst exponent $H_R = 0.74$.

Figure A3: Effect of re-sampling on synthetic self-affine profiles. A: Example of synthetic rough profiles with a standard deviation that scales as $RMS = 0.01 \, L^{0.6}$, analogue of digitized surface rupture traces. The green profile at the top of the left figure is the original ideal synthetic profile with
a regular point spacing $\delta x$ of 500 m. A Gaussian white noise perturbation with a distribution $[0, 250 m]$ is added on the original regular spacing to obtain a noisy profile (pink curve) similar to that of original ruptures maps. This noisy synthetic profile with irregular point spacing is re-sampled with a regular spacing $\delta x$. The inset zooms on the synthetic profiles located on the right. B: Fourier power spectra of the synthetic self-affine profiles shown in A. Colors of each curves correspond to those of profiles. On the right: the spectra have been shifted vertically to improve the visibility.
Figure A2